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A Study on Optimal Sizing of Pipeline Transporting Equi-sized Particulate Solid-Liquid Mixture

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Abstract. Pipelines transporting solid-liquid mixtures are of practical interest to the oil and pipe industry throughout the world. Such pipelines are known as slurry pipelines where the solid medium of the flow is commonly known as slurry. The optimal designing of such pipelines is of commercial interests for their widespread acceptance. A methodology has been evolved for the optimal sizing of a pipeline transporting solid-liquid mixture. Least cost principle has been used in sizing such pipelines, which involves the determination of pipe diameter corresponding to the minimum cost for given solid throughput. The detailed analysis with regard to transportation of slurry having solids of uniformly graded particles size has been included. The proposed methodology can be used for designing a pipeline for transporting any solid material for different solid throughput.

1. Introduction

In case of mineral processing, the ore is generally available in remote areas. At such sites, rail and road facilities are usually non-existent. For such situations it is usually worthwhile to consider the possibility of hydraulic transportation of solids which can help in relocation of the processing plants at more convenient locations. Slurry pipelines are used in various industries for transporting solids in crushed form along with the carrier liquid. These pipelines have proven to be cost effective as compared to other modes of transportation of solids in bulk form. Some other advantages associated with slurry pipe line system over other modes are:

1. Economy of scale
2. Few traffic and accidental hazards
3. Eco-Friendliness etc.

The conventional techniques that are used for hydraulic pipelines for analysis and optimization are also applicable to hydraulic slurry pipelines. Working within the limits of technical and operational constraints, the objective is to arrive at the most economical solution. To achieve this, the different variables involved have to be optimized as per the procedure discussed below:

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Nomenclature

C_1	Levelized net annual cost of pumping energy per watt [£/W]
C_2	Levelized net annual cost of pipe per unit weight of pipe material [£/N]
C_c	Constant of proportionality dependent on expected pressure and diameter range of Pipe [-]
C_D	Coefficient of Drag [-]
C_v	Volume fraction of solids in slurry [-]
D	Diameter of Pipe [m]
d	Diameter of Particles [m]
g	Acceleration due to gravity [m/sec^2]
Δh_S	Head loss due to slurry [m'water'/m'pipe']
Δh_w	Head loss due to water [m'water'/m'pipe']
Q	Total volumetric flow rate [m^3/sec]
Q_w	Volumetric flow rate of liquid phase (m^3/sec)
Q_S	Solid Throughput [Kg/sec]
Re	Reynolds number [-]
ρ	Density [Kg/m^3]
γ	Specific Weight [N/m^3]
S	Specific Gravity [-]
t	Pipe wall thickness [m]
V_o	Settling velocity [m/s]
V_D	Deposition velocity [m/s]
μ	Dynamic viscosity [Pa-sec]
η	Pump's efficiency [%]

Subscripts

L	Carrier fluid
m	Mixture
p	Pipe material

1. Particle size distribution on the basis of slurry preparation, transportation and utilization costs.
2. Slurry concentration on the basis of transportation and dewatering cost.
3. Pipeline diameter on the basis of pumping power costs.
4. Pipe thickness and number of pumping stations on the basis of operational constraints.

In each case the cost of transportation of the solids for the total pipeline system is worked out considering both the capital and operating expenses. Slurry pipelines are generally designed on the basis of specific energy consumption. Thus, for a given solid throughput pressure drop characteristics are calculated based on models available and pilot plant test loop data for various combinations of flow velocities, solid concentrations and pipe diameters. Parameters corresponding to minimum specific energy consumption are taken as optimum.

Several investigators have suggested methods for sizing a pipeline carrying fluids based on minimum cost. These methods with suitable modifications can be applied to slurry pipelines with reasonable accuracy. Albertson [1] considered initial investment cost of pipes and pumps, the annual operational and maintenance costs for the life of the pipes and pumps and the salvage value of the pipeline for sizing a pipeline of minimum cost. Hathoot [2] and Cheremisinoff [3] describe the optimal size of the pipeline as the one for which the total annual cost of the pipe, pumps and power is minimum. Daugherty and Franzini [4] found that the total annual cost C of a pipeline carrying fluid is given by:

$$C = a_1 . D^2 + \frac{b_1}{D^5} \quad (1)$$

where a_1 and b_1 are constants. Later on it was realized that b_1 is not constant as it is a function of friction factor, which further is a function of Reynolds number and pipe roughness.

Swamee [5] suggested a methodology by which a sediment transporting pipeline could be optimally sized using the method of geometric programming. He considered pumping cost as well as the cost of piping system. In this method flow is assumed to be taking place at the deposition velocity. However in actual practice, flow velocity is always kept suitably higher than the deposition velocity. Mishra and Agarwal [6] have suggested a method for optimal sizing of powder handling capsule pipelines by taking suitable models for pressure drop, piping cost and capsule cost. They proposed a methodology for computing optimal size corresponding to least total cost. A similar methodology can be developed for sizing a pipeline handling solid-liquid mixture.

2. Theory

The design procedure for slurry flow pipelines comprises of determination of the pipeline diameter such that the total annual cost is minimum. In a slurry transport pipeline, the total cost is the sum of pumping energy cost and the cost of pipes.

Following assumptions are made towards developing a model for the cost of pumping energy and cost of pipe:

- Particles are uniformly graded.
- Durand's equation [7] for a uniformly graded solid-liquid mixture is reasonably accurate for the prediction of head loss calculations.
- The friction factor can be obtained with reasonable accuracy by Wood's equation [8].

3. Cost of Pumping Energy

In a slurry transport pipeline the pumping energy should be sufficient enough to overcome the losses taking place in the pipeline.

The losses in a pipeline comprises of two parts:

- Major losses
- Minor losses

The major losses in the pipeline are due to the friction between solid and liquid phases. The minor losses are because of the presence of valves, bends and other flow obstruction devices. Major losses are quite high as compared to minor losses because the total length of the pipelines is higher than to be compared with the number of pipe fittings being installed in the system. Here, only the major losses are considered since it is assumed that overall length of the pipeline is sufficiently high.

The friction loss for slurry flow through pipes can be estimated by Durand's equation [7] with reasonable accuracy. Durand, based on his data (pipe diameter varying from 0.02 m to 0.6 m, particle sizes from 0.0001 m to 0.025 m and pipe velocity from 0.6 m/s to 6 m/s) proposed the following equation for the prediction of head loss per unit pipe length in heterogeneous solid-liquid flow:

$$\Delta h_s = \Delta h_w + \frac{40\sqrt{gD}\,(S-1)^{1.5}\,C_v\,f}{V\,{C_d}^{0.75}} \tag{2}$$

Where the head loss due to the carrier liquid alone is expressed as:

$$\Delta h_w = \frac{fV^2}{2gD} \tag{3}$$

We assume that the slurry flow in pipeline is turbulent. According to Wood's equation, friction factor is given by:

$$f = 4(a + b\,Re^{-c}) \tag{4}$$

Here;

$$a = 0.0235\left(\frac{\varepsilon}{D}\right)^{0.225} + 0.1325\left(\frac{\varepsilon}{D}\right) \tag{5}$$

$$b = 22\left(\frac{\varepsilon}{D}\right)^{0.44} \tag{6}$$

$$c = 1.62\left(\frac{\varepsilon}{D}\right)^{0.134} \tag{7}$$

Furthermore, the Reynolds's number can be found by:

$$Re = \frac{\rho_L\,V\,D}{\mu} \tag{8}$$

Various parameter used in Durand's equation [7] have been chosen keeping in view the operational requirements of slurry flow through pipelines. The velocity of flow has been fixed at a value slightly higher than the deposition velocity. The deposition velocity of a given solid (known density and size) for low solid concentration can be calculated using Wick's equation [8]:

$$V_D = 1.87\left(\frac{d}{D}\right)^{\frac{1}{6}}\left[\frac{2\,g\,D\,(\rho_S-\rho_L)}{\rho_L}\right]^{0.5} \tag{9}$$

Flow velocity is assumed to be:

$$V = V_D + 0.2 \tag{10}$$

The solid concentration is calculated from the known value of solid throughput using the equation:

$$Q_s = \frac{\pi D^2 V C v \rho_s}{4} \tag{11}$$

Volumetric flow rate of carrier fluid can be computed as:

$$Q_l = \frac{\pi D^2}{4} \tag{12}$$

Furthermore, total volumetric flow rate in the pipeline is the sum of the individual flow rates of the carrier fluid and the solid phase in the pipeline. This can be computed as:

$$Q = \frac{Q_s}{\rho_s} + Q_l \tag{13}$$

The drag coefficient to be used in Durand's equation [7] can be calculated for the known values of particle size, density and fluid velocity using table 1 as shown below:

Table 1. Expressions for drag coefficient at different Re.

	Settling Velocity	**Drag Coefficient**	**Condition**
	(m/s)	**(-)**	
1	$V_0 = \left[\frac{g(\rho_S - \rho_L)d^2}{18\,\mu_L}\right]$	$C_D = \left(\frac{24}{Re}\right)$	Re < 1
2	$V_0 = \frac{0.2d^{1.18}\left[\frac{g(\rho_S - \rho_L)}{\rho_L}\right]^{0.72}}{\left(\frac{\mu_L}{\rho_L}\right)^{0.45}}$	$C_D = 30Re^{-0.625}$	1< Re <1000
3	$V_0 = \left[4\,g\,d\left\{\frac{\rho_S - \rho_L}{3 \times 0.44\rho_L}\right\}\right]^{0.5}$	$C_D = 0.44$	1000< Re < 2x10^5
4	$V_0 = \left[4\,g\,d\left\{\frac{\rho_S - \rho_L}{3 \times 0.1\rho_L}\right\}\right]^{0.5}$	$C_D = 0.1$	Re > 2x10^5

The volume fraction of the solid phase can be found using equation (11) as:

$$C_v = \frac{4\,Q_s}{\pi\,V\,D^2 \rho_s} \tag{14}$$

4. Pumping Energy

The power required per meter length of the pipe, in terms of the total head loss in the pipeline, is given by:

$$P = \frac{\gamma_m\,Q\,\Delta h_s}{\eta} \tag{15}$$

Where the specific gravity of the slurry can be computed as:

$$\gamma_m = \rho_m\,g \tag{16}$$

The specific gravity of the solid-liquid mixture can be calculated as a function of the volume fraction of the solid phase in the pipeline as:

$$\rho_m = (C_v\, \rho_s) + (1 - (C_v\, \rho_l)) \tag{17}$$

The levelized net annual cost of pumping energy per unit pipe length C_{power} can be computed as:

$$C_{power} = P \times C_1 \tag{18}$$

Where C_1 is the cost of energy consumption per unit watt by the pumping unit installed.

5. Cost of Pipes

The levelized net annual cost of pipe per unit pipe length C_{pipe} is given by Chermisinoff [3] as a function of levelized net annual cost of pipe per unit weight of pipe material C_2:

$$C_{pipe} = \pi\, D\, t\, \gamma_p\, C_2 \tag{19}$$

Mishra and Agrawal [6] have taken the pipe wall thickness t in terms of pipe diameter D and the coefficient C_c for reasonable pressure range as:

$$t = C_c\, D \tag{20}$$

Hence, the cost of the pipe would be:

$$C_{pipe} = \pi\, D^2\, C_c\, \gamma_p\, C_2 \tag{21}$$

6. Total Cost of Pipeline

The levelized total annual cost of the pipeline per unit length C_{total} is given by:

$$C_{total} = C_{power} + C_{pipe} \tag{22}$$

7. Optimal Sizing of Pipeline

The diameter corresponding to total minimum cost is the optimal diameter for a given solid throughput. It has been seen that both the pumping cost and piping cost are functions of diameter of the pipeline. Hence the total cost, as a function of diameter of pipe D, can be written as:

$$C_{total} = f_1(D) + f_2(D) \tag{23}$$

Figure 1 depicts the variations in maintenance, manufacturing and total costs of the hydraulic slurry pipeline as a function of the pipeline diameter at a solid throughput of 50 Kg/s for aluminium particles of size 2mm. The maintenance cost is the cost associated with the pumping requirements as discussed in section 4. The manufacturing cost is the cost of the pipes as discussed in section 5. The total cost as a function of pipeline diameter hence is the sum of these two costs. It can be seen that as the pipeline diameter increases, the maintenance cost decreases due to the fact that for the same solid throughput rate,

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increasing the size of the pipeline results in the decrease of the flow velocity. Hence, the maintenance cost decreases with increase in the pipeline diameter.

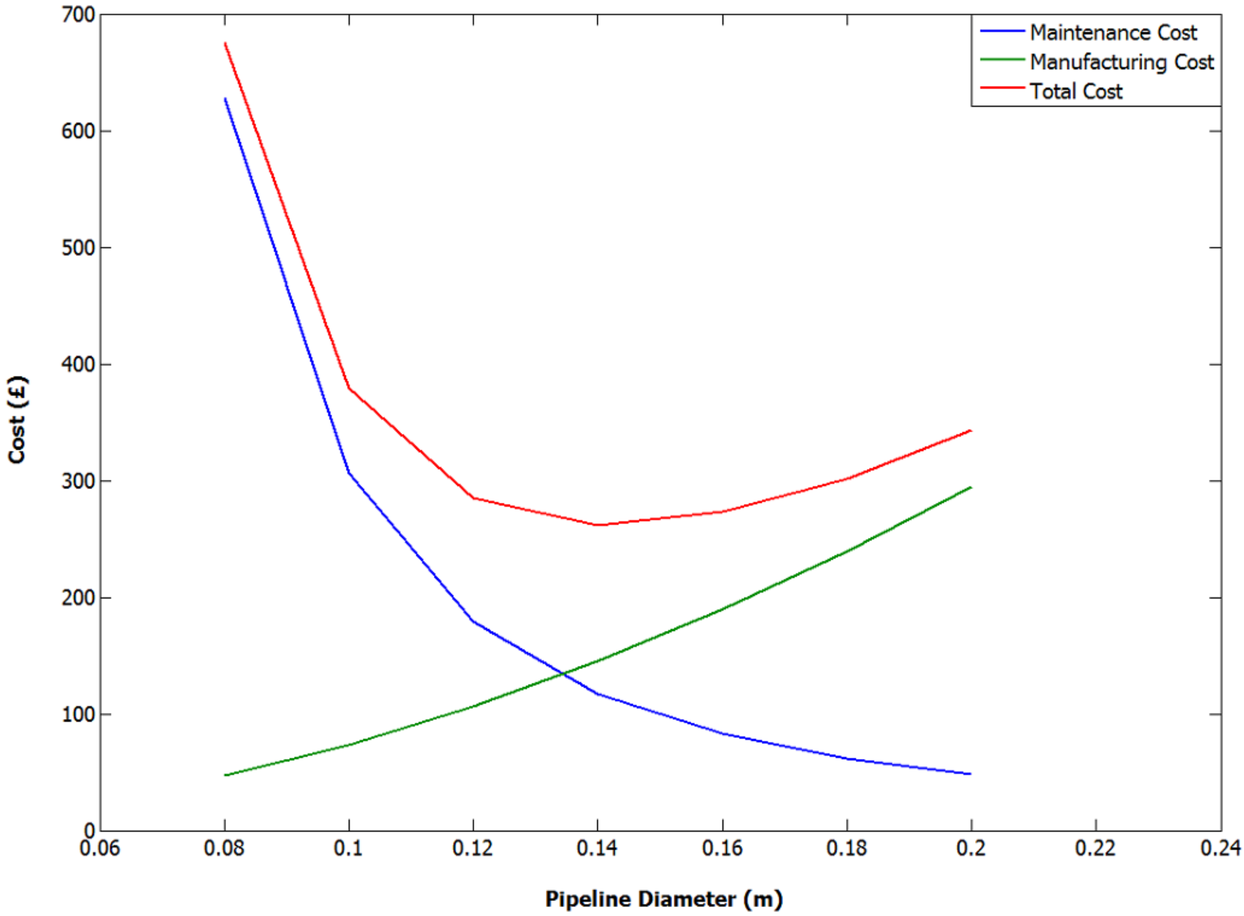

Figure 1. Variation of various costs involved in slurry pipelines for solid throughput of 50 Kg/s.

Furthermore, increase in the pipeline diameter increases the cost of the pipes i.e. manufacturing cost. The cost values shown in the Y-axis of the plot corresponds to the current market price for PVC pipes. It can be clearly seen that the point of intersection of the maintenance and the manufacturing cost curves reflect the optimum diameter for the hydraulic slurry pipeline. At this point of intersection, the total cost curve is at its minimum value. This methodology has been adopted for the optimisation of the slurry pipelines.

8. Design Example

Find optimal size of a pipeline transporting aluminium ore slurry with particles of size 2mm in the solid throughput range from 50Kg/sec to 250Kg/sec.

Computations have been made for Q=50kg/sec up to 250kg/sec for the given design example. Table 2 shows that for particles of diameter 2mm and a throughput of 50 kg/sec of solids, the optimal diameter of the pipeline is 0.14 m. The corresponding values of the flow velocity and solid concentration are 3.17

m/sec and 21.3 % respectively. With increasing solid throughput rate it can be seen that the optimal diameter increases and flow velocities also show an increasing trend. Furthermore, increase in the solid throughput in the pipeline decreases the volume fraction of the solid phase. The average value of the solid phase volume fraction is 20.1%.

Figure 2 shows the variations in optimal diameter and flow velocity with respect to the solid throughput. This plot can be used as a design chart for optimal sizing of pipeline transporting aluminium ore slurry if the required solid throughput rates are known.

Table 2. Variations in optimal diameter for various solid throughputs.

Throughput	**Optimal diameter**	**Velocity of flow**	**Volume fraction of solid phase**
(Kg/sec)	**(m)**	**(m/s)**	**(%)**
50	0.14	3.17	21.3
100	0.19	3.49	21
150	0.23	3.71	20.2
200	0.27	3.9	19
250	0.29	3.99	19

9. Conclusions

The methodology evolved in the present study for optimizing the diameter of the pipeline is found to be applicable over wide range of data for aluminium ore slurry. The optimal size of the pipeline has been calculated from the least cost principle for throughput ranging from 50kg/sec to 250kg/sec for particles of size 2mm. The flow parameters corresponding to the optimal size of the pipeline are in the normal range of operations.

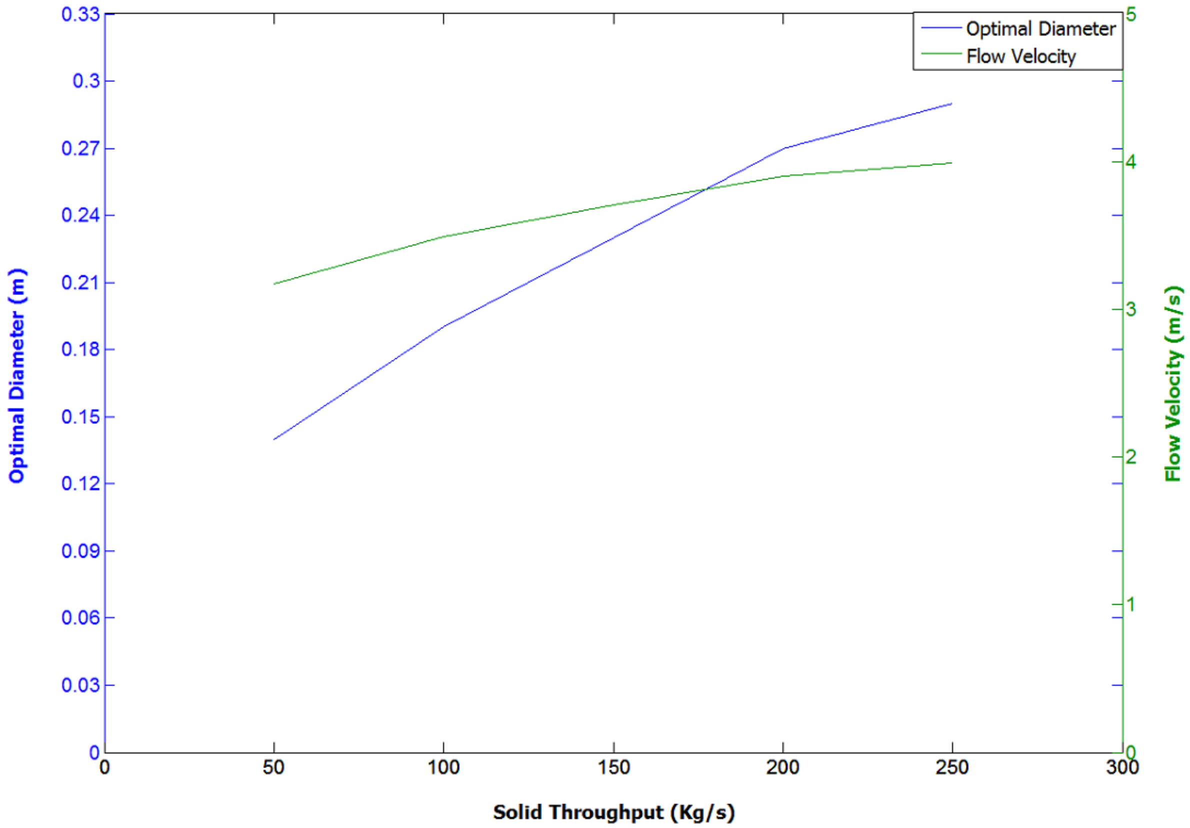

Figure 2. Variation of optimal diameter and flow velocity with solid throughput.

References

[1] M. J. Albertson, Banon and D. Simons, "Fluid for engineers", Prentice Hall, New York, 1960
[2] H. M. Hathoot, I. Ahmad, AL-Amound and S. Fawzi, "Optimal pipeline sizing technique ", Journal Of Transport Engineering, pp. 254-257, 1996
[3] P. N. Cheremisioff, and S. L. Cheng, "Civil Engineering applications", Part 2- Hydraulics /Mechanic, Technomics Publishing Co., 1998
[4] Daugherty and J. Franzini, "Fluid mechanics with engineering application", 5th ed., McGraw Hill Book Co., 1997
[5] P. Swamee and A. Jain, "Explicit equations for pipe flow problems", Journal of Hydraulics Division, pp. 657-664, 1976
[6] R. Mishra and V. C. Agarwal, "Optimal sizing of power handling horizontal capsule pipeline", Journal of Transport Engineering, pp. 185-189, 1998
[7] R. Durand, "The hydraulic transportation of coal and other materials in pipes", College of National Coal Board, London, Nov 1952
[8] M. Wicks, "Transportation of solid at low concentration in horizontal pipes" ASCE International Symposium on solid-liquid flow in pipe, March 1968