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A critical review on coupled geomechanics and fluid flow in naturally fractured reservoirs.

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A critical review on coupled geomechanics and fluid flow in naturally fractured reservoirs

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Abstract

Naturally fractured reservoirs have been a source of challenging issues with regard to field development, well stability, drilling, and enhanced oil recovery, as a connected fracture system can totally dominate the flow patterns. Because of the high degree of heterogeneity in flow characteristics and reservoir geomechanics, several mathematical, numerical and discretization methods are proposed to predict the hydrodynamics behaviour of naturally fractured reservoirs. This paper presents a critical review of the characteristics of naturally fractured reservoirs in terms of geomechanics and fluid flow. In the case of poorly connected fractures and high-density fractured networks, compared to the characteristic length of interest, multi-continuum approaches are widely applicable. The dual continuum approach can handle fracture matrix interaction implicitly much more conveniently than the Discrete Fracture Network (DFN) and Discrete Fracture Matrix (DFM) approaches, but it cannot capture the fracture geometry explicitly where the fracture is the main flow path in the area of interest. Distinct mathematical and numerical modelling of flow and reservoir geomechanics are also addressed in this review paper. In this context, various coupling schemes of reservoir geomechanics and fluid flow are discussed. Recent research challenges related to numerical

modelling of multiphase flow, reservoir geomechanics, coupling schemes, and discretisation are also reviewed. It is concluded that despite several research efforts, coupled geomechanics and multiphase flow is still a challenging issue related to mathematical, numerical models and discretisation schemes to capture the hydrodynamic behaviour, such as fracture deformation and fluid flow behaviour, at fracture matrix interaction in naturally fractured reservoirs and adopting the best modelling approach is very much dependent on the desired hydro-mechanical aspects to be investigated.

Keywords: *Geomechanics, fractured reservoirs, coupling scheme, modelling, fracture-matrix interaction.*

1.0 Introduction

Rigorous numerical modelling of fluid flow in geologically complex reservoirs is a major challenging issue for petroleum reservoir engineers. Conventional and unconventional fractured reservoirs are part of these challenges. Predominantly, fracture network patterns are the main conduits for fluid flow and improve the permeability in tight formations, while the matrix controls the main reservoir storage capacity. Building a modelling framework is a challenging task to describe and understand how single phase and multiphase flow occurs in the fractured reservoirs, and to qualify the hydrodynamic interactions between fractures and adjacent porous matrix under variety of overburden stress levels.

Fractured hydrocarbon reservoirs play a significant role both in the world economy and main energy markets. As a result of high heterogeneity and fabric complexity in naturally fractured reservoirs, accurate hydrocarbon recovery predications is challenging despite the existence of giant hydrocarbon reserves in fractured formations (Geiger et al., 2009). Over 20% of the world's reserves and production are provided by natural hydrocarbon reservoirs, such as the Asmari limestone reservoir in Iran, the vugular carbonate reservoirs in Mexico, the Kirkuk oil field in Iraq, the group of chalk reservoirs in the North Sea, and over 400 billion barrels of hydrocarbon reserves in Canada (Abbas, 2000; Jalali and Dusseault, 2012).

Fractures are ubiquitous in the subsurface (Berkowitz, 2002) and play a significant role in a wide range of engineering applications, such as nuclear waste disposal, groundwater management, unconventional shale gas reservoirs, geotechnical engineering, and enhanced oil recovery (Abass et al. 2007; Lei, Latham, and Tsang 2017; Unsal, Matthäi, and Blunt 2010). The discontinuity of fractures includes complex networks, dominating the geomechanics and hydrogeological behaviour of subsurface rocks (Lei, Latham, and Tsang 2017). The geomechanical analyses play a significant role in demonstrating and characterising phenomena like sand production during the well production, surface subsidence, stability of wells in particular in shale formations, and reservoir compaction where subsurface pressure depletion exists (Jalali and Dusseault, 2012). The main feature of these common phenomena is strongly related to the behaviour of solid interactions with the reservoir fluid flow affected by fractures (Settari and Walters, 2001).

The physical interactions of hydraulic and mechanical processes in the porous bearing formation is called hydromechanical coupling (Rutqvist and Stephansson, 2003). In petroleum reservoirs, hydromechanical interactions are common due to presence of fractures and pores which are deformable and filled by fluid. In general, porous media or fractured rocks are saturated with fluids, and the porous media connectivity and fracture apertures can deform as a result of either change in the external stresses acting on the formation, or change in the internal fluid pore pressure as shown in Fig.1.

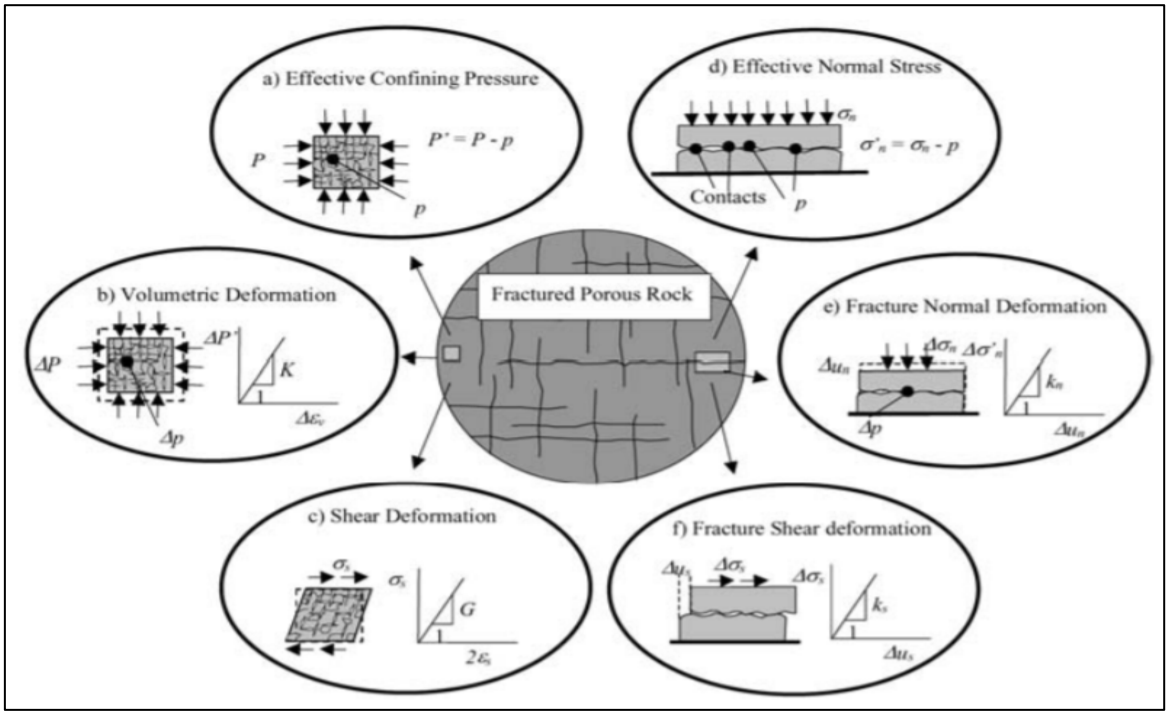

Fig. 1. Overview of the deformation modes in fractured porous media including effects on porous matrix and macro fractures (Rutqvist and Stephansson, 2003).

This paper reviews the most significant fluid flow numerical modelling methods in fractured porous media. It also provides a comprehensive understanding of different coupling schemes between reservoir geomechanics and fluid flow in the fractured porous medium, from physical, conceptual, and mathematical models to discretization approaches. In addition, it shows the flexibility and complexity of different flow modelling approaches in fractured porous media, in terms of computational accuracy and cost. Furthermore, various coupling approaches related to reservoir geomechanics and fluids are summarised based on the geometrical complexity, flexibility, computational efficiency, and types of discretization.

2.0 Modelling of Fluid Flow and Transport in Fractured Porous Media

The flow equations in a small volume of porous media (e.g. rock or geological materials), which is representative of elementary volume (REV), was first derived by Bear (1972) (Bear, 1972). Subsequently, mass and momentum conservation equations were used to describe the fluid flow behaviour and transport in fractured porous media. Fig. 2 illustrates the relationship between volumetric porosity and REV, and the fact that porosity measurement changes with sample volume and the domain of the REV (Bear, 1972; Nordahl and Ringrose, 2008).

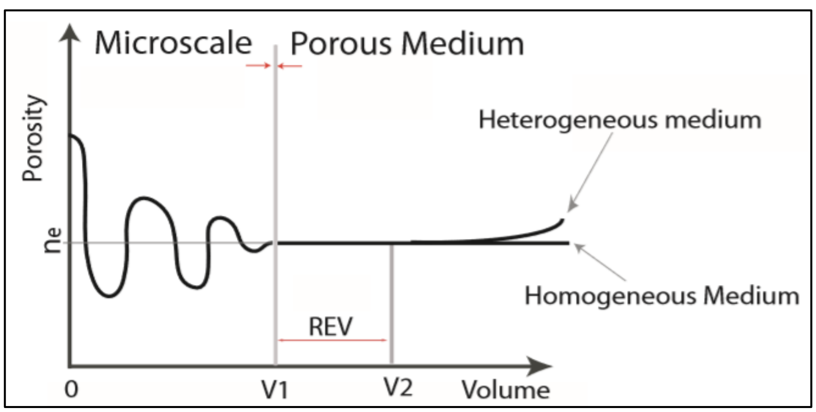

Fig. 2. Illustration of the REV concept for porosity (Nick, 2010).

In this section, the previous track record of governing flow equations for single and multiphase flow reservoirs are introduced. Furthermore, an extension of Darcy’s law in the fractured domain for single-phase flow and physical parameters that govern multiphase flow in fractured reservoirs are highlighted.

2.1 Single Phase Flow

Darcy transport equation is widely applicable for single, two and three phase flow (Bear, 1972). Eq. (1) represents the Darcy transport equation where the pressure gradient is the major driving force (∇P) for single-phase.

$$v = -\frac{k}{\mu}\left(\nabla P - \rho g \nabla D\right) \quad \text{... (1)}$$

The mass conservation equation is normally provided by Eq. (2) for single phase flow in the fractured porous media.

$$\frac{\partial}{\partial t}(\rho\varphi) + \nabla.(\rho v) = q \quad \text{... (2)}$$

Where: v is the Darcy velocity, ρ is the density of the fluid, μ is the viscosity of the fluid, k is the permeability of the formation, g is the gravitational acceleration, D is the depth of the datum, φ is the rock porosity, and q is the source term.

As a result of high flow velocity in the fracture (free channel) domain relative to the porous matrix, Sanaee et al. (2012, 2013) employed the Navier-Stokes equation in fracture volume. In addition, the Brinkman equation, which is an extension of Darcy law (Bars and Worster, 2006), was used to control single phase flow at fracture matrix interactions.

2.2 Multiphase Flow

In the multiphase flow and isothermal condition, multi-mass components and mass balance equations are needed to describe the flow in fracture and matrix reservoir rocks separately for each phase. Mass conservation equation is given by Eq. (3).

$$\frac{\partial}{\partial t}\left(\varphi S_\beta \rho_\beta\right) = -\nabla.\left(\rho_\beta v_\beta\right) + q_\beta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3)}$$

In Eq. (3): $S_\beta, \rho_{\beta,} v_\beta$ is the saturation, density and velocity for the phase of flow respectively (β= g for gas, β= w for water, and β=o for oil).

Darcy law is also widely applied for considering the effect of density, viscosity and pressure gradient for multiphase flow in the fractured porous media as shown in Eq. (4).

$$v_\beta = -\frac{k_a k_{r\beta}}{\mu_\beta}\left(\nabla P_\beta - \rho_\beta g \nabla D\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4)}$$

In Eq. (4): v_β is the Darcy velocity of the phase β, μ_β is the viscosity of the phase, k_a is the absolute permeability of the formation, $k_{r\beta}$ is the relative permeability of the phase, and ∇P_β is the pressure gradient of the phase β.

Multiphase (two and three phases) flow modelling is still a challenging task while the most widely used approach is the same one used for single phase flow. To describe multiphase flow in fractured porous media, some more physics should be introduced before proceeding, such as saturations, relative permeability, and capillary pressure (Karimi-Fard and Firoozabadi, 2001; Monteagudo and Firoozabadi, 2004). In the presence of multiphase flow, fluids jointly fill the porous medium indicates the relation (Eq.5) (Zhangxin et al., 2006).

$$\sum S_\beta - 1 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(5)}$$

When multi-immiscible fluids exist in the porous media, the distinct pressure between the non-wetting and wetting phases is called capillary pressure (Eq. 6); across the interface, pressure arises from the capillary forces and these capillary forces come from the surface and interfacial tension (Pyrak-Nolte et al., 2008; Soares et al., 2015).

$$(P_c) = P_n - P_w \quad \text{...} \quad (6)$$

where: P_c stands for capillary pressure, $P_n\ and\ P_w$ are the non-wetting and wetting phases' pressures respectively.

In immiscible multiphase flow, the presence of the non-wetting phase decreases the cross-sectional area availability to flow of wetting fluid and vice versa. Therefore, the ability of fluid to transport reduces within porous media domain and is defined by relative permeability (Falode and Manuel, 2014; Honarpour et al., 1986; Jerauld and Salter, 1990).

$$k_{r\beta} = \frac{k_\beta}{k_a} \quad \text{...} \quad (7)$$

$$\lambda_\beta = \frac{k_{r\beta} k_a}{\mu_\beta} \quad \text{...} \quad (8)$$

where: k_β is the effect permeability of the phase, k_a is the absolute permeability and λ_β is the mobility of the phase.

Adding Eqs. (7) and (8) to Eq. (2), then Eq. (9) describes multiphase incompressible fluid flow in the porous media.

$$\frac{\partial}{\partial t}\left(\varphi S_\beta \rho_\beta\right) = \nabla.\left(\rho_\beta \lambda_\beta (\nabla P_\beta - \rho_\beta g \nabla D)\ \right) + q_\beta \quad \text{...} \quad (9)$$

However, flow behaviour cannot be investigated alone by Eq. (9) in different continuums. Therefore, it is essential to specify the initial and boundary conditions, and the continuity equation must be detected at different interfaces between distinct continua in terms of pressures, concentrations and mass fluxes (Martin et al., 2017). Fig. 3(a) represents the flow term evaluation, spatial discretization and grid block connections within a multi-continuum system between two neighbour grid blocks (i, j) directly based on the integrated finite difference approach. Fig. 3(b) illustrates the effect of the periodic domain system (Ω) which is the effect of fracture length on the contaminant transport in the fractured porous media at the macroscopic scale where multiple fracture scales exist (Kalinina et al. 2014; Wu et al. 2006).

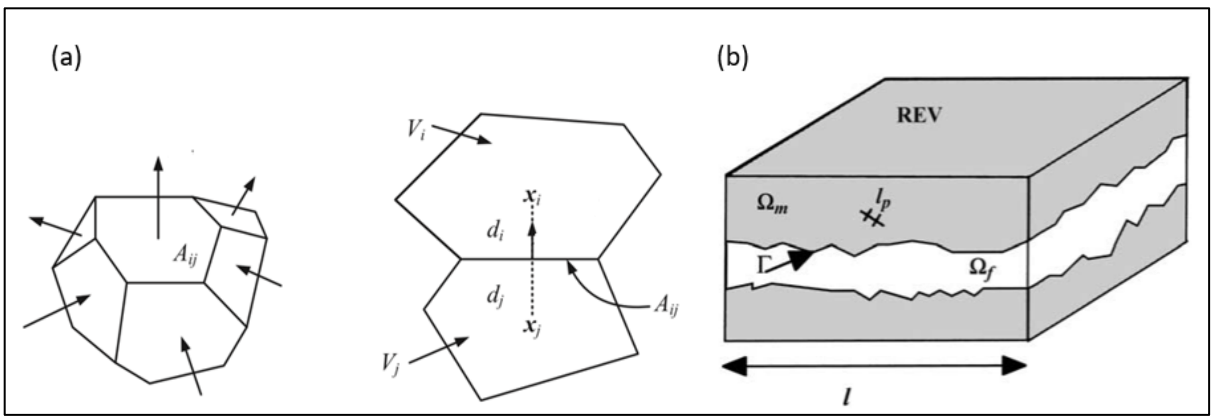

Fig. 3. (a) 2D Finite difference integration for flow term evaluation, spatial discretization and connection (Wu 2016).*(b) Fractured porous media domain, REV (Representative Elementary Volume), Ωf (fracture domain), Ωm (matrix domain), Ω (Periodic domain),* l *(microscopic characteristics length),* l_p *(the pore lengthscale),* Γ *(fracture-matrix boundary or interface)* (Royer et al., 2002).

3.0 Conceptual Flow Modelling of Fractured Rocks

A conceptual model describes the main geological and hydrogeological features of the fractured porous media that control the transport and fluid flow behaviour in the system (Berkowitz, 2002). The realistic development of conceptual models for multiphase flow in fractured rocks is a significant research problem for enhanced oil recovery, nuclear waste disposal, and subsurface contamination (Council, 1996).

In the past few decades, several mathematical modelling approaches have been developed and extended. They mainly depend on continuum approaches involving geometrical information for fracture and matrix formation systems, setting up the domain of the fracture matrix system for mass and energy conservation equations (Lei, Liao, and Zhang 2019; Wu, Liu, and Bodvarsson 2004). These approaches are followed by solving discrete nonlinear algebraic equations numerically which couple fluid flow phases with other physical processes (Warren and Root 1963; Wu 2016).

3.1 Single Continuum Approach

The single continuum approach states that both fracture and matrix are in the same domain, and fractured permeability is adapted by the porous matrix domain. The fluctuation of permeability tensor and its orientation might vary, based on the fracture properties and network. The volume average of this approach is ideally expressed by representative elementary volume (REV) (Bear, 1972; Berre et al., 2019).

In the single continuum method, transport is transfigured from a microscopic to a macroscopic scale when the average of microscopic quantities are used for the problem (Dadzie et al., 2008). The whole domain's exact local characteristics are covered by the use of average quantities of REV.

The heterogeneity size is much smaller than the size of REV and the size of the REV must be smaller than the microscopic length scale. This method allows REV to be determined, and then the continuum approach is applicable in a fractured reservoir. The physical behaviour upscaling method of the macroscopic level can be derived by the REV scale. In general, Darcy's law is applied for flow in the single continuum approach where the flow is essentially influenced by frictional resistance and the pressure gradient is the major driving force in the system as already discussed (Royer et al., 2002).

3.2 Dual Continuum Approach

The dual continuum approach represents the fracture and matrix domains separately, such as with dual porosity and dual permeability values. The dual continuum approach can involve several fractures or fracture networks locally and is expressed by representative elementary volume (REV) (Wu 2016).

The dual continuum method has been advanced and used as the main method for modelling fluid flow, heat transfer, and chemical transportation in fractured porous media (Wu, Pan, and Pruess 2004). Furthermore, the physical process of fluid flow and transport in fractured reservoir rocks are treated separately for each continuum, such as fracture continuum and matrix continuum. The same basic conservation of mass, energy and momentum are governed for flow within each continuum separately (Khalili 2008; Wu and Qin 2009). Although, it should be noted that the dual continuum approach depends on the uniform distribution of denser fracture networks, and detailed information of fracture and matrix characteristics. The dual continuum approach is widely used in petroleum reservoir simulation and in commercial reservoir simulators (Moinfar et al., 2011; Monteagudo and Firoozabadi, 2004).

Multiphase flow is described separately, in naturally fractured formations, for both continua matrix and fracture using doublet flow equations in the dual continuum approach. This method causes a set of partial differential equations for both continua (Wu, Pan, and Pruess 2004).

3.3 Triple Continuum Approach

The triple continuum approach is developed in the same way as Warren and Root's, used for the dual-porosity model; both matrix systems are considered locally uniform and homogenous. There may exist an important heterogeneity within the rock matrix and fractures systems. The concept of double porosity has been extended to explore the effect of heterogeneity on flow in fractured and rock matrix (Wu, Pan, and Pruess 2004). In addition, a number of triple continuum methods have been progressed and used to carry out the effect of rock matrix heterogeneity, small fractures, and for vuggy fractured reservoirs. In general, these multi-continuum approaches have concentrated on distinct levels and scales of the lithological heterogeneity of fractures and rock matrix. This includes subdividing the fractures and matrix systems through two or more subdomains with distinct characterizations for single and multiphase flow in fractured porous media (Wu 2016).

Furthermore, the fracture–matrix system is conceptualized as a triple continuum model to explore the effects of small-scale fractures in fractured reservoirs. This includes single porous rock matrix and two kinds of fractures: (1) large fractures which globally connect to other fractures; and (2) small fractures that are locally connected to the large fractures and the rock matrix (Wu, Pan, and Pruess 2004).

In principle, the triple continuum approach is similar to a dual continuum approach for small fractures. The entire reservoir volume is occupied by three distinct spatial continua (vug, fracture and matrix) that use effective porosity values to approximate the two fracture types (small and large) and rock matrix (Wu 2016).

3.4 Discrete Fracture Network Model (DFN)

In general, the DFN concept is a comprehensive study of fractured rock modelling and it is a group of geometrical planes that represent fractures (Tavakkoli et al., 2009). In DFN modelling, the model domain of the formation includes every fracture pattern's properties (e.g. orientation, size, position, shape and aperture) and flow description, explicitly through fractured systems at fracture matrix interaction. A discrete fracture model is a rigorous approach for the field scale as opposed to other fracture modelling approaches because of computational intensity and large data requirement (Lei, Latham, and Tsang 2017).

In addition, the number of fractures involved in the field simulation is vast, and this approach usually requires the detailed characteristics of fracture and matrix properties and their spatial distributions – which are hardly familiar from the field site in fractured reservoirs. The DFN model commonly disregards the host domain and ignores the flow in the matrix domain, and considers the impenetrability of the matrix domain (Lei, Latham, and Tsang 2017). In the DFN model, it is assumed that only a fracture network can contain and store fluid which is represented by lattice. A DFN model is commonly used and appropriate where (1) the porous media is explicitly presented by entire porosity and permeability of the fracture; and (2) the network represents a model of fractures in low permeable porous media (Berre, Doster, and Keilegavlen 2019; Wu 2016).

3.5 Discrete Fracture-Matrix Model (DFM)

The discrete fracture-matrix model was first introduced by (Noorishad and Mehran, 1982), commonly referred to as the DFM model. The main concept of the discrete fracture-matrix model is based on the equilibrium cross flow between the fluids in the fracture node to the rock matrix node next to the fracture (Monteagudo and Firoozabadi, 2004). The DFM model attempts to create a balance between loss of accuracy and applying geometric complexity by upscaling. Fluid flow is explicitly represented in the fracture and matrix domain, and this model allows the creation of a length of fracture, much less than the size of the matrix domain. Therefore, the DFM model provides secondary permeability in the fracture domain, rather than being part of the explicit fracture network, as in the DFN model (Berre et al., 2019).

Although all the fractures can be incorporated in the fracture domain and within the governing equations, it is not practicable to retain an explicit representation of all fractures because of the computational complexity. Therefore, some of the fractures are preserved, while others are upscaled and replaced by averaged quantities within the matrix domain (Keilegavlen et al., 2017).

The DFM model can be used in the detailed modelling of the interface of two phase flow, capillary pressure and hydromechanical interaction between flow and the matrix medium even if the matrix medium is considered as impermeable (Berre et al., 2019). Fig. 4 highlights a generalized picture of different conceptual models for fractured medium, and how the different conceptual models might relate to examine the specific problem in the fractured porous media. The application of different conceptual models in comparison to others is based on the existence of the separate scales fractured medium, along with the availability of the original information.

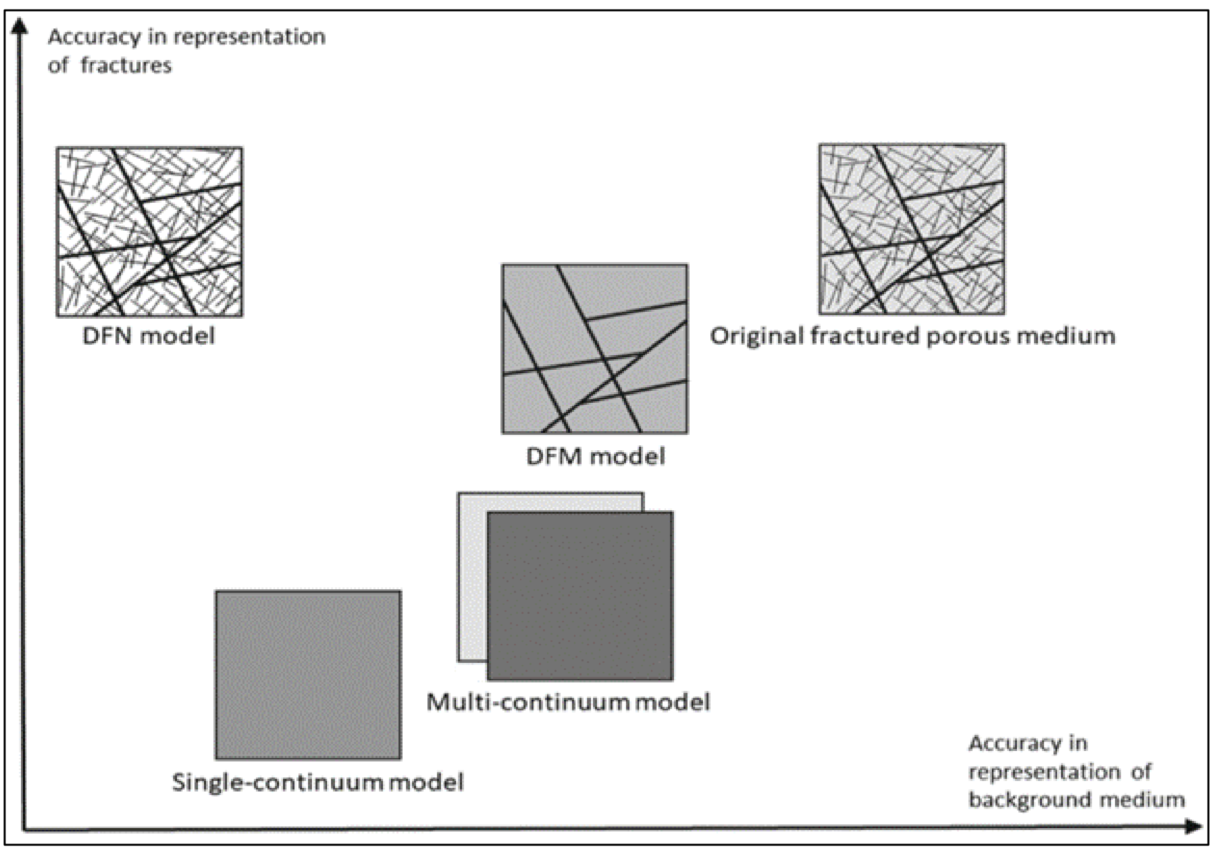

Fig. 4. Illustrating the fractured porous media model concepts (Berre et al., 2019).

4.0 Numerical Flow Modelling of Fractured Rocks

The fractured fluid flow and transport behaviour of the system is normally estimated by numerical models, described by conceptual models such as two and three dimensional (2D, 3D) equivalent fracture matrix networks (Yao et al., 2019). Fig. 5(a) illustrates a simplified 2D representation of the solid red streamline, where the triangular interfaces are diminished to the line segments in the model. A 3D schematic concept of the network interface is also depicted in Fig. 5(b), where the main fluid paths are between tetrahedrons in the triangular interfaces. Flow geometry provides the mathematical formulations of the numerical models specified in the conceptual model (Council, 1996).

In general, several numerical methods have been developed and progressed based on the conceptual models for fluid flow modelling, based on their classification for the complexity and treatment compatibility in fractured porous media (Blunt, 2001; Unsal et al., 2010; Yao et al., 2019; Zhang et al., 2019).

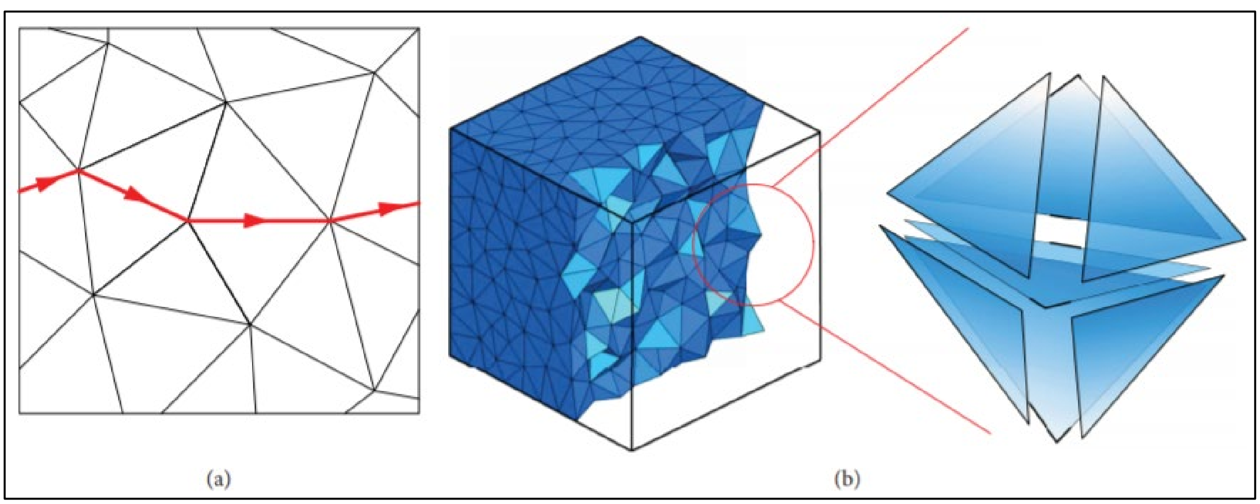

Fig. 5. Conceptual representation fluid flow in equivalent fracture and matrix models in 2D and 3D (Yao et al., 2019).

4.1 *Numerical Modelling of Single Continuum Approach*

The single continuum approach treats the fractured rock system as a continuous body that can be determined implicitly by the old finite difference method and finite element method. In addition, this is the simplest method and gives the single value of porosity and permeability for the fractured domain. The proper application case of the single continuum approach is when a single continuous fracture in the domain is being modelled (Berre, Doster, and Keilegavlen 2019; Jing 2003; Lei, Latham, and Tsang 2017).

4.2 *Numerical Modelling of Dual Continuum Approach*

In the dual continuum implicit approach, finite element method and finite volume or difference discretization schemes are typically used for discretization of the fluid flow process in fractured porous rocks. It is a practical approach to investigate and understand the behaviour of coupled mechanics of multiscale systems (Ashworth and Doster, 2019; Monteagudo and Firoozabadi, 2004). However, these models are limited to sugar cubic representations of a fractured system. Another limitation is the fluid flow exchange term between the fractured domain and matrix, and the exchange term may not be suitably defined by gravity and viscous effects only (Monteagudo and Firoozabadi, 2004).

Furthermore, there is a huge number of advancements of control volume methods for flow discretization such as two-point flux approximations (TPFA), nonlinear two-point flux approximation methods (NTPFA), Multipoint flux approximations (MPFA), mixed finite element (MFE), and mimetic finite difference (MFD) (Arrarás, Portero, and Jorge 2010; Berndtf et al. 2001; Karimi-Fard and Durlofsky 2016;

Klausen and Russell 2004; Lipnikov, Manzini, and Svyatskiy 2011; Wang et al. 2016; Zhang and Abushaikha 2019).

The two-point flux approximation (TPFA) is the straightforward method for discretization where the flux is approximated by using the pressure in the two grids sharing the edge. However, TPFA does not provide a dependable flux, is not convergent, and is not suitable to resolve the flow field accurately. But, it is easy to formulate, employ, and does not suffer from artificial oscillations (Droniou, 2014).

To employ large grid stencils and considering the transmissibilities as a function of the solution of the fluxes, significant progress has been done for the traditional TPFA which involves the computational complexity denoted to nonlinear two-point flux approximation (NTPFA) (Chen et al., 2008).

It is worth mentioning that the flux over an edge cannot be approximated properly by using a linear amalgamation in the two adjacent cells. Therefore, the multipoint flux approximation (MPFA) has been progressed independently by (Aavatsmark et al., 1996) to involve the non-linear approximation, adjust the flux expression, and to consist of more points in the flux stencil (Aavatsmark et al., 2010, 2008, 1998). Furthermore, several local conservative methods have been developed to control volume methods and discontinuity of the permeability tensors such as mixed finite elements (MFE) and mimetic finite differences (MFD) (Klausen and Russell, 2004).

In addition, many of the computational efforts are started solving nonlinear and thereby linear systems in the field of applications. The main significant factor for fast solvers of MPFA is to achieve popularity and to formulate a suitable mathematical framework for the analysis of the methods (Droniou, 2014). The analysis of convergence is still based on the relations between MPFA methods and MFE or MFD discretization (Wheeler and Yotov, 2006).

4.3 Numerical Modelling of Triple Continuum Approach

The main purpose of using this approach is when the vuggy, fractured porous matrix, or two different scales of fractures and rock matrix are available. The triple continuum approach treats the vuggy fracture matrix system separately, as three distinct continua. The finite element methods are used implicitly for modelling of fluid flow, aided with the controlled volume or finite difference method (Wu 2016; Youssef and Alnuaim 2017). The comparisons of the triple continuum approach with the single continuum and dual continuum (dual porosity and dual permeability) approaches are depicted in Fig. 6. In general, the concept of the triple continuum approach is an extension of the dual continuum approach by computing one more network of small fracture between large fractures and matrix blocks as shown in Fig. 6(d).

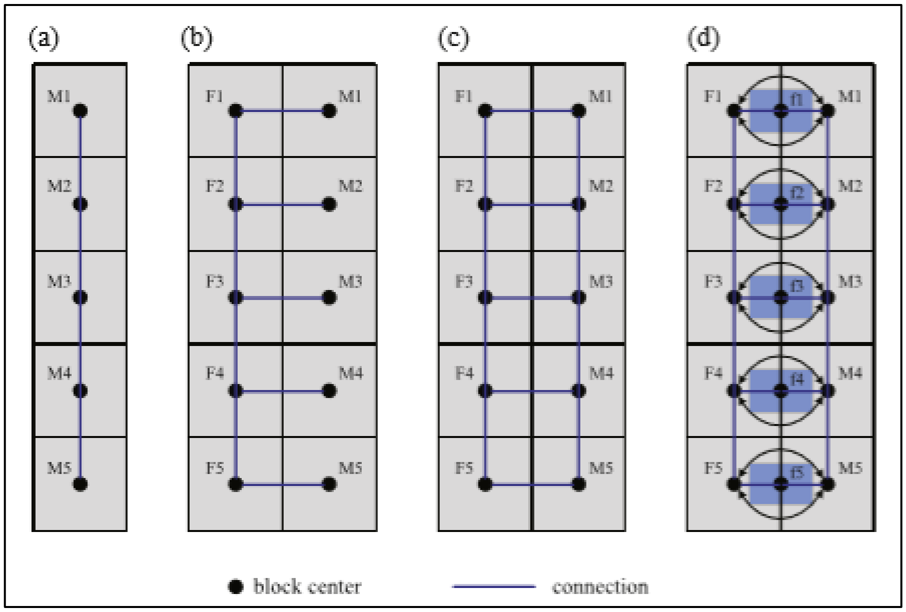

Fig. 6. Illustrating distinct conceptualization for fracture matrix treatment (a) Single Effective Continuum Approach (b) Dual Porosity Approach (c) Dual Permeability Approach (d) Triple Continuum Approach (F=large fracture, f=small fractures, and M=matrix) (Wu 2016; Wu, Liu, and Bodvarsson 2004).

4.4 *Numerical Modelling of Discrete Fracture Network Model (DFN)*

A discrete fracture network (DFN) is based on the premise that the subsurface flow and transport occur mainly within the fractures, and there is no space between the fractures except the location of their intersection. Therefore, the concept of DFN is used to interpret the fracture and fracture set properties. The DFN represents fracture characterization such as orientation, size, spatial distribution, shape, and transmissivity explicitly (Barton et al., 2013). The 3D DFNs provide single aperture values per fracture, because the simplifications of fracture heterogeneity may introduce errors in estimating the hydraulic response in the fractured rock masses as illustrated in Fig. 7 (Huang et al., 2019).

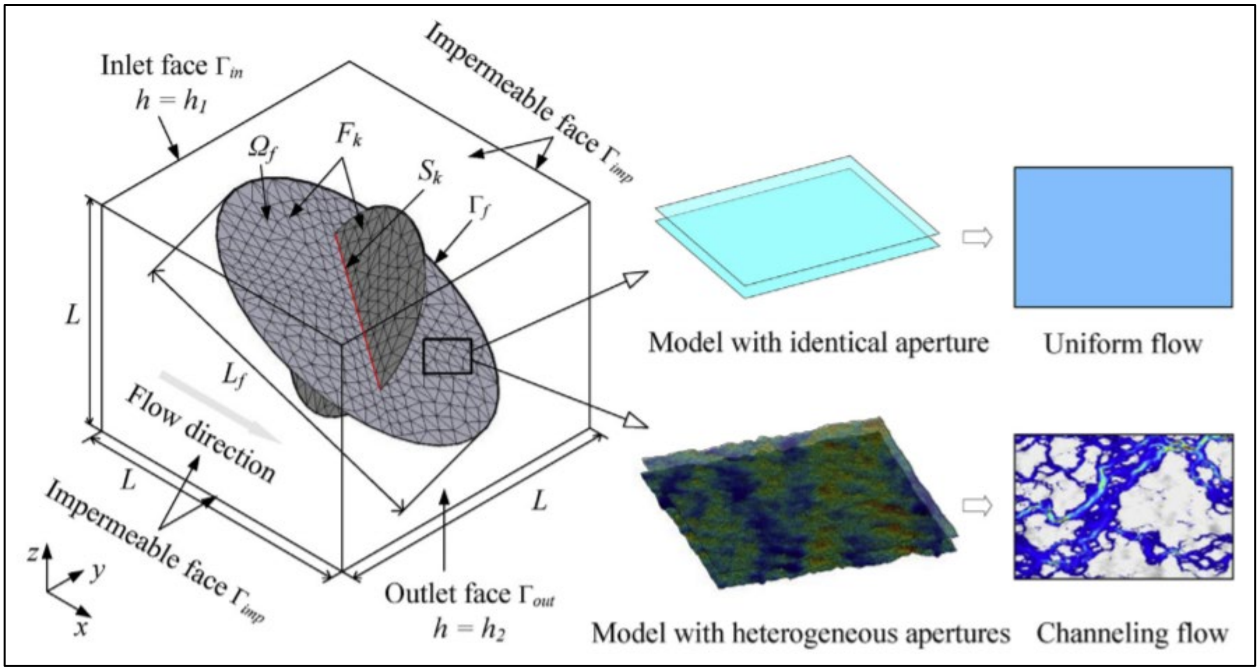

Fig. 7. Representation of 3D DFN fracture aperture distributions and different hydraulic response results, where Lf/L is the fracture length to the domain length, h is the hydraulic head, F_k is the set of fracture intersection, Γ_f is the border of the fracture domain Ω_f , and Γ_S is the fracture intersection (Huang et al., 2019).

4.5 *Numerical Modelling of Discrete Fracture Matrix Model (DFM)*

In the last decade, the discrete fracture matrix model has received considerable attention for simulating naturally fractured reservoirs. Most DFM models focus on unstructured grids to conform to the geometry and the location of the fractures (Moinfar et al., 2014). Fig.8 highlights the unstructured mesh which is triangular based and subdivided into 2D, 1D, and 0D objects. The DFMs can produce the real geometry of the fracture explicitly and each of the fractures has its own size, shape, orientation, and permeability (Yang et al., 2020). Therefore, the DFMs account for the effect of every individual fracture on fluid flow in the fractured porous medium (Karimi-Fard and Firoozabadi 2003). Endeavours toward the development of DFMs based on the finite element method by (Baca et al., 1984), and later (Marcondes and Sepehrnoori, 2010) resulted in the implementation of control volume and finite element methods to simulate fracture and flow in the fractured reservoirs.

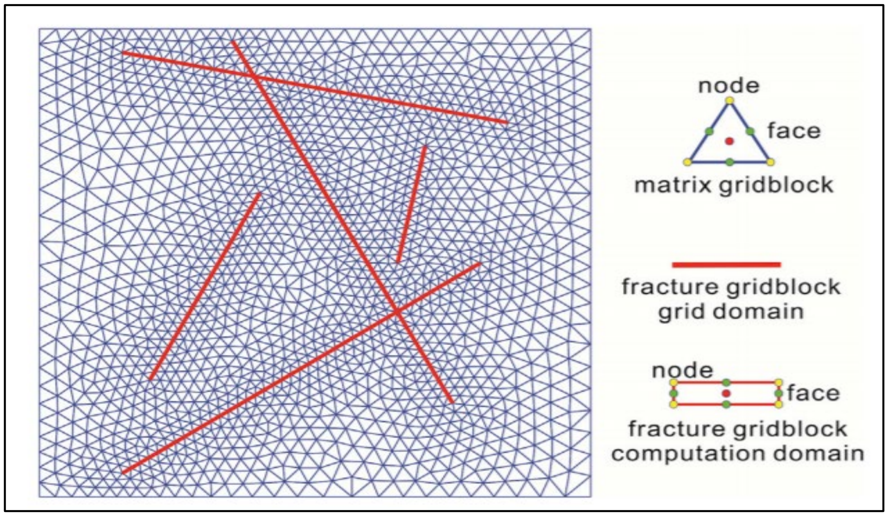

Fig. 8. 2D schematic of DFM (Zhao et al. 2018).

Much research effort was given to the study of outcrop characterisation, and illustration of significant changes in fractures' height, aperture, space, length, and network connectivity (Karimi-Fard, Durlofsky, and Aziz 2003; Lee and Schechter 2015; Lu Li, Shi, and Wang 2019; Monteagudo and Firoozabadi 2004; Xia Yang et al. 2018). However, there was a large discrepancy in terms of realization and uniformity in dual porosity assumptions. The discrete fracture matrix model has been developed to reduce the number of nonphysical notions in the dual continuum approach. The majority of DFMs focused on the unstructured grids to explicitly represent fractured porous rocks. DFM has several advantages, such as the realistic simulation of the geometry of fractures, and the effect of each individual fracture. It is easy to update fracture models in this approach because they are not constrained with grid-defined geometry of fractures. In contrast, the discrete fracture matrix models requires solving discrete systems of equations and hence a complex structure and more difficulty in solving numerically (Moinfar et al., 2011). In addition, the DFMs will provoke a generous number of refined grids around the position where the fractures intersect or densely distributed, which leads to low computational efficiency (Zhao et al., 2018). Therefore, the DFM approach was further integrated and elaborated to deliver an embedded discrete fracture model (EDFM) (Lee et al., 2001; Li and Lee, 2008). In general, EDFMs have straightforward mathematical algorithms for producing grids and will not produce many refined grids compared to DFMs when the fractures have complex geometric features. A structured mesh is also employed to discretize the matrix system in EDFMs and fractures are embedded into the background mesh to be divided into some fracture grids (Yang et al., 2020). It is also stated that the EDFMs have high computational efficiency as a result of avoiding the struggle in creating and recreating high quality conforming mesh (Rao et al., 2019). Furthermore, different types of fracture network models demonstrate different strengths in various aspects but might all suffer from some drawbacks, as listed in Table 1. Table 1 also provides a detailed comparison of distinct fracture network models.

Table 1: Comparison between various numerical flow modelling of fractured rocks.

Numerical Methods (models)	**Input Parameters**	**Advantages**	**Limitations**
Single continuum approach	Material properties of matrix.	• Reduces geometrical complexity. • Suitable for large scale of application. • Uses deterministic or stochastic models. • Productive calculation.	• No consideration of matrix porous media. • No fracture and matrix interaction, displacement and rotation. • Implicit generation of system. • Valid only for using REV (Council 1996; Lei, Latham, and Tsang 2017). • Uniform value of porosity and permeability (Youssef and Alnuaim, 2017).
Dual continuum approach	Properties of fracture and matrix.	• Simplicity of geometry. • Suitable for large scale of application. • Two separate continua (matrix and fracture). • Uses deterministic or stochastic models.	• Implicit generation of system (Ashworth and Doster, 2019). • Continuity of the fractures. • Valid only for using REV (Council 1996; Lei, Latham, and Tsang 2017). • Limited to sugar cube representation of fracture domain. • Needs fluid flow exchange term between fracture and rock matrix (Monteagudo and Firoozabadi, 2004).
Triple continuum approach	Material properties of fractures, matrix rocks and vugs.	• Three separate continua (matrix, fracture and vug or matrix, small fracture and large fracture) (Youssef and Alnuaim, 2017). • Suitability of use for large scales. • Uses deterministic or stochastic models.	• Implicit generation for all systems. • Continuity of the fractures. • Valid only for using REV (Council 1996; Lei, Latham, and Tsang 2017). • Needs fluid flow exchange term between fracture and rock matrix (Monteagudo and Firoozabadi, 2004).
Discrete fracture network model	Material properties of fractures and fracture sets.	• Includes discontinuous fractures. • Explicit generation of the effect of each individual fractures on fluid flow (Monteagudo and Firoozabadi, 2004).	• Fracture is the main store of fluid to flow towards the wellbore. • Rock matrix is impermeable. • Intensive computational time (McClure and Horne, 2013). • Can found only by stochastic and probabilistic models.
Discrete fracture matrix model	Material properties of fractures, fracture sets and matrix blocks.	• Includes discontinuous fractures and matrix blocks. • No need for fluid exchange term between fracture and matrix interface. • Explicit generation of the effect of each individual fractures on fluid flow (Monteagudo and Firoozabadi, 2004).	• Intensive computational time. • Conforming unstructured mesh. • Losing local mass conservation quality while modelling for multiphase flow (Karimi-Fard, Durlofsky, and Aziz 2004). • Can found only by stochastic and probabilistic models. • Numerical instability of the flow computation (Fleishmann et al., 1999).
Embedded Discrete Fracture Model	Material properties of fractures, fracture sets and matrix blocks.	• Includes discontinuous fractures and matrix blocks. • Most popular and accurate numerical flow modelling (Dong et al., 2019). • Explicit generation of the effect of each individual fractures on fluid flow (Monteagudo and Firoozabadi, 2004). • Conforming structured mesh.	• Intensive computational time. • Losing local mass conservation quality while modelling for multiphase flow (Karimi-Fard, Durlofsky, and Aziz 2004). • Numerical instability of the flow computation

5.0 Geomechanical Characterization

Geomechanical record observations trace back to AD 77, when two men observed that the level of water in a well corresponded to the ocean tides, and they recorded this phenomenon in a book (Zhao 2012). Since then, geomechanical research has progressed consistently. As petroleum exploration and production significantly increased, more physical phenomena related to geomechanics have been observed in the oil and gas reservoirs in the early 20th century. For instance, Goose Creek oil field was reported to sink into the water in 1918 and this subsidence was caused by oil and gas extraction (Pratt and Johnson, 1926). Later, the physical hypothesis was developed by Biots, and other scientists behind various observations in the petroleum industry.

5.1 Constitutive Relations

The hypothesis of geomechanics, through a series of laboratory experiments, was first presented by Karl Terzaghi (Terzaghi et al., 1996). In these laboratory experiments, a constant load was applied laterally for a saturated soil sample. This hypothesis was depicted by the following equation:

$$\frac{\partial p}{\partial t} = c\frac{\partial^2 p}{\partial l^2} \quad (10)$$

where p is the pore pressure, t is time, c is the consolidation coefficient and l is the length of the soil sample.

The concept and definition of effective stress were then defined by Karl Terzaghi, as he noticed that effective stress, which is the difference between pore pressure and externally applied stresses, controls the behaviour of the saturated soil samples as depicted in Fig. 9 (Zoback, 2007).

$$\sigma_{i,j} = \sigma'_{i,j} \pm \delta_{i,j} p \quad \text{...} (11)$$

where $\sigma_{i,j}$ is the total stress, $\sigma'_{i,j}$ is the effective stress, $\delta_{i,j}$ is the Kronecker delta in i and j volume respectively.

The + and – signs depend on the direction of externally applied stresses. If the external stresses are defined as positive, then + should be used (Zhao 2012). Then, the Biot Willis Coefficient (α) parameter is identified as the drained and solid bulk moduli of material (Garg and Nur, 1973; Nur and Simmons, 1969).

$$\sigma_{i,j} = \sigma'_{i,j} \pm \alpha\, \delta_{i,j} p \quad \text{...} (12)$$

Biot Willis coefficient can be calculated as in Eq. 13.

$$\alpha = 1 - \frac{K_d}{K_s} \quad \text{...} (13)$$

where K_d is the drained bulk moduli of material and K_s is the solid bulk moduli of material (Biot 1962). The typical value of α is between 0 and 1. For the compacted and nearly solid rocks, the value of α is zero when there are no interconnected pores, and pore pressure has no impact on rock behaviour, such as in quartzite. In contrast, the value of α is equal to one for highly interconnected pores, and indicates pore pressure has a maximum influence such as the case of (uncemented sands) (Zoback, 2007).

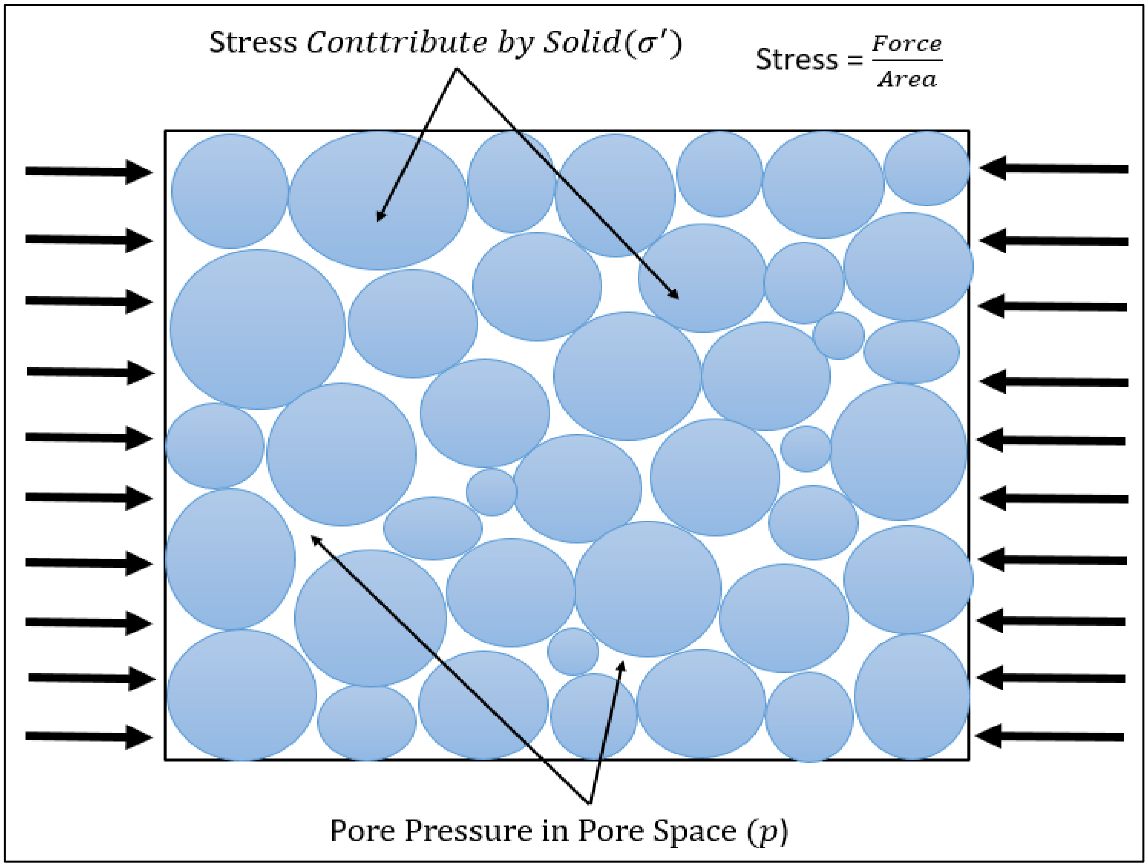

Fig 9. Demonstration of effective stress and pore pressure (Zoback, 2007).

A theoretical governing equation system for three dimensional (3D) consolidation was then developed by Biot (1941). The Biot 3D consolidation model is considered as the basis of computational geomechanics which illustrates pore pressure variation with solid consolidation. This model was then further expanded to describe dynamic behaviour in the soil (Biot 1956). Some theories were developed later to model fluid flow in soil consolidation based on the Biot theory (Green and Naghdi, 1970).

In summary, the basic constitutive relations mentioned above are used to describe porous material deformation behaviour under different stress loading. The major geomechanical constitutive relations are categorised as the elasticity, poroelasticity and thermoporoelasticity models that are widely employed in petroleum reservoir simulation (Zhao 2012). Due to the complexity of the porous material behaviour observations under different stress loading, some simplifications of the geomechanical constitutive relations were made based on natural geomechanical phenomena. The geomechanical constitutive relations are expressed as linear and nonlinear mathematical formulation, based on the requirements of the practical applications (Bemer et al., 2001). The linear poroelasticity model is widely used in fractured porous rocks to investigate the mechanical change due to the pressure depletion in the fractured reservoirs (Bagheri and Settari, 2005; Bai et al., 2019; Garipov et al., 2016; Ren et al., 2018; Sanaee et al., 2013; Sangnimnuan et al., 2018; Yang et al., 2018). Another significant use of geomechanical constitutive relations is to inform when the material reaches plasticity behaviour and eventually collapses. In the reservoir simulation point of view, this phenomenon is significant in recovering oil and gas when the fracture is the main path to flow in the fractured porous rock reservoirs.

5.2 Geomechanical Modelling of Fractured Rocks

In general, numerical approaches in geomechanical modelling are categorized for both continuum and discontinuum methods based on their taxonomy and treatment displacement compatibility (Jing, 2003; Lei et al., 2017). Continuum and discontinuum numerical approaches have been employed to accelerate computer power and defeat the simplified analytical hypothesis (Manouchehrian et al., 2012). The continuum approach has greater efficiency to handle the enormous problems, while the discontinuum approach can more accurately integrate complicated fracture networks and fragmentation processes. In addition, the discontinuum approaches can handle the continuous deformations, whereas there are some advanced continuum techniques to consider the discontinuities which are included contact algorithms and fracture mechanics (Lei et al., 2017). Several numerical methods and codes have been developed and progressed for geomechanical modelling, based on their classification for the compatibility in handling displacements – in classical mechanical problems with little modifications (Cundall, 1988; Figueiredo et al., 2015; Jing, 2003; Potyondy and Cundall, 2004; Tham et al., 2004). The preference for using a method for geomechanical modelling depends on the scale of the problem and the complexity of the fracture network system. There is a huge number of numerical approaches such as finite element method (FEM), finite difference method (FDM), boundary element method for continuum approaches and discrete element method (DEM), and discontinuous deformation analysis (DDA) for discontinuum approaches (Bobet, 2010).

5.2.1 Continuum Approaches

In the conventional continuum approaches, the rock domain is assumed to be continuous and solved by finite difference method (FDM), finite element method (FEM), or boundary element method (BEM) (Bobet, 2010; Pang et al., 2016). The continuum approach is only applicable when a few or a large number of fractures exist (Jing, 2003).

The finite difference method (FDM) is based on the discretization of the governing partial differential equations (PDEs) by following partial derivatives with differences described at neighbour grid points (Jing and Hudson, 2002). The set of linear differential equations are employed by FDM which can be solved by any classical methods. The dynamic problems are solved by FDMs, where the displacements are a function of time and position. In addition, maximum time steps are required explicitly to ensure stability while solving dynamic problems (Bobet, 2010). The localized formulation and its solution are more efficient for memory storage and handling computer applications. There is no local trial in the neighbourhood of sampling points to approximate PDE, as is concluded in FEM and BEM. On the other hand, the conventional FDMs have several shortcomings such as material heterogeneity, complicated boundary conditions, and inflexibility in dealing with fractures (Shojaei et al., 2019; Wang, 2020). In general, the FDMs are incompatible for modelling rock mechanics problems (Nikolić et al., 2016).

The finite element method (FEM) is the most widely employed numerical approach for the analysis of the continuous or quasi continuous media across the science and engineering fields. The term of “Finite Elements” was firstly introduced by Clough (1960). The continuum is discretised into the small-scale aspects in the FEMs. Finite element method (FEM) has proved to be successful in distinct applications for solving geomechanical problems numerically. The indigenous governing equations of geomechanics can convey the various formulations, and the distinct formulation can be expanded, based on the finite element method. The benefit of this approach is that the result is guaranteed to be the exact solution as a result of the discrepancy formulation, which is mathematically identical to the original equation (Zhao 2012). In general, there are well-established numerical methods for solving geomechanical problems. The finite element method (FEM) is the most common and widely implemented due to its flexibility in handling material characterisations of heterogeneity, nonlinearity, and boundary conditions.

However, it is computationally challenging for dynamic related problems (Bobet, 2010). Compared to the FDM, the FEM covers enough flexibility in dealing with fractures, complicated boundary conditions, the treatment of the material heterogeneity, nonlinear deformation, in situ stress, and gravity (Jing and Hudson, 2002). The most significant shortcoming factors are the treatment of the fractures and fracture growth in the implantation of the FEM for the rock mechanics problems. The fracture elements cannot be torn in the general continuum hypothesis during the FEM implementations. Therefore, the FEM suffers from the treatment of block rotation, entire detachment, and large-scale fracture opening (Manouchehrian et al., 2012). The FEM is disabled while the simulation process of the fracture growth due to the required small element sizes, fracture growth continuous re-meshing, and convenient fracture path and element edges. This disadvantage makes the FEM less efficient than the boundary element method (BEM) in dealing with fracture problems (Datas, 2020; Jing, 2003)

The boundaries of the continuum are discretised only in the boundary element method (BEM), whereas the complete domain needs to be discretized in the FDM and FEM methods (Jing and Hudson, 2002; Manouchehrian et al., 2012). In addition, there is no artificial boundaries are required where the common problems in geomechanics are the extended medium to infinity, while the artificial boundaries are required in both FDM and FEM methods (Fahmy, 2021; Mesquita and Pavanello, 2005). BEMs are especially convenient to address the static continuum problems with small boundary to volume ratios, with stress and elastic behaviour or displacements implemented to the boundaries (Gao, 2003). Also, the BEM has much simpler mesh generation and input data preparation with the reduction of the model dimension by one compared with the FDM and FEM (Jing, 2003). Furthermore, the continuous domain inside is applied as a solution, unlike the discontinuous point-wise solution is determined using FDM and FEM. Nonetheless, the BEM is not so sufficient in dealing with material heterogeneity, compared to the FEM because BEM cannot involve as many subdomains as elements in the FEM. Also, the BEM is not powerful in simulating the nonlinear material behaviour compared to the FEM such as in plasticity and damage growth process (Bobet, 2010; Jing and Hudson, 2002). In general, the BEM is more applicable for solving problems of fracturing inhomogeneous and linearly elastic bodies (Kabele et al., 1999). Fig. 10 presents the discretization concepts of finite element methods (FEM), boundary element method (BEM), and finite difference method for fractured systems.

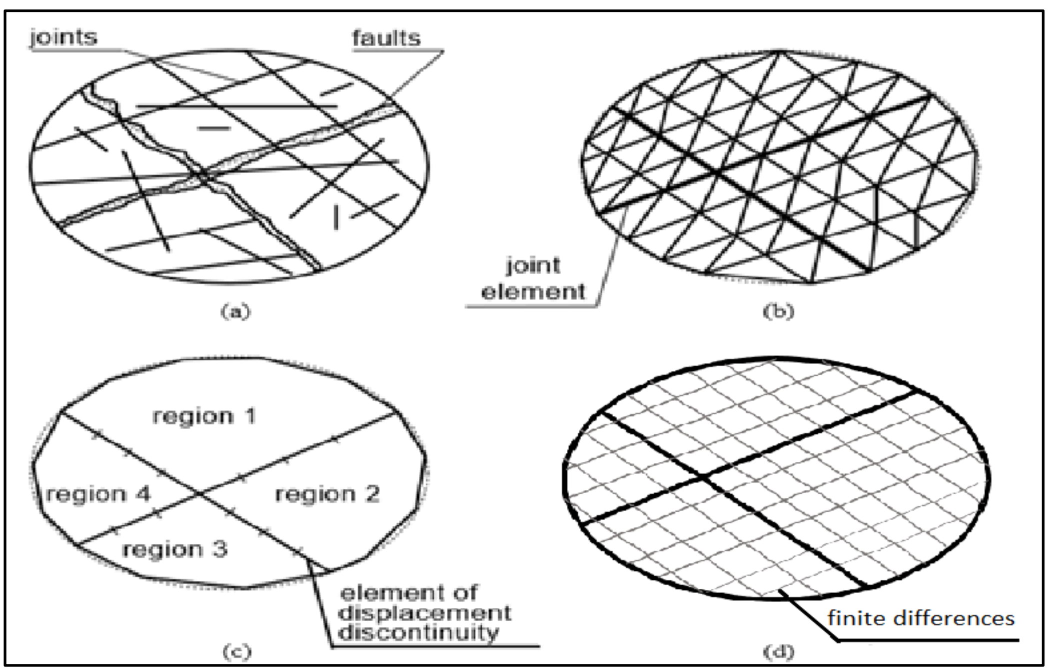

Fig. 10. Illustration of the discretization concepts as (a) shows the representation of the fractured rock mass (b) shows the finite element method (FEM) concept, (c) represents the boundary element method (BEM) concept, and (d) demonstrates the discrete element method (DEM) concept (After Jing, 2003).

5.2.1 Discontinuum Approaches

The discontinuum approach consists of the discrete/distinct element method (DEM) with an explicit solution form and distinct deformation analysis (DDA) with an implicit solution scheme. The discontinuum modelling represents the fractured rock as an assemblage of blocks bounded by a number of intersecting discontinuities (Lei et al., 2017; Lisjak and Grasselli, 2014). The mechanical computation of the fractured geometry can be treated as rigid bodies or deformable subdomains.

The distinct element method (DEM) was originally identified by Cundall (1988). The DEM computation procedures are basically involved (1) the contact between blocks and the contact forces between distinct bodies that care computed based on their relative problems, (2) Newton's second law to calculate acceleration for each distinct element, (3) the velocity and displacement are further developed by time integration with new positions obtained (Hatzor, 2008). The DEM is implemented under an explicit time scheme to solve the problems iteratively until the complement of the block interaction process simulation. Despite the possibility of using a DEM code to model the continuum, the main advantage of the DEM is to model the discontinuities through specific constitutive relations (André et al., 2012). Nonetheless, the proper simulation of the continuous geomechanical materials is the main difficulty for DEM. Although the simulations of the geomechanical problems are simple and accurate, they might need too much computational time for the current computational technology (Ferretti, 2020). Unlike continuous FEM, the DEM can handle the stress-strain characteristics of intact rocks, the shearing/opening of preexisting fractures, the interaction between multiple fractures and blocks (Lei et al., 2017).

The discontinuous deformation analysis is developed to capture the deformation and motion of multi block systems by Shi and Goodman (1985, 1989). Although the discretization method is quite the same for both DEM and DDA, the basic differences between DEM and DDA lies to the computational framework. The DEM employs explicit solution scheme to deal with each blocks separately, whereas the DDA applies the implicit solution form to calculate the displacement field based on a minimization of the whole blocky domain of the potential energy. Compared to the DEM approach, the DDA approach has a significant advantage for fast convergence with unconditional numerical stability that needs a time step smaller than the critical threshold (Jing, 2003).

It is worth mentioning that several efforts have done to combine the finite element analysis of stress/deformation evolution with the discrete element solutions of transient dynamics, contact detection, and interactions in terms of the hybrid finite element- discrete element method (FDEM) (An et al., 2021; Knight et al., 2020; A Lisjak and Grasselli, 2014; Munjiza, 2004). In this discontinuum solution form, the FEM approach is employed to capture the internal stress field of each discrete matrix block while the DEM algorithm is used to calculate the translation, rotation, and interaction of multiple rock blocks (Munjiza, 2004). Fig. 11(b) shows the process of discretization for the solid bodies into a number of elements that deform in accordance with the prescribed boundary and loading conditions using the FEM. On the other hand, the original of the DEM deals with a large number of rigid particles that interact with each other through contact and cohesion laws as shown in Fig. 11(a). When these techniques of FEM and DEM are coupled, the solid bodies are modeled as an assemblage of deformable particles that are bonded with each other as depicted in Fig. 11(c). A series of cohesion points are employed to represent the particle bonds numerically that is located along the boundaries of the deformable particles (Fiore et al., 2013).

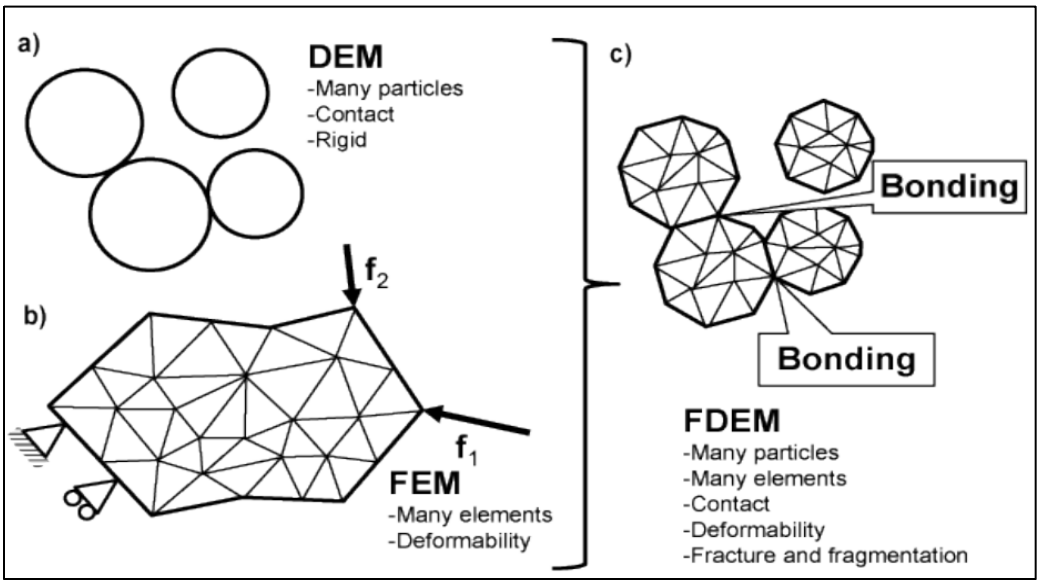

Fig. 11. Illustration of the main feature of the combined finite element method and distinct element method (Fiore et al., 2013).

6.0 Coupled Geomechanics and Fluid Flow

The dynamic coupling of geomechanics and fluid flow is of interest in many areas of science and engineering (Minkoff et al., 2003). Coupling geomechanics and fluid flow have received a great deal of attention in civil and geotechnical engineering for many years (Majorana et al., 2015; McCartney et al., 2016). As a result of the shrinkage and extension of structural deformation, coupled structural mechanics and heat flow also have been investigated in structural science and mechanical engineering (Baran et al., 2017).

In the petroleum engineering discipline, coupled fluid flow and geomechanical interactions can play a major role in dictating the behaviour of fluid flow in fractures and tight rock reservoirs. The characterisation of reservoir geomechanics can play a critical role in enhanced oil recovery (Chiaramonte et al., 2011; Guy et al., 2012), subsidence, stability of wells (Fjær et al., 2008), stress-dependent porosity, the permeability of rock matrix and stress-dependent fracture aperture change (Cao et al., 2019; Mohiuddin et al., 2000; Sanaee et al., 2012) in fractured and unconventional reservoirs. Wellbore stability problems have been the subject of several studies due to the vicinity changes of the stress and strain around the wellbore (Zoback, 2007). Reservoir compaction may lead to serious damage in the wellbore during surface subsidence as a result of reservoir pressure depletion, but can also increase oil recovery and the slow decline in reservoir pressure while production takes place (Muggeridge et al., 2014; Pettersen, 2010).

6.1 Coupling Schemes

Several coupling schemes have been used to model geomechanics and fluid flow interactions (Ahmed and Al-Jawad, 2020; Curnow and Tutuncu, 2015; Doster and Nordbotten, 2015; Jing, 2003; Kim et al., 2013; Lavrov, 2017; Longuemare et al., 2002; Rutqvist and Stephansson, 2003; Sanaee et al., 2013; Weishaupt et al., 2019; Xiong et al., 2011; Zhao et al., 2017). Coupling methods are generally divided into two different categories: volume coupling and coupling through flow properties (Settari and Mourits, 1998). Volume coupling requires the same pore volume changes in reservoir geomechanics and flow models which are the functions of stress, pressure, and temperature (Lee and Schechter 2015). In the coupling through flow properties, permeability and relative permeability are varied as a result of changes in stress and displacements (Settari and Mourits, 1998).

Furthermore, methods of coupling between geomechanics (solid deformation) and reservoir flow, in mathematical terms, are generally categorized into four types: one-way coupling, two-way coupling, iteratively coupling and fully coupling (Tran et al., 2004).

1. One-way coupling: In one-way coupling schemes, two separate essential sets of equations are solved independently for fluid flow and geomechanical deformation over the same time period (Minkoff et al., 2003). This type of coupling is also called explicit coupling because the data is merely conveyed in a one-way direction from a fluid flow simulator to a geomechanical model. This means that changes in pore pressure lead to changes in stress, strains, and displacements, but changes in stress and strain do not impact the pore

pressure changes (Tran et al., 2004). Although there is the weakest link between geomechanical deformation and fluid flow (Tran et al., 2004), one-way coupling has provided valuable insight into a physical situation, and it is obviously desirable for fluid flow alone, where the mechanical situation is important (Minkoff et al., 2003). For instance, a one-way coupling experiment was employed successfully that involved 200 fluid flow simulations to predict well failure rates in the Belridge Field, California (Fredrich et al., 2000, 1996). Although the numerical modelling of one-way coupling is simple and does not require the magnificent development for the modelling of field scale, the one-way coupling is less desirable from the physical point of view compared to the other types of coupling (Fredrich et al., 2000).

2. Two-way coupling: The fundamental concept of two-way coupling is an extension of one-way coupling and sometimes called loose coupling or pseudo-coupling (Chin et al., 2002; Tran et al., 2004). In two-way coupling, reservoir geomechanics and fluid flow simulators are run sequentially (Jin et al., 2000) based on two distinct sets of equations that are solved independently, and the information is conveyed in both directions between the two simulators (Minkoff et al., 2003). Two-way coupling is relatively as simple as one-way coupling, but its main advantage is the fact that it captures much more non-linear physics between geomechanical and reservoir flow simulators, very close to fully coupling (Dubinya et al., 2015; Kim et al., 2012). The primary drawback of explicit coupled schemes (one-way and two-way coupling) is that the explicit nature of the coupling can enforce time step restrictions on runs because of

concerns about stability and accuracy (Dean et al., 2006). Fig.12 defines how distinct numerical coupling schemes work.

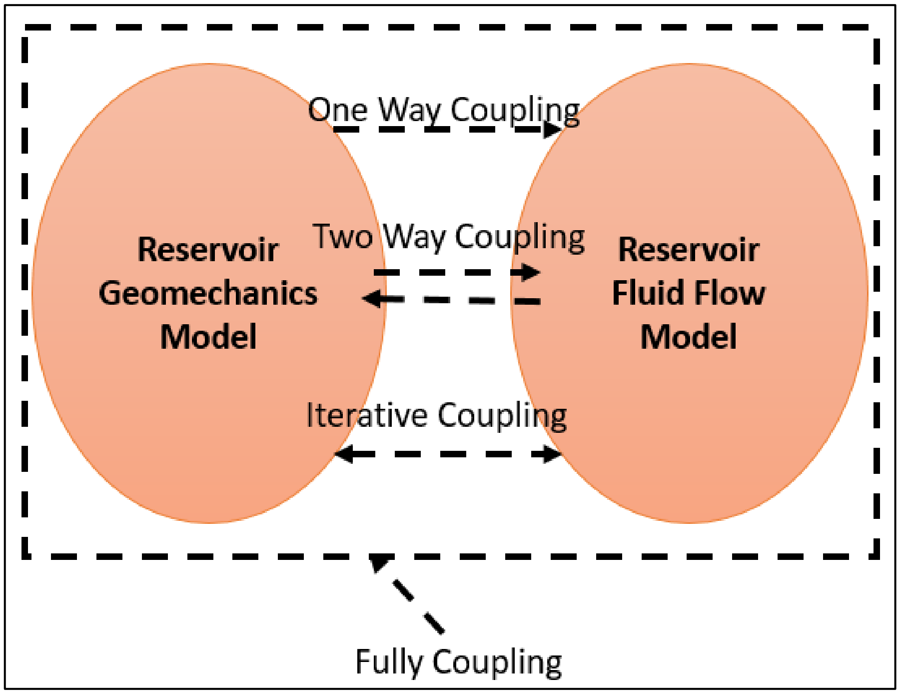

Fig. 12. Representation of various types of simulation coupling schemes.

3. Iterative coupling: In this type of coupling, either reservoir flow variables or geomechanics variables are solved first, and then the other variables are solved sequentially at each time step iteration (Lee and Schechter 2015; Tran, Settari, and Nghiem 2004). The governing equations of the flow simulator and reservoir geomechanics model subsystems are decomposed by a coupled system of equations. While the reservoir flow simulator and geomechanics model solve their governing subsystems of equations separately, the coupled system of equations is solved iteratively using data variables between both subsystems as shown on the lower part of Fig.13, through a data exchange interface (Chin et al., 2002). The main advantages of iterative coupling is the privilege that they are easy to implement between an existing reservoir flow

simulator and geomechanics model through a data exchange interface (Mikelić et al., 2014). The primary shortcoming to the iteratively coupling scheme is the display of the first-order convergence rate calculation in the nonlinear iterations and then may require a huge number of iterations for complex issues (Cervera et al., 1996; Dean et al., 2006).

4. Fully coupling: the governing equations of reservoir flow variables (such as saturation, pressure, and temperature) and the geomechanical response (such as displacements) are solved simultaneously (Charoenwongsa et al., 2010; Giani et al., 2018; Pan et al., 2009, 2007; Settari and Walters, 2001; Stone et al., 2000). This coupling scheme is sometimes referred to as an implicit coupling, because the whole system is calculated simultaneously and can be discretized on one grid domain (Tran et al., 2004), as depicted on the upper part of Fig. 13. The advantage of a fully coupling approach is internal consistency and the accuracy of the solution (Giani et al. 2018; Yang, Moridis, and Blasingame 2014). Another advantage of the fully coupling approach is the stability and also conservation of the second-order convergence for non-linear iterations. However, the coupling of flow and geomechanics conservative relations are complicated to adopt and modelling of the fully coupling multiphase flow simulator is extraordinarily difficult to models of inelastic mechanical deformation and nonlinear (Minkoff et al., 2003; Osorio and Chen, 1999). In addition, the fully coupling approach requires more code development techniques, becomes slower than other approaches, and requires utilisation of iterative methods in some situations (Rocca, 2009).

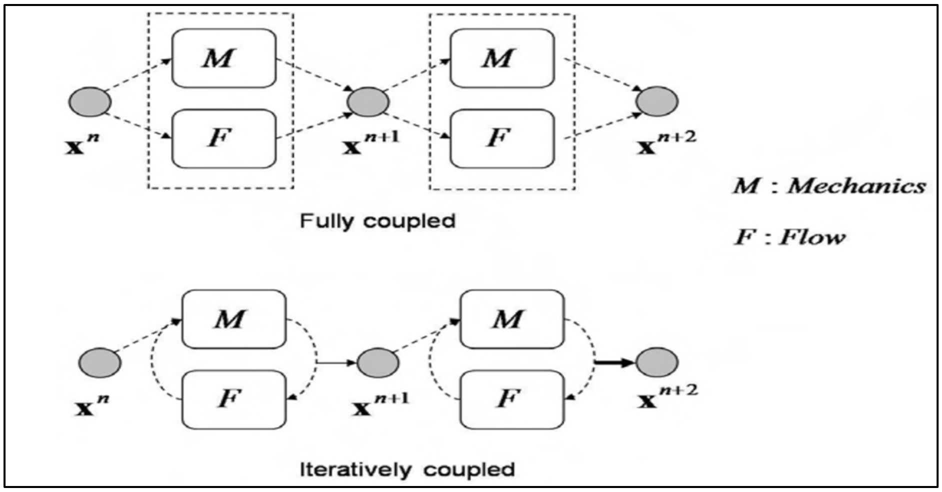

Fig. 13. Schematics of fully and iterative coupling methods (Kim, 2010).

6.2 Numerical Discretization

Discretization is used to transfer the mathematical model into an algebraic equation system or numerical model (Liu, 2018). The main principle of discretization is to divide the whole domain of the reservoir into a number of discrete subdomains (elements and control volumes), each of which is represented by a discrete number of points (grids, nodes) (Nordbotten et al., 2019). Subsequently, the objectives of discretization is to convert partial differential equations into a set of algebraic equations which are valid at each of these discrete points (grid points, nodes), followed by solving the system of the algebraic equation to determine the values of the dependent variables at each of the discrete points that cannot be solved analytically (BinZubair et al., 2010).

Several distinct temporal and spatial discretization methods are employed in the literature for coupling reservoir geomechanics and fluid flow in fractured porous media. The first order differential equation method is applied for temporal discretization, while various distinct methods are employed for spatial discretization, such as the finite difference method (FDM), finite element method (FEM), finite volume method (FVM), and so on.

a) Review of Reservoir Geomechanics Discretization

In fractured geomechanical modelling, the finite element methods are widely employed by nodal enrichment for the discontinuous displacement modes (Ren et al., 2018). Although finite difference methods are often applied to special cases of one-dimensional problems, such as the simplified geometries of crystal layers, finite element methods are more applicable to a multidimensional differential equation with complicated geometries (Saikia et al., 2018). To capture displacement discontinuity jumps in the fractured porous rocks, finite element methods are enhanced with some local degree of freedom, the outcome of which is referred to as the embedded finite element method (EFEM). Alternatively, the hypothesis of partition unity is employed in the extended finite element method (XFEM), with nodal enrichment used to capture the various feature of fractured porous media discontinuity (Armero and Linder, 2008). Oliver, Huespe, and Sánchez (2006) (Oliver et al., 2006) compared EFEM and XFEM, with an emphasis on accuracy and computational cost. However, it is demonstrated that both approaches present the same quantitative and qualitative results for enough refined meshes. In general, EFEM showed higher accuracy and relatively lower cost than XFEM (Oliver et al., 2006).

b) Review of Fluid Flow Discretization

Extended finite element methods (XFEM) are also used to solve the single and two-phase flow in the fractured reservoirs and is less recommended to model coupled fluid flow and geomechanics compared to other methods (Boon et al., 2018; Fumagalli and Scotti, 2013). Despite their simplicity, finite difference methods (FDM) are also applied to solve fluid flow in fractured reservoirs with the aid of other discretization methods (Antonietti et al., 2016). In addition, the finite volume methods are used to capture the degree of freedom of more physical unknowns that may exist in the centre of the matrix blocks, and also to capture fracture discontinuities for multiphase flow, based on the discrete fracture matrix models (Stefansson et al., 2018). The main advantages of finite volume methods are that they preserve the locality of mass conservation as well as computational efficiency and flexibility (Ahn, 2019; Profito et al., 2015; X. Yang et al., 2020).

c) Review of Coupled Reservoir Geomechanics and Fluid Flow Discretization

Extensive amount of research has focused on implementing various discretization schemes based on the length scale of the problems and coupling schemes (Chin et al., 2002; Duran et al., 2020; Garipov et al., 2016; Garipov and Hui, 2018; Jalali and Dusseault, 2012; Minkoff et al., 2003) as listed in Table 2. There is a lack of physical control in the fracture subdomain due to the use of the Darcy flow equation. Researchers (Ren et al., 2018) also highlighted the coupling between reservoir geomechanics and two-phase flow using the extended element method (XFEM) and finite volume (FVM) discretization based on the discrete fracture matrix model. It is noted that no literature has been published on coupling reservoir geomechanics using FEM, and multiphase flow using FVM, for non-Darcy flow equations in the fracture subdomain though.

Table 2: Comparison of distinct coupling and discretization schemes.

Type of Coupling	Fluid Phases	Discretization Method		Flow Domain
		Flow	Geomechanics	
Two-way coupling (Minkoff et al., 2003)	Two Phase	FEM	FEM	Full Domain
Fully coupling (Garipov et al., 2016)	Single Phase	FVM	FEM	Full Domain
Fully Coupling (Yang, Moridis, and Blasingame 2014)	Two Phase	FEM	FEM	Full domain
Iterative Coupling (Chin et al., 2002)	Two Phase	XFEM	XFEM	Full Domain
Iterative Coupling (Duran et al., 2020)	Single Phase	FEM	FEM	Full Domain

7.0 Concluding Remarks

In this review, the most recent mathematical, numerical, and discretization approaches for reservoir geomechanics and fluid flow are investigated. Various types of coupling schemes are described and compared, related to their application, computational efficiency, and cost. The available literature has shown, in terms of modelling and simulation, there is mature research in the field for the single-phase flow in fractured porous media. The remaining challenges for fluid flow are the suitable selection of the conceptual models for the flow governing equations, and the necessity of the upscaling procedure and discretization in the fracture network. This is particularly important in capturing the fluid properties in the fractures and the surrounded porous matrix. In contrast, if the local fracture networks play as the dominant flow path in the area of interest, the fractures must be represented explicitly; in such cases, DFN and DFM are the most preferable methods to capture the flow characteristics in the fracture and the surrounded porous media. The dual continuum approach is conceptually simpler and computationally much less demanding than the DFN and DFM approaches (Kumar et al., 2016). In addition, modelling purposes and the quality of the data in the specific application are major influencing parameters used to select the proper modelling schemes.

Multiphase flow occasionally occurs in many applications including the fractured porous media. This complex type of flow is governed by many physical parameters, such as saturation, wettability, relative permeability, and capillary pressure, and subsequently causes non-linearly coupled flow. The experimental and numerical multiphase flow data in fractured porous media context is exceptionally sparse for the constitutive relations of relative permeability and capillary pressure, compared to the non-fractured porous media. Nonetheless, significant signs of progress have been achieved to model multiphase flow in the fractured porous media (Helmig et al., 2013; Sabti et al., 2016) since the first multiphase experimental (Kazemi and Merrill, 1979) and numerical modelling work (Wu 2016). In addition, as a result of having distinct relative permeability and capillary pressure in the constitutive relations at the fracture and matrix interface in the fractured porous media, these strong heterogeneities in the capillary barriers and resistance of flow leads to the main numerical challenges (Antonietti et al., 2016; Brenner et al., 2018). Despite several studies, modelling of multiphase flow is still an active area of research in fractured porous media, in terms of the conceptual and constitutive relations of multiphase flow, conventional mathematics and numerical representation of multiphase Darcy law in fractured rocks (Berre et al., 2019).

There is a strong relationship between fractured flow patterns and fracture structure itself. As a result of fluid flow processes, the fracture characteristic configurations change, and then the fracture deformations affect the fluid flow indirectly in the fractured porous media. Therefore, several kinds of coupling schemes have been used to couple reservoir geomechanics characteristics and fluid flow in the fractured porous media (Jiang and Yang, 2018; Rutqvist and

Stephansson, 2003; Xue et al., 2014). One-way coupling is widely used for subsidence analysis, because it is more straight forward and less expensive, while it has a lower degree of coupling compared to other coupling methods (Angus et al., 2015; Giani et al., 2018). Two-way coupling is far more appropriate than one-way coupling to predict flow behaviour in fractured reservoirs (Kim et al. 2012). Despite a high resolution of results and convergence, a fully coupled method requires a unified hydromechanical simulator to provide sufficient definition for both reservoir geomechanics and fluid flow compared to iterative coupling scheme (Ashworth and Doster, 2019; Zareidarmiyan et al., 2018). In addition, the fully coupled scheme requires enormous software development efforts and large computational costs (Kim et al., 2013) whereas the iterative coupling scheme combines the conventional reservoir geomechanics and fluid flow simulators through a data exchange interface (Chin et al., 2002). Despite several research efforts (Ashworth and Doster, 2019; Martin et al., 2017; Weishaupt et al., 2019), coupled geomechanics and multiphase flow is still a challenging issue related to mathematical, numerical models and discretization schemes to capture the hydrodynamic behaviours, such as fracture deformation and fluid flow interaction at fracture matrix interface in naturally fractured reservoirs.

Nomenclature and units

P = Effective confining pressure, $MPa = \frac{N}{m^2}$

p = Pore pressure, MPa

σ_n = Effective normal stress, MPa

K = Bulk modulus of the material, MPa

ε_v = Volumetric strain

u = Velocity, m/s

G = Shear modulus of the material, GPa

v = Darcy velocity, m/s

k = Effective permeability of the rock, m^2

μ = Viscosity of the fluid, $Pa.s$

ρ = Density of the fluid, kg/m^3

g = gravitational acceleration, m/s^2

D = Depth of the datum, m

φ = Porosity of the rock, $fraction$

q = Source term, $kg/m^2.s$

k_a= Absolute permeability, m^2

k_r= Relative permeability, $fraction$

β = Subscript of the phases (β= g for gas, β= w for water, and β=o for oil)

S = Saturation of the fluid, $fraction$

P_c= Capillary pressure, Pa

P_w= Pressure of wetting fluid, Pa

P_n= Pressure of non-wetting fluid, Pa

λ = Mobility of the fluid, $dimensionless$

c = Consolidation coefficient

l = Length of the sample, m

σ = Total stress, MPa

σ' = Effective stress, MPa

δ = Kronecker delta

α = Biot Willis Coefficient, $fraction$

K_d = Drained bulk moduli of the material, MPa

K_s = Solid bulk moduli of the material, MPa

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