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Fuzzy adaptive singular value decomposition cubature Kalman filtering algorithm for Lithium-ion battery state-of-charge estimation

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Abstract: To solve the problem of the slow convergence speed for the battery state-of-charge estimation of cubature Kalman filter algorithm, the ternary lithium-ion battery is taken as the research object, and an algorithm combining the fuzzy self-adaptation and singular value decomposition cubature Kalman filtering is proposed. The algorithm takes the system innovation and its change rate as the fuzzy input, and the output as the adjustment factor, which is used to adjust the process noise covariance matrix R . The Kalman gain is adjusted through the fuzzy control of R . To ensure the stability of the algorithm in the calculation process, the singular value decomposition is applied to cubature Kalman algorithm. Then, a second-order RC equivalent circuit model with double internal resistance is built and tested under different conditions to verify the rationality of the improved algorithm. The verification results show that under the simple condition, the convergence speed of the proposed algorithm in the different initial state-of-charge values increased by 40.00% and 25.00%, respectively, the maximum estimation error of the state-of-charge is 2.52% and 2.51%, the Mean Absolute Error is 0.816 and 0.880%, the Root Mean Square Error is respectively 1.276% and 1.380. When the initial state-of-charge value is 0.8, the convergence speed in the complex condition is increased by about 30.00%, the maximum estimation result error, Mean Absolute Error and Root Mean Square Error are 2.21%, 0.222%, and 1.327%, respectively. When the initial state-of-charge value is 0.6, the convergence speed in the complex condition is increased by about 10.00%, the maximum estimation result error, Mean Absolute Error, and Root Mean Square Error are 2.72%, 0.941%, and 1.327%, respectively. Without reducing the estimation accuracy, the improved algorithm can significantly increase the convergence speed of predictive value tracking, which provides a theoretical basis for the wide application of lithium-ion batteries.

Keywords: lithium-ion battery; state-of-charge; convergence speed; fuzzy adaptive algorithm; singular value decomposition; double internal resistance

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1. Introduction

With the rapid economic development, the problems of resource shortages and environmental pollution are increasingly serious [1-4], the green energy and environmental protection development have become common concerns of all countries in the world [5-8]. Due to the advantages of high energy density, long service life, and low environmental pollution, lithium-ion batteries are widely used in new energy vehicles, special robots, and

1
2 aerospace fields. With the continuous expansion of its application range, the research on battery management
3 systems (BMS) has received increasing attention for the reliable operation of lithium-ion batteries [9-13]. In the
4 BMS, the accurate estimation of the state-of-charge (SOC) is very important, which is a key factor for safe
5 charging-discharging and accurate life estimation of the battery pack [14-17].
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10 Lithium-ion batteries mostly work in complex environments [18-22], these working environments and sensor
11 measurement errors will increase the difficulty of accurate SOC estimation [23-27]. As a state variable, SOC cannot
12 be directly measured by sensors, but can only be obtained through the calculation of related parameters. The
13 methods for SOC estimation can be roughly divided into ampere-hour (Ah) integration, open-circuit voltage
14 (OCV) method, model-based estimation method, and the way based on data-driven. The Ah integration is simple
15 in principle and easy to implement, but as an open-loop estimation system, it cannot realize the correction of the
16 initial value, and the error will continue to accumulate. Common improvement methods for the Ah algorithm
17 include the combination of Ah integration and adaptive extended Kalman filter (EKF) [28]. In this method, the
18 adaptive EKF algorithm is used to obtain the initial SOC value, and switch to the Ah algorithm for SOC
19 estimation when the estimated value converges. If the estimation error reaches the set value, it switches to the
20 adaptive EKF algorithm for SOC estimation again. This algorithm effectively avoids the error accumulation of
21 Ah and the correction of the initial error, which reduces the estimation time of the adaptive EKF algorithm,
22 obtains a higher SOC estimation accuracy, and has strong practicability. The OCV method estimates the SOC
23 value through the function curve of the OCV and SOC value, however, it is difficult to obtain the OCV value of
24 the lithium-ion battery in actual working conditions, so it is impossible to directly apply this method to
25 engineering practice. Wang et al. proposed a fusion OCV estimation method [29], which obtains the fused OCV
26 data through the first-order backward differential integration of the small current test results, and fits the data
27 through a neural network to obtain the OCV-SOC curve. This method reduces the influence of the battery
28 polarization effect and has a high SOC estimation accuracy.
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47 There are many model-based estimation methods, and the most widely used equivalent circuit models are
48 Thevenin equivalent model, second-order RC equivalent model, etc. After the equivalent model of the battery is
49 established, parameter identification is performed online or offline, and then, related algorithms are used to
50 estimate the SOC. Jiang Cong et al. proposed an adaptive square root extended Kalman SOC estimation method
51 based on the Thevenin model [30]. This method combines the adaptive algorithm with the square root algorithm,
52 which improves the accuracy of SOC estimation while maintaining the stability of the calculation process, and it
53 has good results at different temperatures. Peng N et al. proposed a SOC estimation method based on online
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parameter identification and improved adaptive dual unscented Kalman filter (ADUKF) algorithm^[31], in which the singular value decomposition (SVD) algorithm is used to solve the problem of divergence of the noise covariance matrix, then, two UKF algorithms are used to realize the co-estimation of parameters and SOC, while an adaptive algorithm is adopted to improve the estimation accuracy. The verification results show that the SOC estimation accuracy of this method is much higher than that of the ordinary DUKF. The data-driven SOC estimation method does not rely on an accurate battery model, it has great advantages when the system model is difficult to describe. Common data-driven methods include artificial neural network (ANN), support vector machine (SVM), and extreme learning machine (ELM). For example, Ma, L et al. proposed a method for co-estimation of SOC and state-of-energy (SOE) based on long short term memory (LSTM) neural network^[32], in which good estimation results can be obtained under different working conditions.

Aiming at the goal of shortening the convergence time of the cubature Kalman filtering algorithm (CKF), an algorithm that combines the fuzzy adaptive algorithm and SVD method with CKF is proposed to estimate the lithium-ion battery SOC, and a second-order RC model with double internal resistance is established. On the premise of ensuring the stability of the algorithm and accuracy of the estimation, this method shortens the convergence time of the SOC estimation. In addition, the application of double internal resistance can significantly improve the accuracy of the second-order RC model.

2. Mathematical analysis

2.1. Second-order RC equivalent circuit with double internal resistance modeling

A reasonable battery equivalent model is the basis for accurate SOC estimation. Common lithium-ion battery equivalent models include Thevenin equivalent model, PNGV model, and second-order RC model^[33-38]. Taking into account the requirements of battery simulation accuracy and the principle of minimizing the amount of calculation, a second-order RC equivalent model is selected to perform equivalent simulations of lithium-ion batteries. For the internal resistance of the battery is different during charging and discharging, the internal resistance of the battery for charging and discharging is calculated separately to form a double internal resistance second-order RC model^[39-41]. The equivalent circuit is shown in Figure 1.

[Insert Figure 1]

Figure 1. The second-order RC equivalent model of lithium-ion battery with double internal resistance

In Figure 1, U_{OC} represents the open-circuit voltage, R_1 and R_2 are polarization resistances, C_1 and C_2 are polarization capacitances, the R_1C_1 loop is used for the concentration polarization phenomenon of the equivalent circuit, and to characterize the process of rapid voltage changes when the current changes suddenly. The R_2C_2 loop is used for the electrochemical polarization phenomenon of the equivalent circuit, and to characterize the

process of slow voltage changes. U_L is the terminal voltage of the circuit, $I(t)$ is the internal current of the battery, R_{01} and R_{02} represent the internal resistance of the battery during discharging and charging, respectively. Taking the discharge direction as the reference, the KVL equations are shown in Equation (1).

$$\begin{cases} U_L = U_{oc} - U_0 - U_1 - U_2 \\ \frac{dU_1}{dt} = \frac{I}{C_1} - \frac{U_1}{R_1 C_1} \\ \frac{dU_2}{dt} = \frac{I}{C_2} - \frac{U_2}{R_2 C_2} \end{cases} \Leftrightarrow \begin{cases} U_0 = IR_{01}, (\text{discharge}) \\ U_0 = -IR_{02}, (\text{charge}) \end{cases} \quad (1)$$

In Equation (1), U_1 and U_2 represent the voltage across $R_1 C_1$ and $R_2 C_2$, respectively. Taking the state variable of the circuit as $x = [SOC, U_1, U_2]^T$, and the terminal voltage U_L as the observation variable, the state equation and observation equation of the model can be obtained as shown in Equation (2).

$$\begin{cases} \begin{bmatrix} SOC_{k+1} \\ U_{1,k+1} \\ U_{2,k+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-\Delta t/\tau_1} & 0 \\ 0 & 0 & e^{-\Delta t/\tau_2} \end{bmatrix} \begin{bmatrix} SOC_k \\ U_{1,k} \\ U_{2,k} \end{bmatrix} + \begin{bmatrix} -\frac{\eta \Delta t}{Q_N} \\ R_1 \left(1 - e^{-\Delta t/\tau_1}\right) \\ R_2 \left(1 - e^{-\Delta t/\tau_2}\right) \end{bmatrix} I_k + w_k \Leftrightarrow \begin{cases} \tau_1 = R_1 C_1 \\ \tau_2 = R_2 C_2 \end{cases} \\ U_{L,k+1} = \begin{bmatrix} \frac{\partial U_{oc}}{\partial SOC} & -1 & -1 \end{bmatrix} \begin{bmatrix} SOC_k \\ U_{1,k} \\ U_{2,k} \end{bmatrix} - U_0 + v_k \Leftrightarrow \begin{cases} U_0 = IR_{01}, (\text{discharge}) \\ U_0 = -IR_{02}, (\text{charge}) \end{cases} \end{cases} \quad (2)$$

In Equation (2), Q_N is the actual capacity of the battery, Δt is the sampling time, τ is the time constant, w_k and v_k respectively represent the system observation error and measurement error, η is the Coulomb efficiency, which is generally equal to one.

2.2. Parameter identification of the equivalent model of lithium-ion battery

High-precision model parameters are the prerequisite for accurate lithium-ion batteries SOC estimation [42, 43]. With the method of curve fitting, the Hybrid Pulse Power Characteristic (HPPC) condition data is selected for model parameter identification. Due to the inconsistency of model parameters in the charging and discharging process, to improve the accuracy of the model, it is necessary to identify the two processes separately. Taking the parameter identification of the discharge process as an example, the voltage and current curves of one cycle for HPPC condition are shown in Figure 2.

[Insert Figure 2]

Figure 2. The current and voltage curve of the HPPC working condition

When the battery is charged and discharged in the HPPC condition, the instantaneous change of current will cause the change of the internal resistance terminal voltage of the battery, that is, the sudden changes in the voltage curve of the AB and CD segments in Figure 2 is caused by the internal resistance of the battery. To improve the accuracy, the internal resistance of the two voltages is calculated separately, and the average value is taken as the final result. The calculation process is shown in Equation (3).

$$R_{01} = \frac{U_A - U_B + U_D - U_C}{2I} \quad (3)$$

The DE part in Figure 2 shows the discharging process of the polarizing capacitor to the polarizing resistor, and $I(t)$ is equal to zero, which is a zero-input response. For a simplified calculation, this curve is used for fitting to obtain the relevant parameter values. The response function of DE segment and its simplified expression are shown in Equation (4).

$$\begin{cases} U_L = U_{OC} - IR_1 e^{-\frac{t}{\tau_1}} - IR_2 e^{-\frac{t}{\tau_2}} \\ U_L = U_{OC} - ae^{-bt} - ce^{-dt} \end{cases} \quad (4)$$

According to the corresponding relationship between the parameters and coefficients in Equation (4), the relevant parameters of the model can be obtained. Taking into account the hysteresis voltage characteristics of lithium-ion batteries, the open-circuit voltage during the charging and discharge process are respectively fitted and averaged. The open-circuit voltage curves are shown in Figure 3.

[Insert Figure 3]

Figure 3. The fitting curve of the U_{OC} -SOC

In Figure 3, U_{OC1} is the OCV curve of the charging process, U_{OC2} is that of the discharging process, and U_{OC3} is the mean voltage fitting curve of the charging and discharging. It can be seen from the figure that the difference in OCV between charge and discharge is small, so the average value of the two curves is selected as the final fitting result.

2.3. Fuzzy adaptive algorithm

Lithium-ion batteries mostly work in complex environmental conditions. For the influence of environmental factors, the statistical characteristics of the observation noise will keep changing, which leads to the continuous change of the observation noise covariance matrix R [44-47]. To improve the convergence speed of the CKF, a fuzzy adaptive algorithm is introduced. The innovation e_k and its rate of change Δe_k are selected as the input of the fuzzy controller, and the adjustment factor α of R as the output. The input expressions of the system are shown in Equation (5).

$$\begin{cases} e_k = y_k - y_k^* \\ \mathbf{V}e_k = e_k - e_{k-1} \end{cases} \quad (5)$$

In Equation (5), y_k represents the measured value of the terminal voltage, y_k^* represents the estimated value of the terminal voltage. In the fuzzy adaptive system, under ideal conditions, the system innovation is zero, when the innovation and its rate of change are large, the output value will increase, and thereby the Kalman gain is reduced. When the innovation and its rate of change are small, the output value decreases, making the Kalman gain larger and increasing the proportion of the measured value, to achieve the purpose of making the algorithm stable and converging quickly. The theoretical residual covariance matrix expression after introducing the adjustment factor is shown in Equation (6).

$$P_{zz,k|k-1} = \frac{1}{2n} \sum_{i=1}^{2n} (z_{i,k|k-1} z_{i,k|k-1}^T - \hat{z}_{i,k|k-1} \hat{z}_{i,k|k-1}^T) + \alpha R \quad (6)$$

It is assumed that the innovation interval is $[-0.04, 0.04]$, and divide it into six segments: very small, small, medium, big, and very big. The domain of innovation change rate is set as $[-0.035, 0.035]$, which is divided as small, medium, big, and very big. The membership functions of e_k and Δe_k are shown in Figure 4.

[Insert Figure 4(a)]

(a) The membership function graph of e_k

[Insert Figure 4(b)]

(b) The membership function graph of Δe_k

Figure 4. The membership function graph of the input variable

Assuming that the output domain of the fuzzy system is $[0.2, 2]$, the output of the fuzzy system is divided into five intervals of very small, small, medium, big, and very big according to the actual situation, the domain of α is shown in Figure 5.

[Insert Figure 5]

Figure 5. The membership function graph of the output variable

Reasonable fuzzy rules can reduce the amount of calculation [48, 49]. The larger the number of fuzzy rules, the greater the amount of calculation, considering the trade-off between computational complexity and accuracy, the fuzzy rules are established as shown in Table 1.

[Insert Table 1]

Table 1. The table of fuzzy rule

A logical defuzzification method is very important for the fuzzy control system. Common methods include the membership average method, the area centroid method, and the area equal division method [50]. To obtain good output results, the area centroid method is chosen for defuzzification. The fuzzy input-output surface is shown in

Figure 6.

[Insert Figure 6]

Figure 6. The graph of fuzzy rules

2.4. Fuzzy adaptive singular value decomposition cubature Kalman filter algorithm

For the state covariance matrix is prone to be non-positive definite when calculates the cubature point in the CKF algorithm, the SVD method is introduced [51, 52]. This method does not need to meet the condition that the covariance matrix must be positive definite, which can make the algorithm more stable.

The SOC estimation based on fuzzy adaptive SVD-CKF (FASVD-CKF) can be divided into three parts, namely initialization, time update, and measurement update. The initial state variables and value of the error covariance matrix are defined as shown in Equation (7).

$$\begin{cases} \hat{x}_{0|0} = E(x_0) \\ P_0 = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T] \end{cases} \quad (7)$$

In Equation (7), E represents the expectation of the variable. The time update process includes the prediction of state and error covariance matrix through the cubature point calculation. The calculation process is shown in Equation (8).

$$\begin{cases} P_{k-1|k-1} = U_{k-1} S_{k-1} V_{k-1}^T \\ x_{i,k-1|k-1} = U_{k-1} \sqrt{S_i} \xi_i + x_{k-1|k-1} \\ x_{i,k|k-1}^* = f(x_{i,k-1|k-1}, u_{k-1}) \\ \hat{x}_{k|k-1} = \frac{1}{2n} \sum_{i=1}^{2n} x_{k|k-1}^* \\ P_{k|k-1} = \frac{1}{2n} \sum_{i=1}^{2n} x_{k|k-1}^* x_{k|k-1}^{*T} - \hat{x}_{k|k-1} \hat{x}_{k|k-1}^T \end{cases} \quad (8)$$

The measurement update process mainly includes the calculation of auto-covariance and cross-covariance matrices and obtains the Kalman gain, state variable estimates, and state estimation error covariance matrix. The calculations of the self-covariance and the cross-covariance matrix are shown in Equation (9).

$$\begin{cases} P_{k|k-1} = U_{k-1}^* S_{k-1}^* (V_{k-1}^*)^T \\ x_{i,k|k-1} = U_{k-1}^* \sqrt{S_{k-1}^*} \xi_i + \hat{x}_{k|k-1} \\ P_{zz,k|k-1} = \frac{1}{2n} \sum_{i=1}^{2n} (z_{i,k|k-1} z_{i,k|k-1}^T - \hat{z}_{i,k|k-1} \hat{z}_{i,k|k-1}^T) + \alpha R \\ P_{xz,k|k-1} = \frac{1}{2n} \sum_{i=1}^{2n} (x_{i,k|k-1} z_{i,k|k-1}^T - \hat{x}_{i,k|k-1} \hat{z}_{i,k|k-1}^T) \end{cases} \Leftarrow \begin{cases} z_{k|k-1} = h(x_{k|k-1}) \\ \hat{z}_{k|k-1} = \frac{1}{2n} \sum_1^{2n} z_{k|k-1} \end{cases} \quad (9)$$

The Kalman gain can be calculated by the auto-covariance and cross-covariance matrices. The expressions of Kalman gain, state variable estimated value, and state estimation error covariance matrix are shown in Equation (10).

$$\begin{cases} K_k = P_{xz,k|k-1} / P_{zz,k|k-1} \\ \hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (z_k - \hat{z}_{k|k-1}) \\ P_{k|k} = P_{k|k-1} - K_k P_{zz,k|k-1} K_k^T \end{cases} \quad (10)$$

Equations (7) to (10) are a complete SOC estimation process. At this point, the state estimation at time k is completed, and the time point $k=k+1$ is set to realize the state estimation at the next time. The flow chart of SOC estimation based on FASVD-CKF is shown in Figure 7.

[Insert Figure 7]

Figure 7. The flow chart of FASVD-CKF

Based on the cubature criterion, the CKF algorithm approximates the state mean and covariance of the nonlinear system through $2n$ cubature points with the same weight. This can avoid the process of linearizing the nonlinear system, and theoretically has a higher estimation accuracy than the EKF algorithm. The fuzzy adaptive algorithm improves the convergence speed of the CKF, and the application of the SVD algorithm solves the problem of instability of the state estimation covariance matrix during the operation.

3. Analysis of experimental results

A ternary lithium-ion battery with a rated capacity of 70Ah is taken as the experimental object. To obtain relevant experimental data, an experimental platform is built with the BTS200-100-104 battery test device and temperature control equipment. The platform is shown in Figure 8.

[Insert Figure 8]

Figure 8. Experimental platform

All data are carried out at a constant temperature of 35°C. Measured with the cyclic discharge experiment, the actual capacity of the experimental battery is 68.27Ah.

3.1 Analysis of parameter identification results

The curve fitting method is used to identify the parameters of the equivalent model. The average curve expression of the open-circuit voltage function obtained by the charge and discharge of HPPC test is shown in Equation (11).

$$U_{OC} = 2.269 * SOC^5 - 9.252 * SOC^4 + 13.51 * SOC^3 - 8.073 * SOC^2 + 2.45 * SOC + 3.275 + 0.5 * (-3.011 * SOC^4 + 7.233 * SOC^3 - 5.264 * SOC^2 + 1.915 * SOC + 3.307) \quad (11)$$

Taking into account the differences in model parameters during the charging and discharging process, parameter identification is carried out separately during the two processes. The identification results are shown in Table 2 and Table 3.

Table 2. Model parameters for the discharging process

[Insert Table 2]

Table 3. Model parameters for the charging process

[Insert Table 3]

To simplify the calculation process, R_{01} and R_{02} are the average values under different SOC values during the discharge and charge process, respectively. For R_1 , R_2 , C_1 , and C_2 , all charge-discharge data are selected and the average value is taken as the final result. The accuracy of the model is verified under HPPC condition, and the results are shown in Figure 9.

[Insert Figure 9 (a)]

[Insert Figure 9 (b)]

(a) The voltage comparison of HPPC working condition

(b) Identification error of HPPC working condition

Figure 9. Comparison of model voltage effect

In Figure 9(a), UL1 and UL2 represent the actual voltage and analog voltage respectively. It can be seen from Figure 9(b) that the maximum error of the analog voltage is 0.0266V, overall, the error is small and fluctuates in a small range. The adopted method can obtain high-precision model parameters, which lays the foundation for the accurate estimation of the SOC value.

3.2 Analysis of experimental results of DST working condition

To verify the SOC estimation effect of the proposed method, an experimental analysis is carried out under the dynamic stress test (DST) condition. This condition includes charging, discharging, and shelving processes of different lengths of time, which can effectively simulate the actual working conditions of lithium-ion batteries.

The process noise covariance matrix is set as $Q = \text{diag}(10^{-6} \ 10^{-6} \ 10^{-6})$, and the observed noise covariance matrix R is 0.05. The initial value of actual SOC is 1, and the initial SOC value of the other three algorithms is 0.8. The voltage curve of DST and the SOC estimation results of different algorithms are shown in Figure 10.

[Insert Figure 10 (a)]

[Insert Figure 10 (b)]

(a) The voltage curve of DST condition

(b) SOC estimation under DST condition

[Insert Figure 10 (c)]

[Insert Figure 10 (d)]

(c) The error of SOC estimation under DST condition (d) Convergence process of different algorithms under DST condition

Figure 10. The data and SOC estimation results of the DST condition when $SOC_0 = 0.8$

In Figure 10, S_0 represents the SOC reference value, S_1 is the SOC estimation result of the EKF algorithm, S_2 and S_3 are the SOC values estimated by the CKF algorithm and FASVD-CKF algorithm, respectively. $Err_1 \sim Err_3$ are the estimation errors corresponding to S_1 , S_2 , and S_3 . It can be seen from Figure 10 (c) that all three algorithms have large estimation errors at the end of discharge, which is caused by the severe chemical reaction inside the battery. In the stable discharge stage, the accuracy of the estimation results of the CKF and FASVD-CKF algorithms is almost the same, the maximum errors of SOC estimation for them are about 2.52%, and the maximum error for that of the EKF algorithm is about 2.11%. In Figure 10 (d), the convergence time of the FASVD-CKF, CKF, and EKF is about 30 s, 50 s, and 65 s, respectively. Compared with CKF, the convergence time of the improved algorithm is shortened by about 40.00%, indicating that the introduction of the fuzzy adaptive algorithm can improve the convergence speed of the algorithm SOC estimation.

For a more intuitive comparison of the three algorithms SOC estimation results, the Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) of the estimation results are compared and analyzed as shown in Table 4.

Table 4 Comparison of performance indicators of various algorithms under DST operating conditions

[Insert Table 4]

The MAE of the FASVD-CKF algorithm is 0.816%, which is smaller than that of the CKF and EKF algorithms. With a small difference, the RMSE value of FASVD-CKF is between the data of EKF and CKF algorithms. In Table 4 and Figure 10, it can be seen that the FASVD-CKF can significantly improve the convergence speed, and the introduction of the fuzzy adaptive algorithm does not significantly reduce the accuracy of the CKF algorithm. In general, the algorithm has high stability and there is no divergence in the SOC estimation process, which proves that the SVD algorithm can effectively preserve the stability of the improved algorithm.

To further verify the estimation effect of the proposed algorithm under different initial SOC conditions, the initial SOC value is set to 0.7, and other conditions are consistent with the previous ones. The estimation results are shown in Figure 11.

[Insert Figure 11 (a)]

[Insert Figure 11 (b)]

(a) Convergence process of different algorithms under DST condition

(b) The error of SOC estimation under DST condition

Figure 11. The SOC estimation result under DST condition when $SOC_0 = 0.7$

Figure 11(a) shows the convergence process of the three algorithms, and the legend is consistent with Figure 10. In Figure 11(a), the convergence time of S_1 , S_2 is 120 s and 60 s respectively. The time of S_3 is about 45 s, which is about 25% shorter than that of S_2 . It means that the fuzzy adaptive algorithm can still improve the convergence

1
2 speed of the algorithm in different SOC initial values. In Figure 11(b), Err2 and Err3 show a high degree of overlap,
3 the maximum SOC estimation error of the two algorithms is about 2.51% except for the end of discharge, and the
4 maximum error of the EKF is about 2.23%. The MAE and RMSE comparisons of the three algorithms are shown
5 in Table 5.
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10 Table 5. Comparison of performance indicators of various algorithms under DST operating conditions

11 [Insert Table 5]

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13 In Table 5, the MAE of the improved algorithm is 0.880%, which is slightly smaller than that of EKF, but greater
14 than the value of the CKF algorithm, indicating that the estimation error of the FASVD-CKF algorithm is smaller
15 than that of EKF. Compared with CKF, there is almost the same SOC estimation accuracy of FASVD-CKF. The
16 RMSE value of the improved algorithm is 1.380%, which is slightly larger than that of the other two algorithms.
17 The error fluctuation range of FASVD-CKF is larger than that of EKF and CKF.
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23 Compared with the CKF and EKF, the SOC estimation error of the FASVD-CKF algorithm increases slightly,
24 but the final estimation result still has a higher estimation accuracy, so it is feasible to improve the convergence
25 speed of the CKF algorithm SOC estimation by the fuzzy adaptive algorithm.
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29 3.3 Analysis of experimental results of BBDST working condition

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31 Lithium-ion batteries are widely used in many fields such as energy storage, new energy vehicles, and aerospace.
32 To further verify the SOC estimation effect of the FASVD-CKF algorithm under complex working conditions,
33 the experimental verification is carried out under Beijing Bus Dynamic Stress Test (BBDST) condition. It is
34 designed to reduce the power according to the actual operating conditions of the car, which includes 19 working
35 steps such as starting, coasting, braking, and rapid acceleration, so it can effectively simulate the actual working
36 conditions of the battery.
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43 The process noise covariance is set as $Q = \text{diag}(10^{-6} \ 10^{-6} \ 10^{-6})$, the observed noise covariance is $R = 0.05$, and
44 the initial value of SOC in each algorithm is 0.08. The BBDST working condition voltage curve and SOC
45 estimation results are shown in Figure 12.
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48

49 [Insert Figure 12 (a)]

50 (a) The voltage curve of BBDST condition

[Insert Figure 12 (b)]

(b) SOC estimation under BBDST condition

51 [Insert Figure 12 (c)]

52 (c) The error of SOC estimation under BBDST condition

[Insert Figure 12 (d)]

53 (d) Convergence process of different algorithms under BBDST
54 condition

55 Figure 12. SOC estimation results and errors of BBDST experimental when $\text{SOC}_0 = 0.8$

56
57 In Figure 12, S0 represents the actual SOC value, S1 represents the SOC estimation value based on the EKF
58 algorithm, S2 is the SOC estimation value based on the CKF algorithm, S3 is the SOC estimation result based on
59
60

the FASVD-CKF algorithm, and Err1~Err3 are S1~S3 corresponding SOC estimation error, respectively. It can be seen from Figure 12 (b) that the SOC estimation curves of the CKF and the FASVD-CKF algorithm have a higher degree of overlap, and the estimation results of the EKF are slightly different from the two. In Figure 12 (c), Err2 and Err3 are highly consistent in the stable stage of SOC estimation with a maximum error of 2.21%. Overall, the fluctuation range for Err3 is small, while Err1 is somewhat different from the two Errs. For the fierce internal chemical reaction of the battery, at the end of discharge, the accuracy of the SOC estimation is reduced. In Figure 12 (d), the convergence time of S1 and S2 is about 60 s and 50 s, respectively, and the time of S3 is about 35s, which is 30.00% higher than the convergence speed of S2. The convergence speed of S3 is faster than that of the other two algorithms, so the FASVD-CKF algorithm still has good results under complex working conditions. Therefore, the fuzzy adaptive algorithm can improve the convergence speed of the algorithm effectively. To further visually compare the SOC estimation effects of the three algorithms, the MAE and RMSE of the algorithms are compared as shown in Table 6.

Table 6. Comparison of performance indicators of various algorithms under BBDST operating conditions

[Insert Table 6]

In Table 6, the MAE and RMSE values of the FASVD-CKF algorithm are 0.222% and 1.327%, respectively, which are slightly different from the CKF algorithm and smaller than the values of the EKF algorithm. Combining Table 6 with Figure 12, it can be seen that the fuzzy adaptive algorithm can effectively improve the convergence effect of the CKF algorithm without reducing the accuracy of SOC estimation. In the entire estimation process, except for the divergence of the algorithm due to the chemical reaction inside the battery at the end of discharge, the other parts of the curve are relatively stable, indicating that the SVD algorithm can still maintain the stability of the algorithm under complex working conditions. To verify the estimation effect of the FASVD-CKF algorithm under different initial values, the SOC_0 is 0.6, and other parameters remain unchanged, the estimation results are shown in Figure 13.

[Insert Figure 13 (a)]

[Insert Figure 13 (b)]

(a) Convergence process of different algorithms under BBDST condition

(b) The error of SOC estimation under BBDST condition

Figure 13. SOC estimation results under BBDST condition when $SOC_0 = 0.6$

Figure 13 (a) shows the convergence effects of the three algorithms, S_0 is the reference value, S_1 is the SOC estimation result of the EKF algorithm, S_2 , S_3 are the SOC estimation results of the CKF and FASVD-CKF algorithms, respectively. The convergence time of S_1 and S_2 is about 200 s, and the time of S_3 is about 180 s, which means that the convergence speed of the improved algorithm is increased by 10.00%. Compared with S_1 and S_2 , S_3 has a better convergence effect, and the FASVD-CKF still has a good convergence effect when

1
2 estimating the SOC of lithium-ion batteries under complicated working conditions. Figure 13 (b) shows the SOC
3 estimation error curve, and Err1~Err3 are the SOC estimation errors corresponding to S1~S3, respectively. It can
4 be seen from the figure that the error curves of Err2 and Err3 almost overlap, and there is a slight difference
5 between Err1 and the other two errors. The comparison results of MAE and RMSE of the three results are shown
6 in Table 7.
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11 Table 7. Comparison of performance indicators of various algorithms under BBDST operating conditions

12 [Insert Table 7]

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15 It can be seen from Table 7 that there is no significant difference between the estimation effects of CKF and
16 FASVD-CKF algorithms, and the estimation results of both algorithms are better than that of EKF. It is feasible
17 to improve the convergence speed of the algorithm through the fuzzy adaptive algorithm. Except for the part at
18 the end of the discharge, the algorithm stability is relatively high, therefore, the SVD algorithm can effectively
19 ensure the stability of the algorithm.
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25 Compared with the traditional CKF algorithm, the convergence speed of the FASVD-CKF algorithm is
26 improved by the introduction of the fuzzy adaptive algorithm, through the SVD algorithm, the stability of the
27 algorithm is guaranteed. However, in the fuzzy adaptive algorithm, the more fuzzy rules, the greater the
28 computational complexity, which increases the time required for the entire process, and it can be further improved.
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33 4. Conclusions

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35 The real-time accurate estimation of SOC provides an important guarantee for the safe operation of the BMS,
36 and it is also a hot and important topic of research. Aiming at improving the convergence speed of the SOC
37 estimation, the FASVD-CKF algorithm is proposed based on the double internal resistance model, and it is
38 experimentally verified through DST and BBDST conditions under different initial states. The verification results
39 show that it is feasible to construct a fuzzy system that uses innovation and its change rate as input and output
40 coefficients to adjust R to improve the convergence speed of the algorithm. In the DST condition, when the initial
41 SOC value is 0.8, the convergence time of the FASVD-CKF algorithm under DST working condition is 30 s, the
42 maximum estimation error is 2.52%, and the MAE and RMSE are 0.816% and 1.216% respectively. Compared
43 with the estimation results of the CKF algorithm, the convergence speed of the proposed algorithm is increased
44 by about 40.00% while the accuracy remains almost unchanged. When the initial SOC value is 0.7, the verification
45 results are completely consistent with the initial SOC value of 0.8. Under the BBDST condition, when the initial
46 SOC value is 0.8, the convergence time is 35 s, which is 30.00% faster than that of the CKF algorithm. The
47 maximum estimation error, the MAE and RMSE values are 2.21%, 0.222%, and 1.327%, respectively. When the
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2 initial SOC value is 0.6, almost consistent verification results can be obtained. Overall, the accuracy of the
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4 algorithm is high, and the SVD algorithm can effectively ensure the stability of the algorithm. The FASVD-CKF
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6 algorithm has a high convergence speed under the premise of ensuring accuracy, which provides a basis for
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8 optimizing the battery management system, and is of positive significance to the application and promotion of
9
10 lithium-ion batteries.

11 The Fuzzy rules will increase the computational complexity of the algorithm, which can be further optimized.
12
13 In the future, based on this algorithm, a dual-adaptive algorithm will be proposed to reduce the computational
14
15 complexity, and achieve the improvement of state estimation accuracy and convergence speed.

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17
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21
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23
24 Technology (No.17zx7110, 18zx7145).

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Table 1. The table of fuzzy rule

α Δe_k	e_k	VS	S	M	B	VB
S		VS	VS	S	M	M
M		S	S	M	M	B
B		M	M	M	B	VB
VB		M	M	B	VB	VB

Table 2. Model parameters for the discharging process

SOC(1)	$R_{01}(m\Omega)$	$R_1(m\Omega)$	$R_2/m\Omega$	$C_1(F)$	$C_2(F)$
1	1.626	0.062	0.121	22422.135	145931.520
0.9	1.614	0.083	0.130	22007.309	132337.603
0.8	1.610	0.079	0.158	17419.126	100709.370
0.7	1.605	0.094	0.156	24299.792	173727.939
0.6	1.612	0.070	0.137	21131.495	150309.693
0.5	1.610	0.054	0.108	31975.677	163412.341
0.4	1.624	0.043	0.116	29247.483	129855.272
0.3	1.634	0.042	0.115	33843.308	135943.908
0.2	1.654	0.059	0.118	37889.002	163810.323
0.1	1.683	0.097	0.166	21339.936	144377.786

Table 3. Model parameters for the charging process

SOC(1)	$R_{02}(m\Omega)$	$R_1(m\Omega)$	$R_2(m\Omega)$	$C_1(F)$	$C_2(F)$
0.05	1.631	0.055	0.723	12646.866	18922.522
0.15	1.588	0.041	0.650	18367.969	34076.462
0.25	1.550	0.057	0.572	16758.178	47415.830
0.35	1.535	0.064	0.480	6666.147	38222.889
0.45	1.533	0.066	0.571	11434.338	38551.944
0.55	1.525	0.057	0.669	22566.684	33122.077
0.65	1.501	0.079	0.447	9798.706	35546.444
0.75	1.509	0.065	0.720	10410.944	39177.485
0.85	1.501	0.025	0.346	36210.835	27223.755

Table 4 Comparison of performance indicators of various algorithms under DST operating conditions

Algorithms	MAE	RMSE
EKF	0.882%	1.146%
CKF	0.860%	1.340%
FASVD-CKF	0.816%	1.276%

Table 5. Comparison of performance indicators of various algorithms under DST operating conditions

Algorithms	MAE	RMSE
EKF	0.887%	1.181%
CKF	0.864%	1.373%
FASVD-CKF	0.880%	1.380%

Table 6. Comparison of performance indicators of various algorithms under BBDST operating conditions

Algorithms	MAE	RMSE
EKF	0.248%	1.448%
CKF	0.223%	1.326%
FASVD-CKF	0.222%	1.327%

Table 7. Comparison of performance indicators of various algorithms under BBDST operating conditions

Algorithms	MAE	RMSE
EKF	0.966%	1.338%
CKF	0.938%	1.329%
FASVD-CKF	0.941%	1.327%

Figure 1. The second-order RC equivalent model of lithium-ion battery with double internal resistance

Figure 2. The current and voltage curve of the HPPC working condition

Figure 3. The fitting curve of the UOC-SOC

Figure 4. The membership function graph of the input variable

Figure 4 (a). The membership function graph of e_k

Figure 4 (b). The membership function graph of Δe_k

Figure 5. The membership function graph of the output variable

Figure 6. The graph of fuzzy rules

Figure 7. The flow chart of FASVD-CKF

Figure 8. Experimental platform

Figure 9. Comparison of model voltage effect

Figure 9 (a). The voltage comparison of HPPC working condition

Figure 9 (b). Identification error of HPPC working condition

Figure 10. The data and SOC estimation results of the DST condition when $SOC_0 = 0.8$

Figure 10 (a). The voltage curve of DST condition

Figure 10 (b). SOC estimation under DST condition

Figure 10 (c). The error of SOC estimation under DST condition

Figure 10 (d). Convergence process of different algorithms under DST condition

Figure 11. The data and SOC estimation results of the DST condition when $SOC_0 = 0.8$

Figure 11 (a). Convergence process of different algorithms under DST

Figure 11 (b). The error of SOC estimation under DST condition

Figure 12. SOC estimation results and errors of BBDST experimental when $SOC_0 = 0.8$

Figure 12 (a). The voltage curve of BBDST condition

Figure 12 (b). SOC estimation under BBDST condition

Figure 12 (c). The error of SOC estimation under BBDST condition

Figure 12 (d). Convergence process of different algorithms under BBDST condition

Figure 13. SOC estimation results under BBDST condition when $SOC_0 = 0.6$

Figure 13 (a). Convergence process of different algorithms under BBDST condition

Figure 13 (b). The error of SOC estimation under BBDST condition

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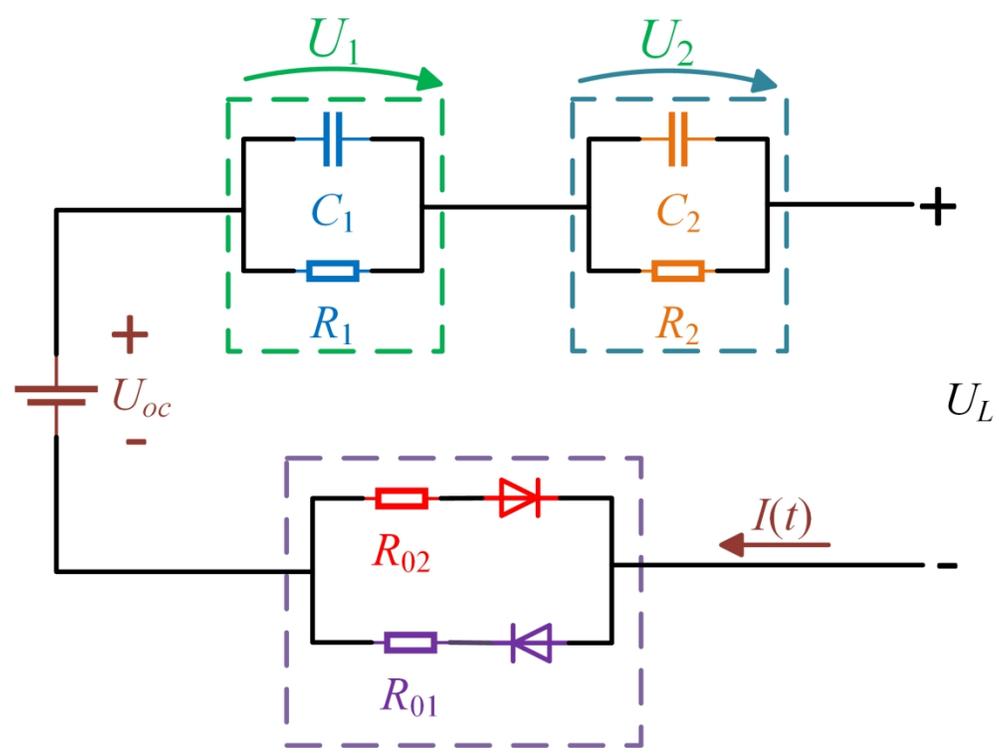


Figure 1. The second-order RC equivalent model of lithium-ion battery with double internal resistance

74x56mm (800 x 800 DPI)

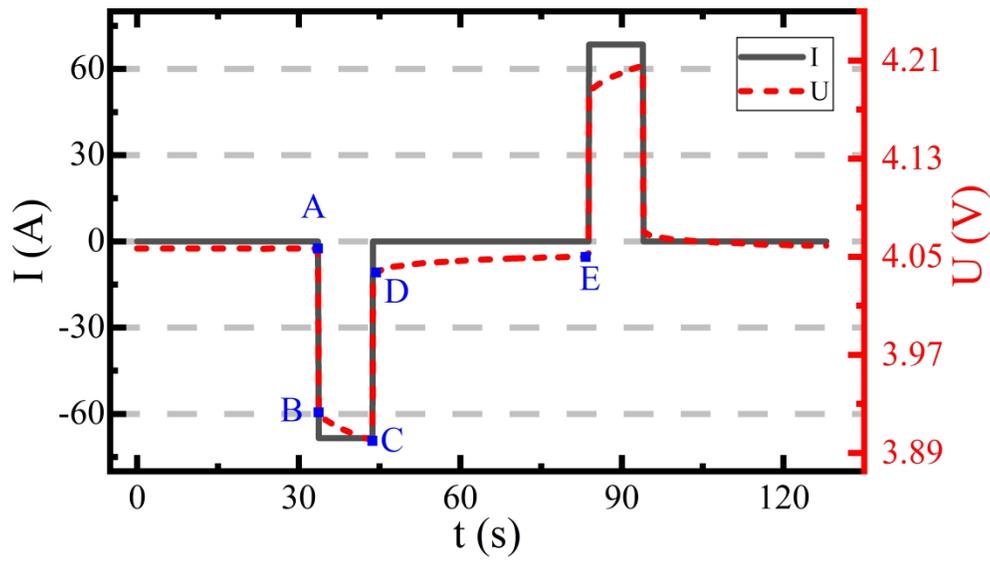


Figure 2. The current and voltage curve of the HPPC working condition

179x99mm (800 x 800 DPI)

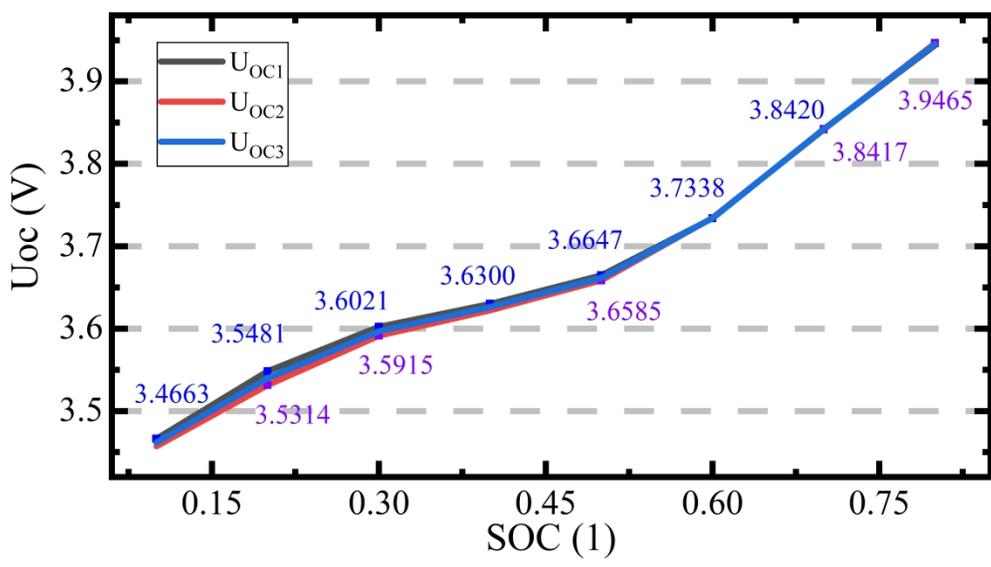


Figure 3. The fitting curve of the UOC-SOC

179x99mm (800 x 800 DPI)

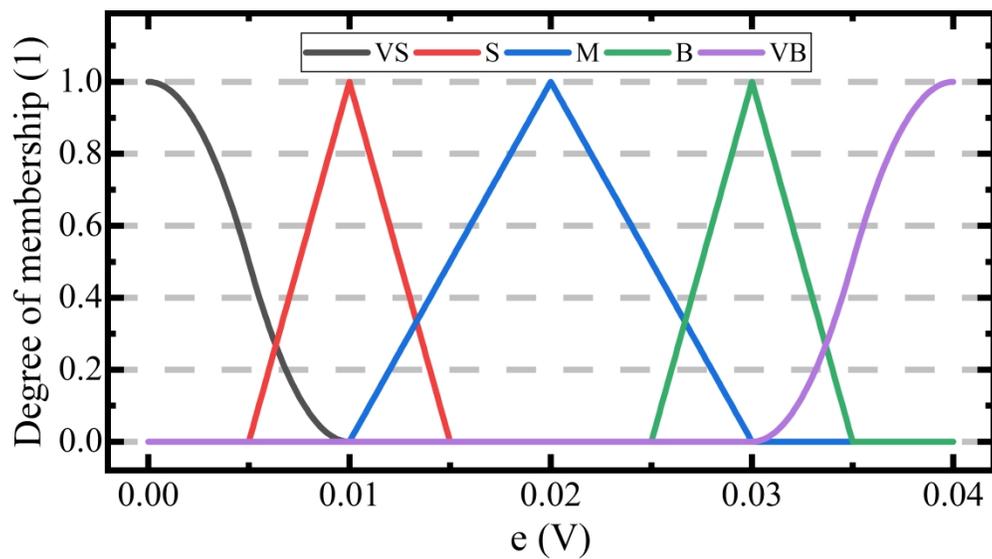


Figure 4 (a). The membership function graph of ek

179x99mm (800 x 800 DPI)

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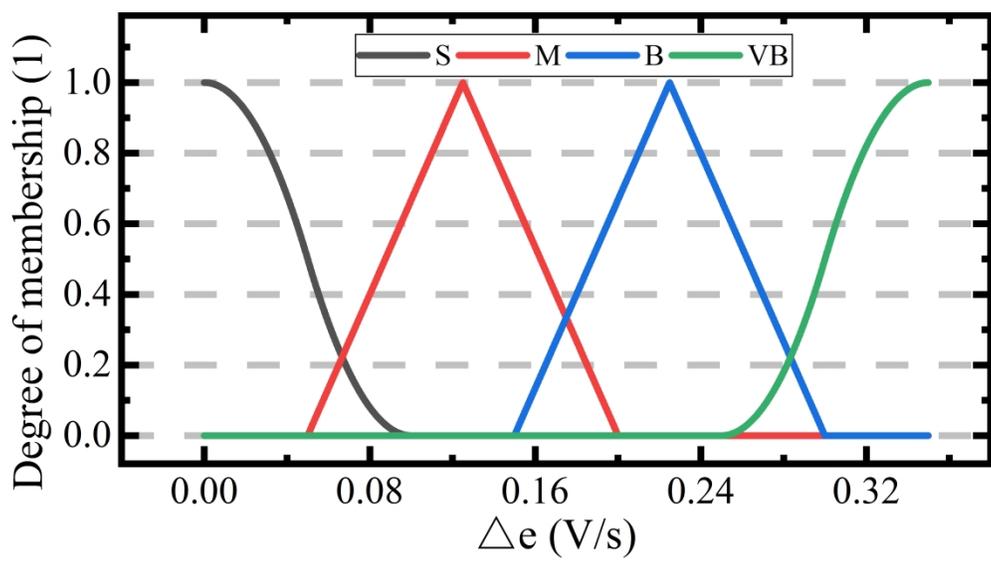


Figure 4 (b).The membership function graph of Δe_k

179x98mm (800 x 800 DPI)

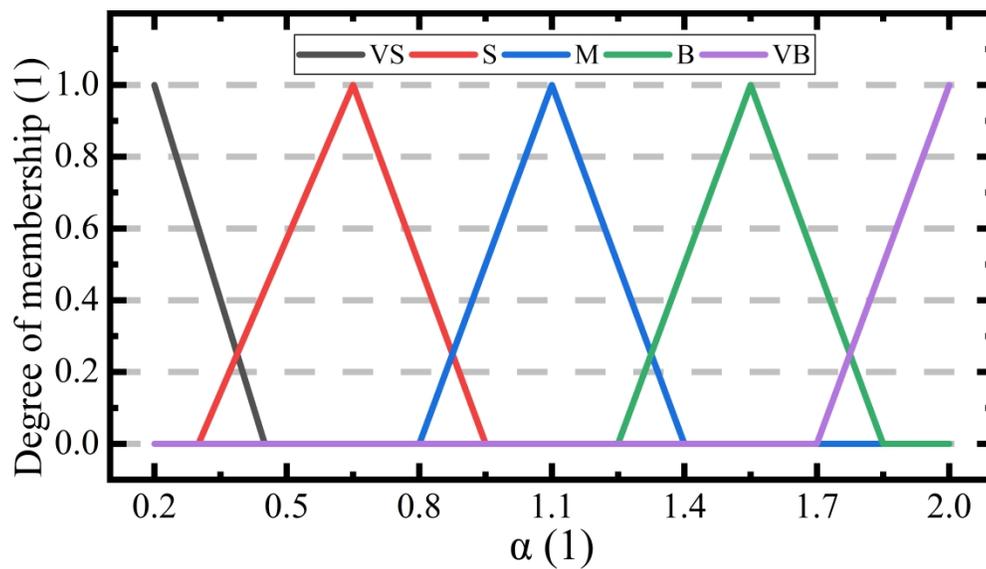


Figure 5. The membership function graph of the output variable

176x99mm (800 x 800 DPI)

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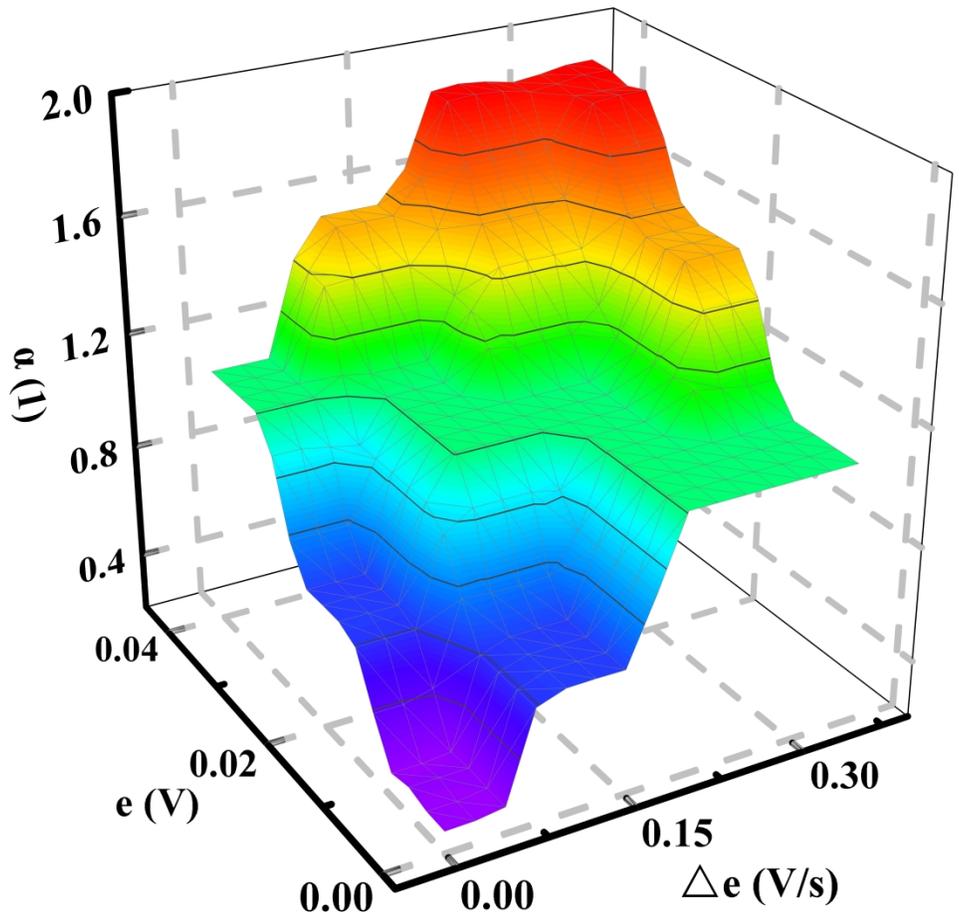


Figure 6. The graph of fuzzy rules
144x134mm (800 x 800 DPI)

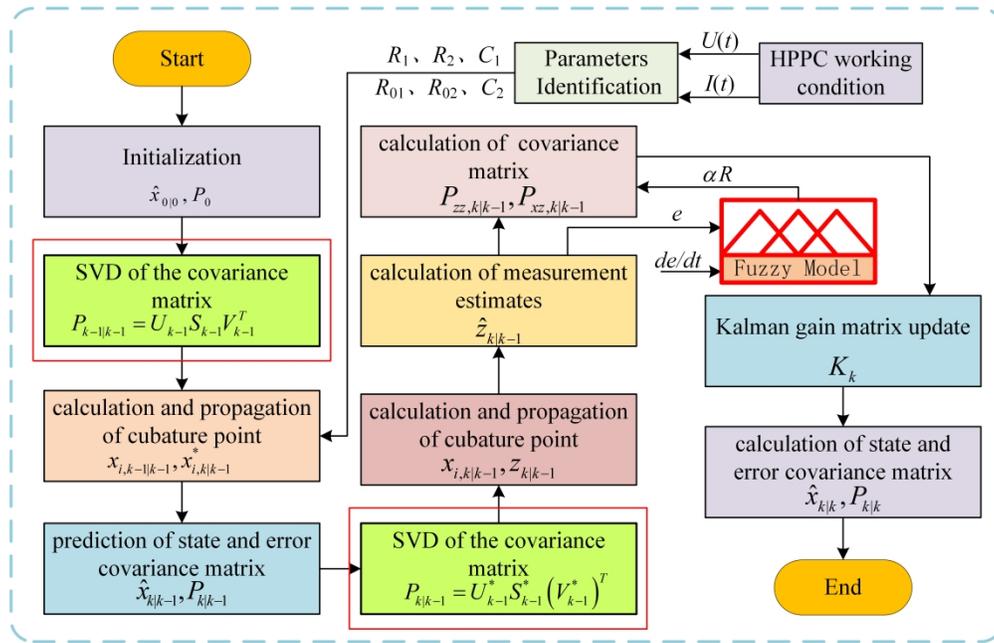


Figure 7. The flow chart of FASVD-CKF

180x115mm (800 x 800 DPI)

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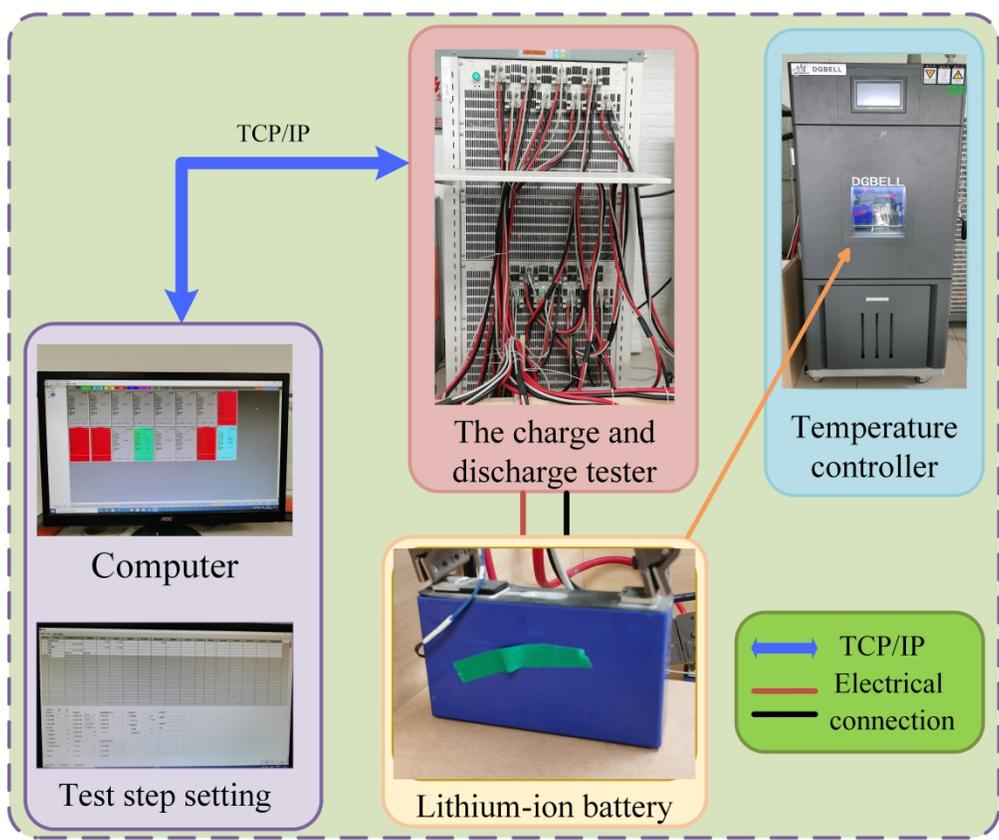


Figure 8. Experimental platform
171x142mm (800 x 800 DPI)

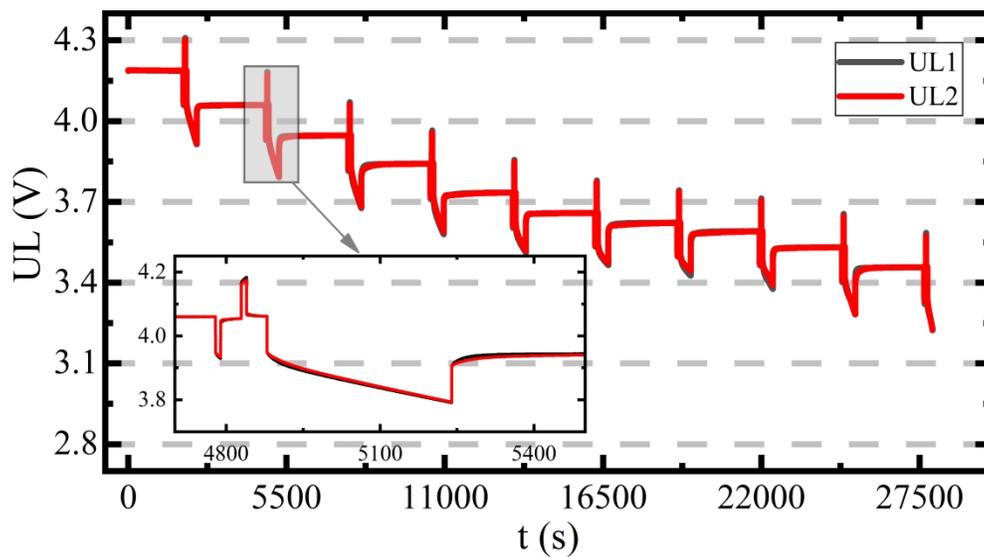


Figure 9 (a).The voltage comparison of HPPC working condition

178x98mm (800 x 800 DPI)

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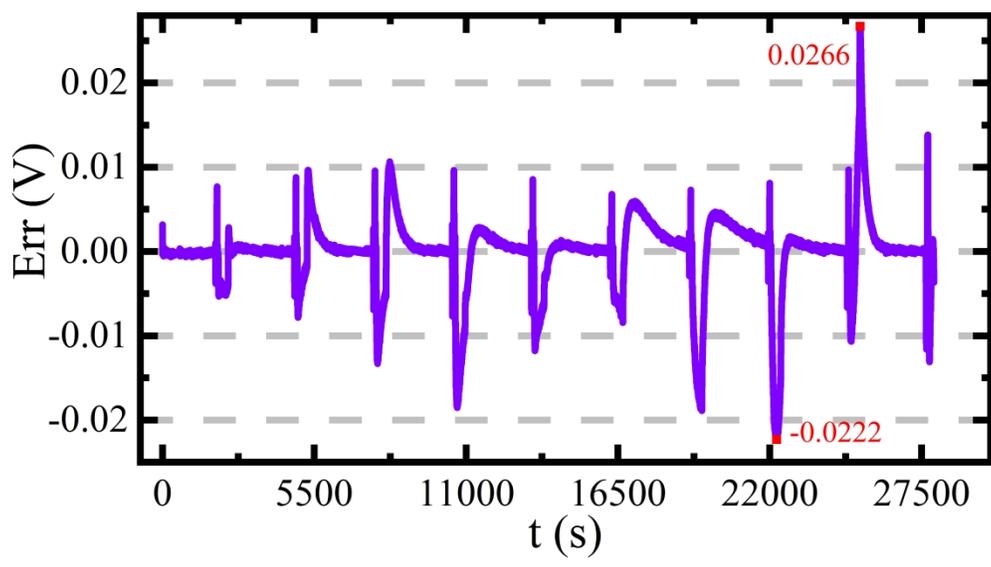


Figure 9 (b). Identification error of HPPC working condition

179x99mm (800 x 800 DPI)

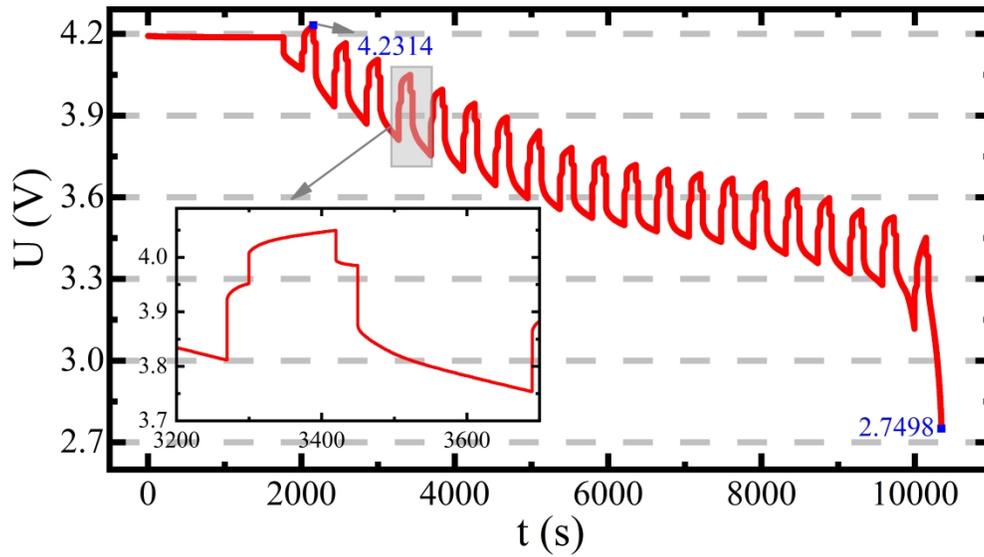


Figure 10 (a).The voltage curve of DST condition

179x99mm (800 x 800 DPI)

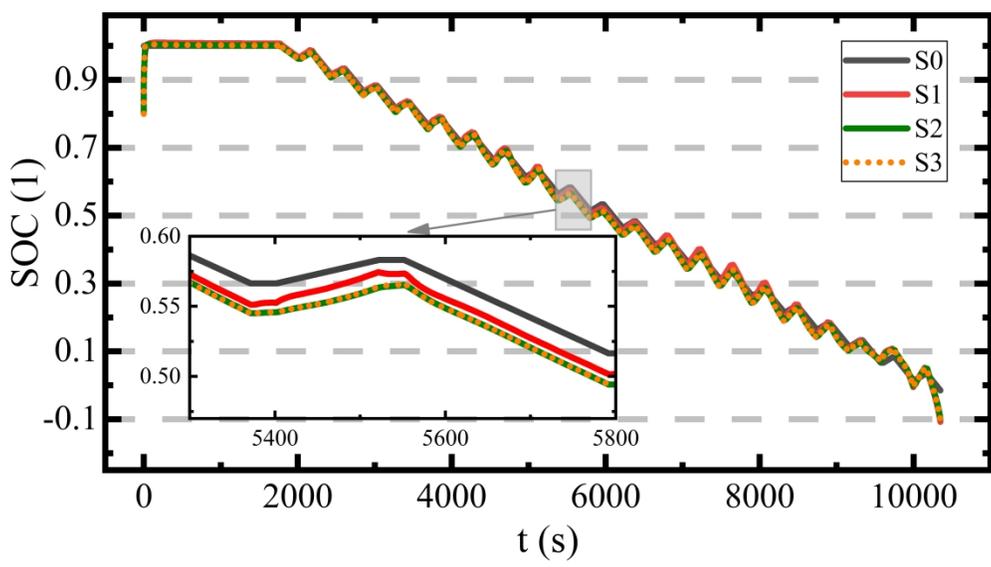


Figure 10 (b). SOC estimation under DST condition

179x99mm (600 x 600 DPI)

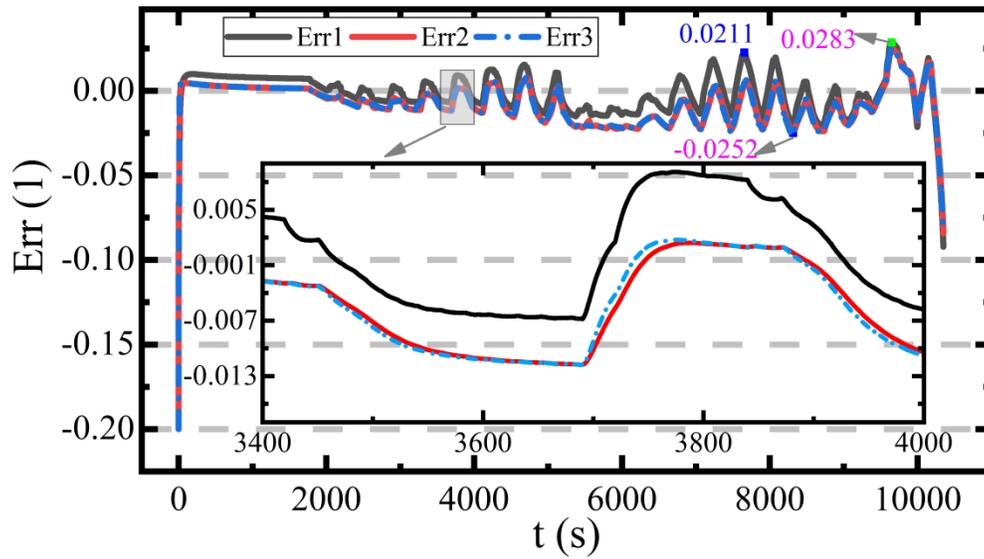


Figure 10 (c). The error of SOC estimation under DST condition

179x99mm (800 x 800 DPI)

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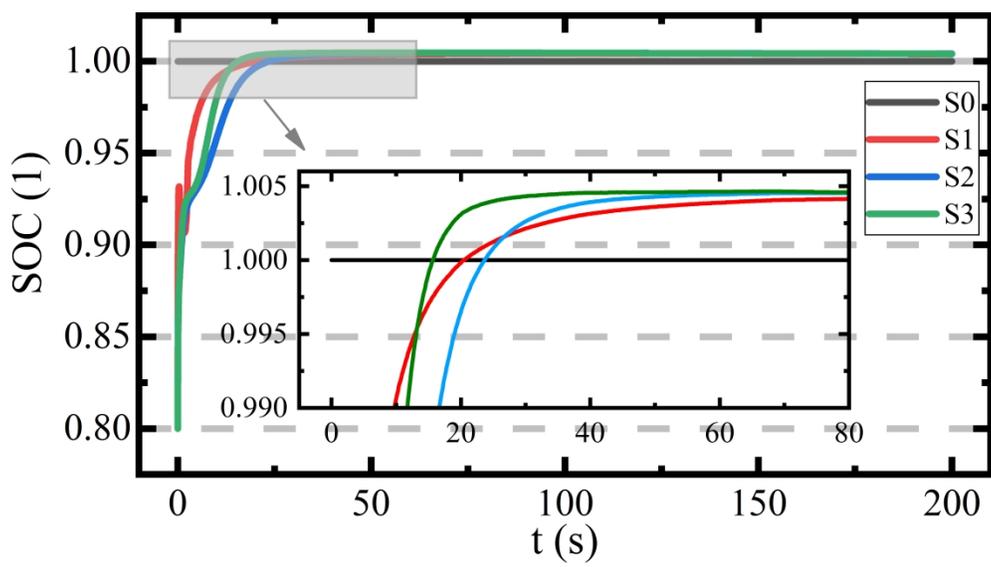


Figure 10 (d). Convergence process of different algorithms under DST condition

181x100mm (800 x 800 DPI)

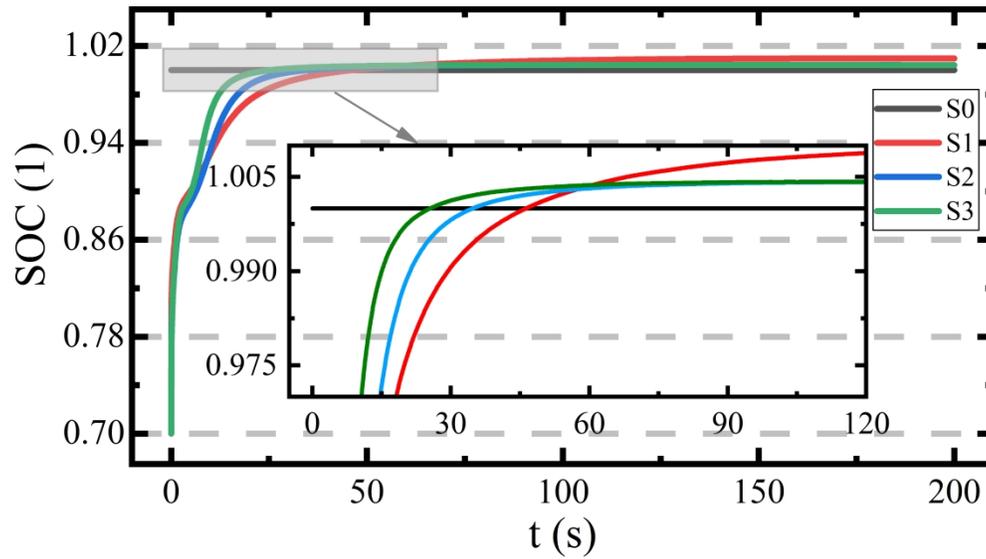


Figure 11 (a).Convergence process of different algorithms under DST

179x99mm (800 x 800 DPI)

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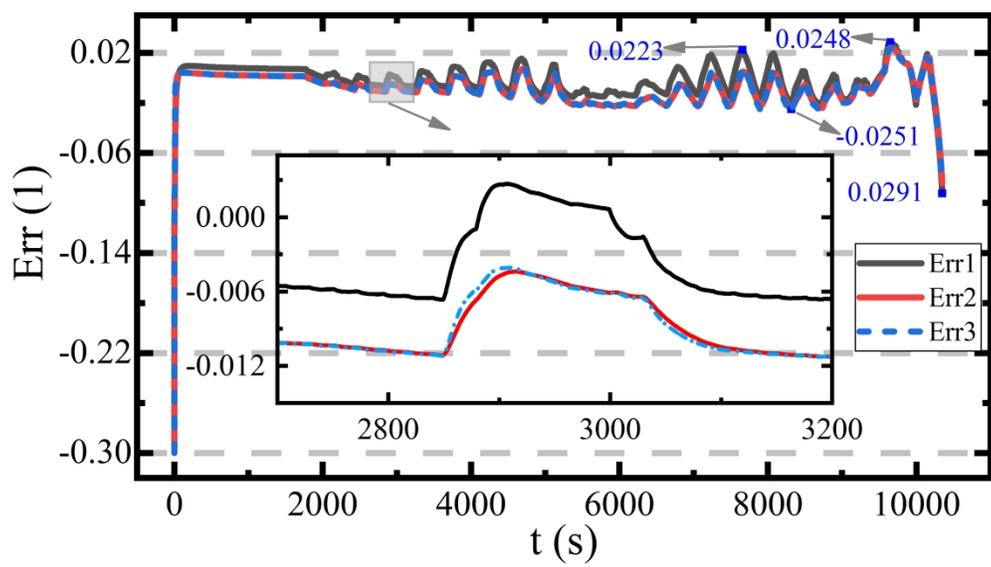


Figure 11 (b).The error of SOC estimation under DST condition

178x99mm (800 x 800 DPI)

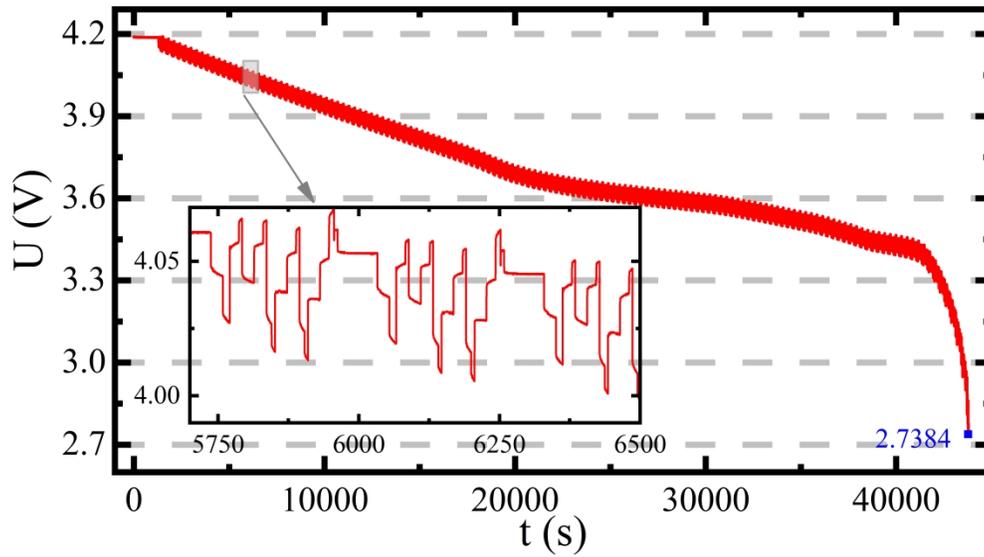


Figure 12 (a). The voltage curve of BBDST condition

179x99mm (800 x 800 DPI)

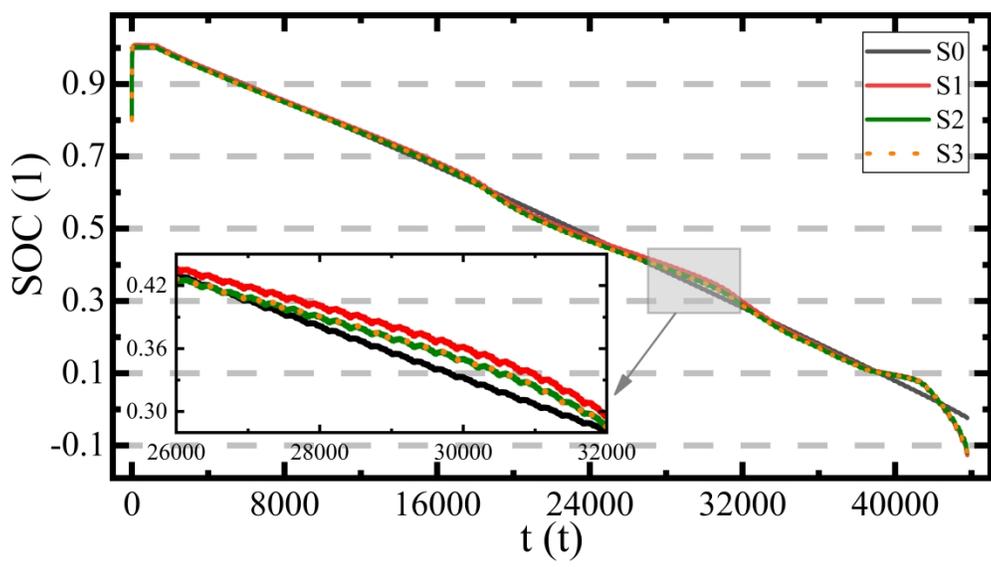


Figure 12 (b). SOC estimation under BBDST condition

179x99mm (600 x 600 DPI)

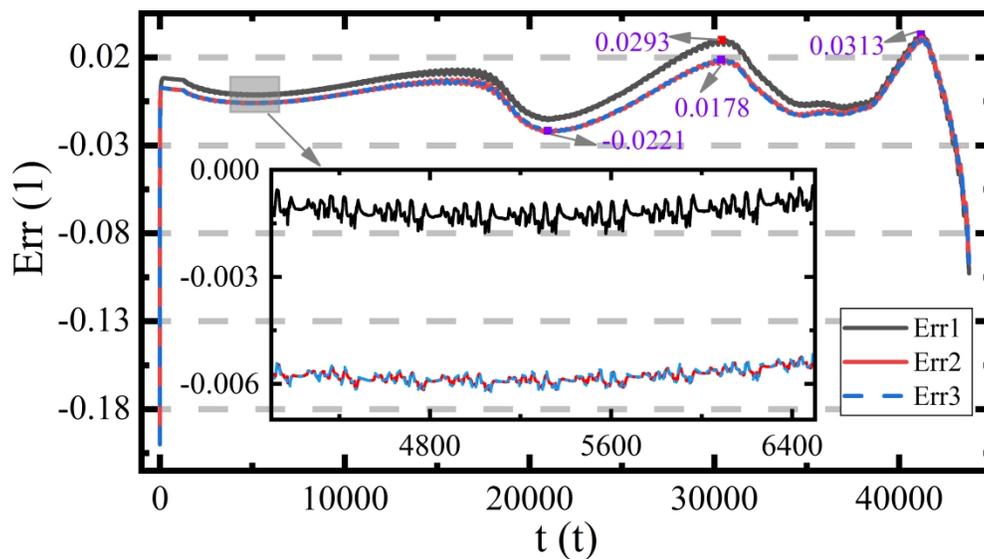


Figure 12 (c). The error of SOC estimation under BBDST condition

179x99mm (800 x 800 DPI)

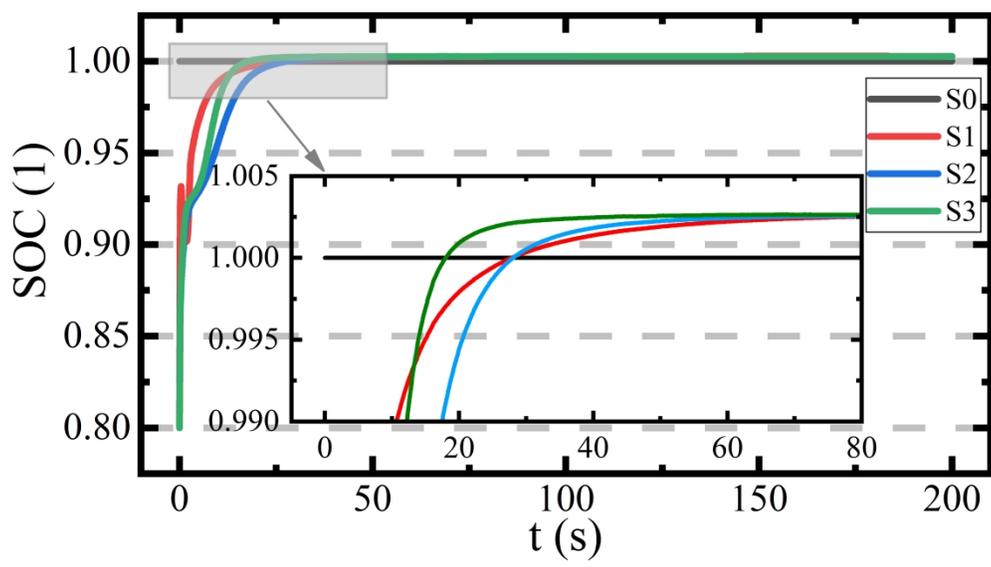


Figure 12 (d). Convergence process of different algorithms under BBDST condition

180x100mm (800 x 800 DPI)

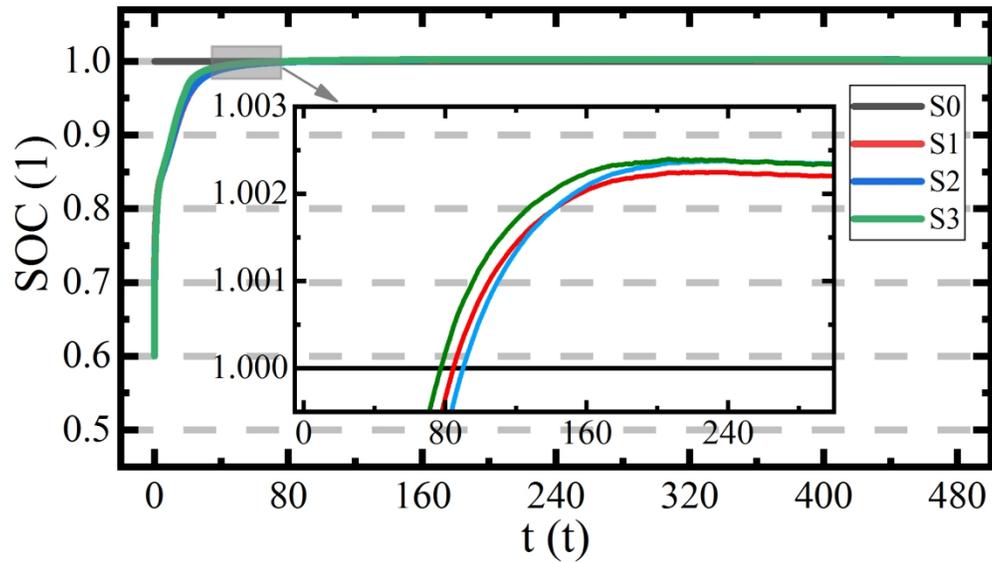


Figure 13 (a). Convergence process of different algorithms under BBDST condition

179x101mm (300 x 300 DPI)

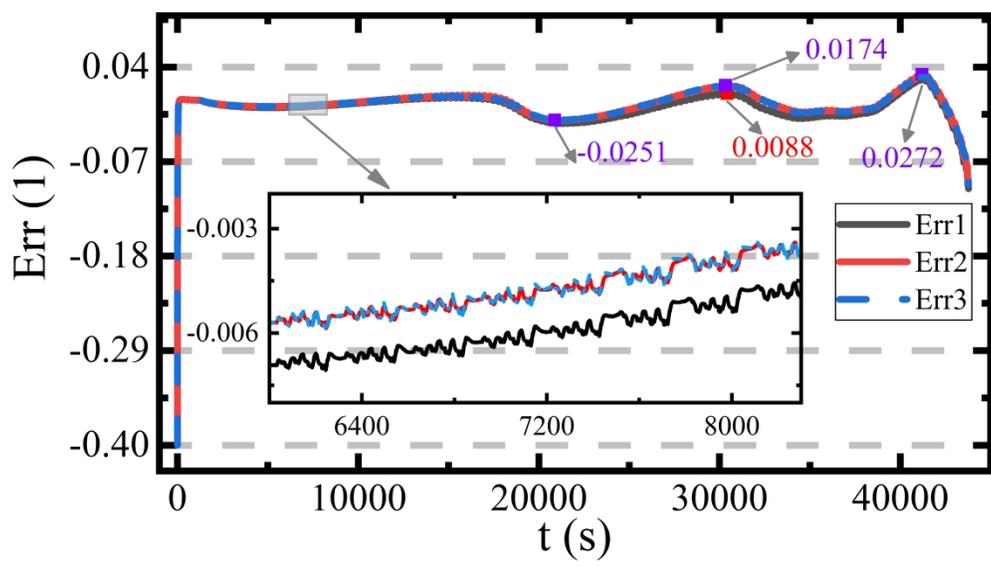


Figure 13 (b). The error of SOC estimation under BBDST condition

180x99mm (800 x 800 DPI)

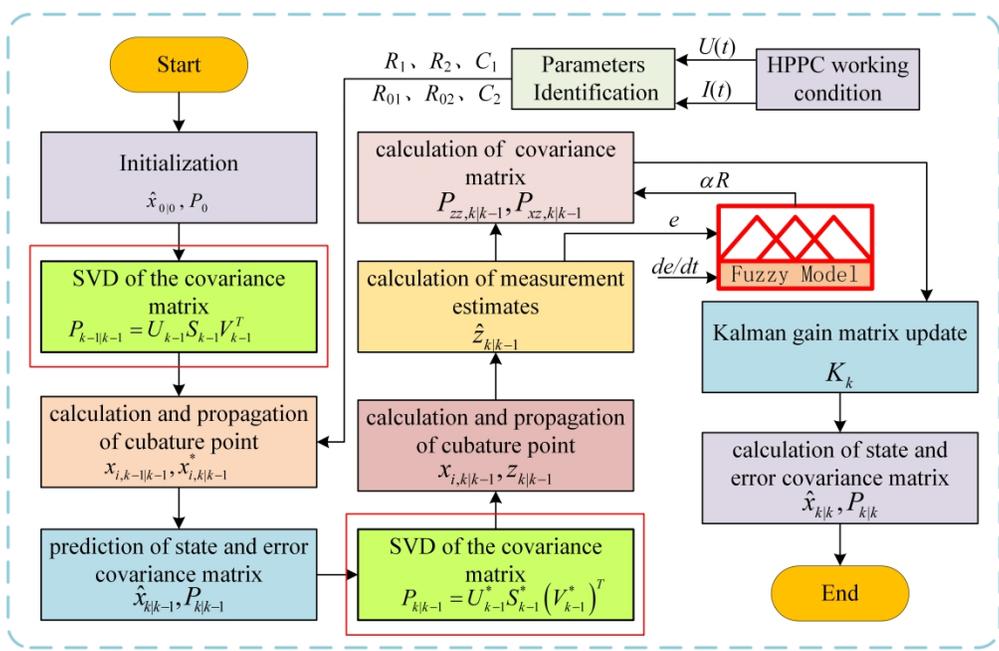
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4 **Fuzzy adaptive singular value decomposition cubature Kalman**
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6 **filtering algorithm for Lithium-ion battery state-of-charge**
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8 **estimation**
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10 **Xiao Yang¹, Shunli Wang^{1*}, Wenhua Xu¹, Jialu Qiao¹, Chunmei Yu¹, Carlos Fernandez²**

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18 **Abstract:** To solve the problem of the slow convergence speed for the battery state-of-charge
19 estimation of cubature Kalman filter algorithm (CKF), a Fuzzy adaptive singular value decomposition
20 cubature Kalman filtering algorithm (FASVD-CKF) is proposed. And a double internal resistance
21 second-order RC model is established to simulate the lithium-ion batteries. Finally, the proposed
22 algorithm is verified under two different working conditions. The results show that the proposed
23 algorithm has fast convergence speed, good stability, and high accuracy in SOC estimation.
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180x115mm (800 x 800 DPI)