ZAMANI, E. and DOUNAS, T. 2022. Discretisation design strategies: strategies to integrate design and fabrication through discretization. *In Kouider, T. and Saleeb, N. (eds.) Proceedings of the 9th International congress on architectural technology (ICAT 2022): digitally integrated cities: closing the chasm between social and physical, 19-20 May 2022, London, UK.* Aberdeen: Robert Gordon University [online], pages 1-15. Available from: https://drive.google.com/file/d/17KEcD4GshS1PpSyH6dTPGRwNfsBtYtan/view

Discretisation design strategies: strategies to integrate design and fabrication through discretization.

ZAMANI, E. and DOUNAS, T.

2022





٠,,

DISCRETISATION DESIGN STRATEGIES

strategies to integrate design and fabrication through discretization

ERFAN ZAMANI AND THEODOROS DOUNAS <u>e.zamanigoldeh@rgu.ac.uk</u> <u>t.dounas@rgu.ac.uk</u>

Abstract. In the present paper, we introduce a classification system, for discretisation strategies, based on the procedural differences. This paper has a particular focus on strategies explicitly positioned towards an integration between digital design, robotic fabrication and robotic assembly. In the first step, the paper introduces and analyses previous methods from the literature and built case studies and proposes a classification for discretisation approaches. This classification is based on three basic designing strategies: Top-Down, Bottom-Up and Hybrid, in a parametric design manner. The second step defines a general parametric framework for each approach based on the classification analysis. Due to the specifications and functions, these approaches can be synced and combined with other parametric design tactics, such as panelising, subdivision, or generative design. We describe and analyse the possibilities of connecting other parametric features with our discretisation definitions in each category. In the end, this paper introduces several alternative implementation avenues for each category, including a logical design strategy, without considering any specific software or tool.

Keywords: Discretisation, Digital Design, Robotic Fabrication, Parametric Architecture, Geometric Complexity.

1. Introduction

Computer-aided design tools bring new opportunities to produce complex geometric shapes with minimum mathematical inputs. Correspondingly, this progress has affected the architecture field, and digital modelling is becoming increasingly popular proportionately with other industries (5). In tandem, there has been a shift in architectural design representation from ortho-plane and regular shapes toward the forms that are often associated with more geometric complexity and design flexibility (9). Not only, this aesthetic revision poses a challenge in design stage, but it also demands an adjustment in the manufacturing and construction procedure (1).

The present paper assumes discretisation as a step-by-step design procedure that could simplify the complex geometry into simpler geometries by dividing it into understandable roles and relations (9). Discretisation can be used to break down an architectural volume or surface into smaller and buildable pieces (14) or can be used to generate a 2D or 3D volume (30).

Additionally, discretisation can bring new features to design. For instance, in discretisation, we can start the design process from the module instead of the overall geometry. This feature gives the designer enough flexibility and opens new possibilities to integrate the digital file with fabrication machines.

This paper proposes a classification system for discretisation strategies in architecture by reviewing the existing discretisation methods. This study scrutinises case studies to find digital procedural differences and similarities. Due to the specifications and functions, the proposed strategies can be synced and combined with other parametric design tactics, such as panelising, subdivision, or generative design. In each category, we describe and analyse the possibilities of connecting other parametric features with our discretisation definitions.

2. Literature Review

Kolarevic in (10) discusses design data digitalisation and integration of design, analysis, manufacture, and assembly. He explains how this integration optimises the construction time and reduces human errors.

Currently, buildings constructors and designers are working in two fragmented sectors and designers are not getting involved in the construction process after the delivery of the documents (31).

However, during a discretisation process (as a digital design strategy), by using analytical methods, designers can consider and generate a lot of fabrication and assembly options, such as interlocking joints, modular orientations or even picking and placing strategies (5). This feature brings designers the possibility to come over into both design and construction stages (10).

In (9), authors explain discretisation as a reverse engineering process in a parametric manner. In their proposed framework, a non-orthogonal/free form first parametrically delineates and readjusts the surface. Secondly, the parametric surface can be discretised (9). Katrin & Penn introduce a discretisation method using subdivided surfaces. This method enables designers to produce an "approximation" of the freeform surface and break it into a family of discretised elements.

Some studies, like (9) and (14), have considered discretisation as a solution to manage free form's geometry complexity by digitally re-formulating the surfaces and evaluating them down into smaller components.

However, discretisation can be used as a general digital design strategy to fulfil the requirements for fabrication and assembly (13). In (32), different subdivision schemes have been analysed for semi-regular geometries, and the discretised units have been applied to the form by point mapping controls. Similar method has been used in (33), where authors, propose geometric calculations for wave-shape formats based on cross-ratios of identical tasselling parts.

Another approach to discretise a surface is to apply a curvature network on it. This technique can be manipulated with different geometric meshes like conical or hexagon shape; like (12) or (25). There have been more studies on conical meshes and curvature network techniques. The articles (16) and (17) study and extend the previous knowledge of quadrilateral meshes and their functionality to discrete large scale-free forms. In these papers, the researchers examine the possibility of a relation between circular and conical meshes with other parametric capabilities (e.g., offsetting tools) (17). Also, their research can validate the glass multilayer buildings design and

categorise the construction elements (16). Likewise, another geometric approach is proposed in (14) to discrete double-curved surfaces based on the intersection of tangent planes.

3. Methodology

This study develops a classification system, for discretisation strategies, based on the procedural differences. This paper has a particular focus on strategies explicitly positioned towards an integration between digital design, robotic fabrication and robotic assembly. This classification makes a comprehensive basis for a deep procedural understanding of current discretising methods in architectural design toward generalising our methods for a wide range of structures.

In the first step, this paper introduces and analyses the previous discretisation approaches. These analyses lead us to elicit each case study's logical and algorithmic specifications. In the second step, the methods are classified according to their design logic and conceptual and procedural differences (Figure 1).

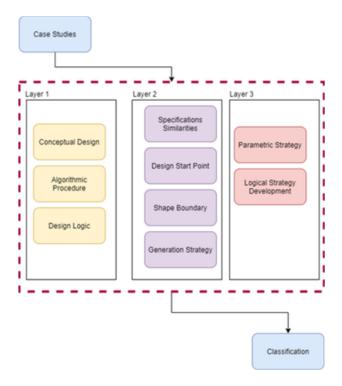


Figure 1 Methodology Layers

4. Case Studies

This paper first reviews several discretisation cases studies from the literature and built structures and secondly, re-checks their parametric design mechanisms and assort the algorithmic steps.

4.1. LITERATURE CASE STUDIES

In the table below, 14 discretisation related case studies/papers are introduced (Table 1). This classification is used to generalise the discretisation methods and generate a parametric framework for each category.

Table 1 Literature Discretisation Case Studies, Credit by Erfan Zamani

Paper	
1	Geometric modelling with conical meshes and developable surfaces
2	A parametric strategy for free-form glass structures using quadrilateral planar facets
3	Ornamental Discretisation of Free-form Surfaces
4	Generative Agent-Based Design Computation
5	Meso-Scale Digital Materials: Modular, Reconfigurable, Lattice-Based Structures
6	Project DisCo: Choreographing Discrete Building Blocks in Virtual Reality
7	Aggregated Structures: Approximating Topology Optimized Material Distribution with Discrete
Paper	
Building Blocks (Er	ror! Reference source not found.)
8	Interlocking of Convex Polyhedra: towards a Geometric Theory of Fragmented Solids
9	Topological Interlocking Assemblies
10	A generalized framework for designing topological interlocking configurations
11	Geometry as Interface: Parametric and Combinatorial Topological Interlocking Assemblies (
12	A Model for Intelligence of Large-scale Self-assembly (
13	Voxelcrete - Distributed voxelized adaptive formwork
14	Discretized Fabrication of Geometries Generated with Cellular Growth Simulations

4.1.1. Planner Surface/Mesh & Curvature Network

Liu and his colleagues in (12) and (25) propose a dynamic strategy based on curvature network and planer mesh. In their methodology, the quad mesh may be recognised either as planer or conical. This feature makes it possible to optimise the mesh and combine it to other parametric settings such as subdivision or panelising (12). Their research was updated in 2009 for hexagonal shape meshes. They proposed a top-down approach to approximate a surface and evaluate it to be broken down into discrete hexagons (25).

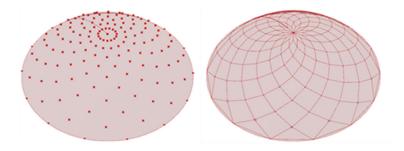


Figure 2 Curvature Network, Credit by Erfan Zamani

The other similar relevant example is (26) worked particularly for glass roofing structures. This paper uses the proposed method to bring the necessary geometric principles in a parametric framework. The parametric tool, CATIA, approximates the given structure format and generates discretized quadrangular meshes in a double-curved surface (26).

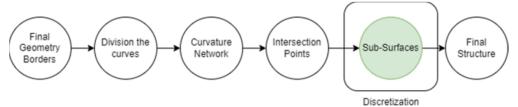


Figure 3 Curvature Network Parametric Logic, Credit by Erfan Zamani

4.1.2. Intersection of Tangent Planes

There is another top-down geometric approach to discrete double-curved surfaces based on the intersection of tangent planes. The paper (14) explores the possibility of two-dimensional mapping points over a 3D shape and generating planes by these points. By changing the tangent of these planes (the angle between planes and freeform), different shapes would be generated that result from the confluence of planes (14). Baharlou and Menges (1) follow the same method in the manner of "Constrained Generating Procedures "(GCP's), considering fabrication constraints. In their method, complex patterns can be used for planner locating and have more generated options (1).



Figure 4 Intersection Validation, Credit by Erfan Zamani

4.1.3. Algorithmic Growth

The new developments in fabrication methods, for example, 3d printers, bring more options for the modular designs based on generative and bottom-up approaches (7). The Project DisCo (Discrete Choreography) made a foundation for spatial aggregation modelling with beam shape modules in Virtual Reality (VR). This methodology is mainly proposed for gaming, and the discretised elements use sensorial physical/gamer body interactions to instantly assemble the structure (27). The current digital design approach is mainly used to bridge the gap between digitalisation and fabrication to take advantage of novel and progressive production techniques. This digital to analogue translation needs to consider the natural geometry and specifications of the modules (19). Cubic modularity because of its simple geometry can streamline the process.

Rossi & Tessmann developed the tools WASP to explore different computational generative strategies to generate and assemble a mass of discrete elements from a manual designed module. Their methodology enables the designer to define a diverse variety of rules for modules to grow. This bottom-up aggregation can be transferred into digital design workflow and can be synced with robotic assembly systems (19, 20,21,24).

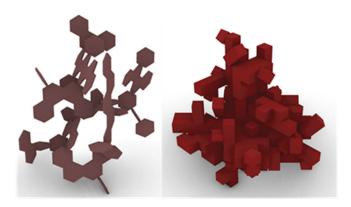


Figure 5 Algorithmic Growth, Credit by Erfan ZamaniGoldeh

4.1.4. Moving Cross-Section Procedure

The cross-section methodology had been presented in (8), based on polyhedron shapes. This model includes a network of planner square grides and polyhedron modules in which each edge of square grids plays the leading role to define modules' placement. This process that named "Moving Cross-Section Procedure" (8) & (15) can translate the planners into the modules by defining the modules' side angle. The pair of parallel sides of each square gride specifies the module's side locations. In this method, polyhedron modules are not predefined, and they are constructed along and in harmony with panner grides. Although cross-section is based on angle and pattern, the resulting modules are still controllable.

Tessmann (23) validates the "Cross-Section" method with an assembly system for topological interlocking blocks. In this interlocking validation that has been examined as a part of the research design studio, tetrahedron shape modules can fill a predesigned planner or curvature. Although the design of the modules might be varied, they can still be categorised as tetrahedron families.

The modules' dimensions and sides' geometry can bring new options for interlocking joints. This method constructs the base of (15) research. Their research simplified Tessmann's methodology by making a pattern of square grids on a surface and aligning tetrahedron modules to it. Then, they developed this method by applying hexagonal shape modules on a predefined cells network.

Bejarano and Hoffmann, in (2) generalised "Moving Cross-Section Procedure" by a "Topological Interlocking Configuration" (TIC). Their configuration is an assembly system based on the repetition of single modules on tasselling surfaces or mesh. The angular surfaces are still critical, but by analysing the modules' structural behaviour, the authors add central point and height values that make the modular parametric control more flexible. These behaviours include rotation, movements and slipping to the front, back and sides.

The Cross-Section method generally makes a topological structure in which each module is surrounded by several neighbours, depending on the number of modules' sides (1) & (15) & (25). The modules should be designed to prevent X and Y motions. To keep the entire structure, the designer may need to design the supporting frames or different designs in the border parts (2). This modular volumetric support and resulting interlocking system bring a high structure resistance that counteracts the external and internal forces (34) & (8).

The digital and software technology progress brings new features to the Cross-Section and tasselling-based methods such as more complexity in overall form and modular shapes. Also, there is more variety in the designs of the modules and interlocking joint options that enable us to consider different types of fabrication and assembly alternatives. In the figures below, we explored the Cross-Section Method.

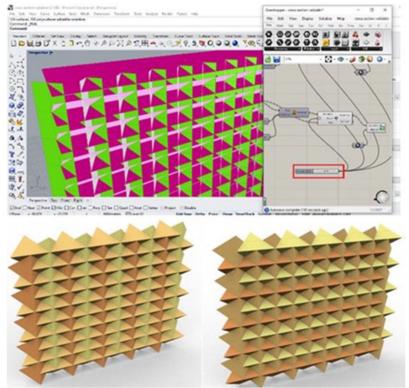


Figure 6 Cross-Section Validation, Credit by Erfan Zamani

4.1.5. Aggregation and Interlocking System

Rossi and Tessmann (20) propose a spatial assembly approach for an architectural formation aggregated by discretised modules. This aggregation can be resulted from the relation of the module with itself or with surrounding modules. They advance an aggregation model in 3Dimension based on the growth method (19). In this method, the 3D outline of the final geometry specifies the boundary of a modules' density. This reversible procedure can integrate the complexity of modules' shapes with their geometry relations and specifications. This method enables the designer to manage or even define the attribution of the modules and bring new options for fabrication and assembly levels. Their developed plugin WASP can link the discretised digital model and physical worlds.

Another example of modular aggregation configuration is (11) in which authors present a method as coverage for non-standard concrete structures with dodecahedron shape modules. The modules can be assembled to make a closed form for concrete to be cast. This temporary structure can be disassembled, and modules can be reused. Due to its nature and the high number of sides, Dodecahedrons can be constrained among several same shape neighbours and brings more interface options (11).

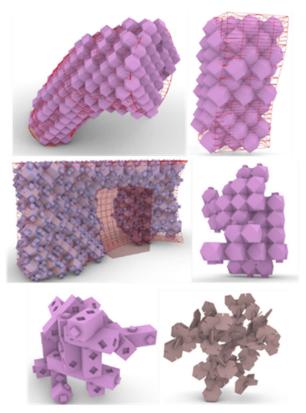


Figure 7: WASP Top-Down & Bottom-Up Validation, Credit by Erfan Zamani

4.2. BUILT STRUCTURES

In conjunction with literature proposed approaches, this paper parametrically recreate 6 built structures.

Name	Parametric Validation Method	
1	(Traditional) Brick wall	Pattern and Modular
		Orientation
2	ArboSkin	Panelising
3	Serpentine Pavilion 2016	Pattern and Modular
		Orientation
4	British Museum Court Roof	Sub-division and Triangular
		Panels
5	80Hz Pavilion	Point's Network and Point
		Attraction
6	TLDC Tsumiki Pavilion	Modular Morphing

Table 2 Built Case Studies and methodologies, Credit by Erfan Zamani

In 1999, Norman Foster designed a roof cover with a grid shell structure for Great Court (Figure 5) in National British Museum. Foster's technique approximates the degree of curvature network on a surface and applies triangular planers (9) & (13). Not only did architectural design and structural mechanism Foster consider, but he

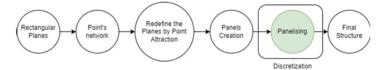
also observed the fabrication process. For example, the average area of designed glass triangular panels is less than 0.7 square metres as larger sizes could not be manufactured (13). Although Foster used curvature, in this paper, we validate his approach in Grasshopper and curtail the algorithm by two times discretisation.

This paper recreates the brick wall as the simplest structural element. In this paper, the standard cubic brick has been used as the module. The form of the surface (wall) can be changed either parametrically or not parametrically. This algorithm is based on a planner curvature network, and the size of the bricks and the distance between them are parametrically changeable. This algorithm also can be applied to more complex geometries.

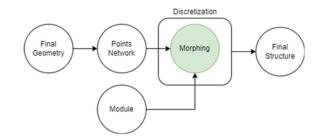


Figure 8: Built Discretization Case studies Revalidation, Credit by Erfan Zamani

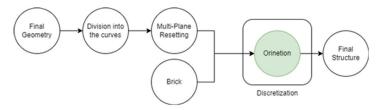
The other examined case study is ArboSkin. First, the overall geometry is outlined with curves, and then it has discretised into smaller panels. In the case study, 80Hz Pavilion, a set of points in U and V dimensions are used to locate the panels and point attraction technique is used to define the angular adjustment. Tsumiki Pavilion had been designed by Kengo Kuma and this paper validate its structure in Grasshopper. For this case study a network of points is applied on the final geometry (Pyramid) and the designed modules have morphed into the points.



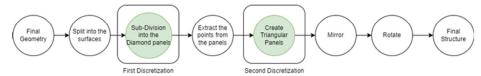
1: 80Hz Pavilion Parametric Logic



2: TLDC Tsumiki Pavilion Parametric Logic



3: Parametric Brick Assembly Logic



4: British Court Roof, Two-times Discretisation Logic



5: ArboSkin Parametric Logic

Figure 9 Parametric Logics of each built case study, Credit by Erfan Zamani

5. Classification

To classify the case studies, we consider the design's starting point and algorithmic logic of each. According to the case study validation and analysis, the parametric design starting point is either overall geometry or modular design or a combination of both. In addition, the parametric specifications and the potential of algorithmic development have been considered too.

Figure 1Table 3 Classification of Discretisation Approaches, Credit by Erfan Zamani

Approach	Design Starting Point	Parametric Specifications
Top-Down	Overall Geometry	The modules are constrained inside the geometry.
Bottom-Up	Modules	The modules follow the algorithmic rules to aggregate and form a geometry.
Hybrid	Overall Geometry & Modules	The modules parallelly follow the algorithmic rules and geometries (either surface or solid mesh). The closest match is the aggregation that is formed inside or extraneous of the larger geometry.

In the Top-Down approach, the resulted shape is being designed first, and then a parametric tool would come to create the nearest match. Hudson in (6) named this method "post-rationalisation". Hudson describes that the "Final Geometry" may be designed either in or out of the parametric tools and framework. This parametric recreation can link the discretisation algorithms with fabrication and assembly machinery.

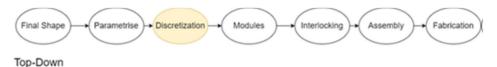


Figure 10 Top-Down Logic, Credit by Erfan Zamani

In the Bottom-Up approach, the module's design is the start point so that the discretised elements are defined first, and then rule-based algorithms form the overall structure. Hudson named this method "Pre-rationalisation" (6). As a result, a parametric rule-based setting will be achieved that can be adjusted according to the required application, dimensions, and fabrication possibilities.

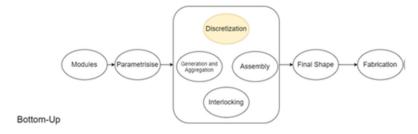


Figure 11 Bottom-Up Logic, Credit by Erfan Zamani

There is another approach, eliciting from the combination of mentioned approaches above. This Hybrid approach can include a top-down approach for overall geometry (large scale) in parallel with the bottom-up approach for modules (small scale). However, hybrid, like other parametric methodologies, creates a revisable system in which the different parameters and sub-parameters will be changed by changing the parameters on any scale (large or small).

The discretised hybrid digital models act in two stages: first, creating the outline of the final structure and second, making the modules. In the first stage, the outline can be parametric or not parametric border sketches of the resulted shape, but there are more options to create a map for modules to follow. For example, it may be a closed curve or a parametric pattern. Indeed, in this approach, the outline acts like a bag filled with several fruits(modules) or a string that connects several beads.

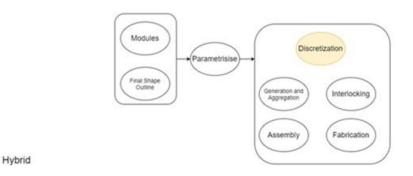


Figure 12 Hybrid Logic, Credit by Erfan Zamani

6. Discussion

The research presented is original in the sense that, addresses a classification for logical parametric discretisation. The research is also significant in the sense that it establishes the legitimacy, constraints and pragmatic aspects of discretisation and provides the toolset and a comprehensive foundation to be expanded in terms of fabrication and assembly.

On the one hand, this paper provides a systematic classification across discretisation approaches and, on the other hand, highlights the parametric design methodology as an efficient tool for innovative and intelligent construction.

Each category of proposed parametric logic can be developed with different interlocking options. This possibility brings new potentialities to offer innovative assembly methods. The design and assembly are the parts of a circular and dynamic process in which pre-programmed, and computer-controlled machines collaborate with digital models, parallelly with implementation.

Furthermore, this research is expandable to design structures specifically ideal for robotic assembly. The proposed logical classification can validate the assembly system by examining the joints and structural stability under the scale.

References

BAHARLOU, EHSAN and MEGNES., 2013. Generative Agent-Based Design Computation. http://papers.cumincad.org/cgi-bin/works/paper/ecaade2013 081

BEJARANO, A., & HOFFMANN, C., 2019. A generalized framework for designing topological interlocking configurations. International Journal of Architectural Computing, 17, 53–73. https://doi.org/10.1177/1478077119827187

- BRANKO KOLAREVIC, & MALKAWI, A., 2005. Performative architecture: beyond instrumentality. Spon Press.
- GU, N., WATANABE, S., ERHAN, H., & HAEUSLER, M., 2014. DESIGNING WITH DISCRETE GEOMETRY (pp. 513–522). https://doi.org/10.1111/j.1531-314x.2006.00068
- HENRIKSSON, V., & HULT, M., 2015. Rationalizing freeform architecture Surface discretization and multi-objective optimization.
- HUDSON, R., 2010, Strategies for parametric design in architecture. (p. 274).
- JENETT, B., CELLUCCI, D., GREGG, C., & CHEUNG, K., 2016. Meso-Scale Digital Materials: Modular, Reconfigurable, Lattice-Based Structures. V002T01A018. https://doi.org/10.1115/MSEC2016-8767
- KANEL-BELOV, A., DYSKIN, A., ESTRIN, Y., PASTERNAK, E., & IVANOV-POGODAEV, I., 2010. Interlocking of Convex Polyhedra: towards a Geometric Theory of Fragmented Solids. Moscow Mathematical Journal, 10, 337–342. https://doi.org/10.17323/1609-4514-2010-10-2-337-342
- KATRIN, J., PENN, A., & SHEPHERD, P., 2014. Designing with Discrete Geometry. http://papers.cumincad.org/cgi-bin/works/paper/caadria2014_177
- KOLAREVIC, B., 2004. Architecture in the digital age: design and manufacturing (pp. 280-304).
- LEDER, S., 2020. Voxelcrete Distributed voxelized adaptive formwork. http://papers.cumincad.org/cgi-bin/works/paper/ecaade2020_172
- LIU, Y., POTTMANN, H., WALLNER, J., YANG, Y.-L., & WANG, W., 2006. Geometric modeling with conical meshes and developable surfaces. ACM Transactions on Graphics, 25, 681–689. https://doi.org/10.1145/1141911.1141941
- MALEK, SAMAR RULA., 2012. The Effect of Geometry and Topology on the Mechanics of Grid Shells (p. 131).
- MANAHL, M., STAVRIC, M., & WILTSCHE, A., 2012. Ornamental Discretisation of Free-form Surfaces. International Journal of Architectural Computing, 10(4), 595–612. https://doi.org/10.1260/1478-0771.10.4.595
- PFEIFFER, A., LESELLIER, F., & TOURNIER, M., 2020. Topological Interlocking Assemblies Experiment (C. Gengnagel, O. Baverel, J. Burry, Ramsgaard Thomsen, Mette, & S. Weinzierl, Eds.; pp. 336–349). Springer International Publishing. https://doi.org/10.1007/978-3-030-29829-6 27
- POTTMANN, H., BRELL-COKCAN, S., & WALLNER, J., 2007. Discrete Surfaces for Architectural Design. Curves and Surface Design: Avignon 2006, 213–234.
- POTTMANN, H., & WALLNER, J., 2008. The focal geometry of circular and conical meshes. Advances in Computational Mathematics, 29, 249–268. https://doi.org/10.1007/s10444-007-9045-4
- RETSIN, G., 2019. Toward Discrete Architecture: Automation takes Command.
- ROSSI, A., & TESSMANN, O., 2017. Integrating design and fabrication with discrete modular units.
- ROSSI, A., & TESSMANN, O., 2017a. Aggregated Structures: Approximating Topology Optimized Material Distribution with Discrete Building Blocks (p. 10).
- ROSSI, A., & TESSMANN, O., 2017B. Collaborative Assembly of Digital Materials.
- STAVIC, M., WILTSCHE, A., & FREIßLING, C., 2011. GEOMETRIC AND AESTHETIC DISCRETIZATION OF FREE FORM SURFACES.
- TESSMANN, O., 2012. Topological Interlocking Assemblies.
- TESSMANN, O., & ROSSI, A., 2019. Geometry as Interface: Parametric and Combinatorial Topological Interlocking Assemblies. Journal of Applied Mechanics, 86, 111002. https://doi.org/10.1115/1.4044606
- WANG, W., & LIU, Y., 2009. A Note on Planar Hexagonal Meshes (Emiris, Ioannis Z, F. Sottile, & T. Theobald, Eds.; Vol. 151, pp. 221–233). Springer New York. https://doi.org/10.1007/978-1-4419-0999-2 9
- GLYMPH, J., SHELDEN, D., CECCATO, C., MUSSEL, J., & SCHOBER, H., 2004. A parametric strategy for free-form glass structures using quadrilateral planar facets. Automation in Construction, 13(2), 187–202. https://doi.org/10.1016/j.autcon.2003.09.008

- DRUDE, J. P., ROSSI, A., & BECKER, M., 2020. Project DisCo: Choreographing Discrete Building Blocks in Virtual Reality. In C. Gengnagel, O. Baverel, J. Burry, M. Ramsgaard Thomsen, & S. Weinzierl (Eds.), Impact: Design with All Senses (pp. 288–299). Springer International Publishing. https://doi.org/10.1007/978-3-030-29829-6_23
- TIBBITS, S., 2011. A Model for Intelligence of Large-scale Self-assembly. 8.
- KLEMMT, C., PANTIC, I., GHEORGHE, A., & SEBESTYEN, A., 2019. Discretized Fabrication of Geometries Generated with Cellular Growth Simulations. 11.
- KAIJIMA, S., MICHALATOS, P., TAYLOR, A., & LONDON.,2007. Discretization of Continuous Surfaces as a Design Concern.
- FARMER, M., 2016. The Farmer Review of the UK Construction Labour Model Modernise or Die Time to decide the industry's future.
- SRINIVASAN V, AKLEMAN E, MANDAL E., 2005. Remeshing schemes for semi-regular tiling's. in ieee compute. soc; 2005. p. 44–50. Available from: http://ieeexplore.ieee.org/document/1563209/
- PINKALL, U. BOBENKO, A.,1996. Discrete isothermic surfaces. Journal für die reine und angewandte Mathematik (Crelles Journal), 1996(475), pp.187–208. doi:10.1515/crll.1996.475.187.
- PASTERNAK, E., DYSKIN, A.V., ESTRIN, Y., KHOR, H.C. AND KANEL-BELOV, A.J., 2003. Fracture Resistant Structures Based on Topological Interlocking with Non-planar Contacts. Advanced Engineering Materials, 5(3), pp.116–119. doi:10.1002/adem.200390016.