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An improved long short-term memory based on global optimization square root extended Kalman smoothing algorithm for collaborative state of charge and state of energy estimation of lithium-ion batteries

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Summary

State of charge and state of energy are essential performance indicators of the battery management system and the key to reflecting the remaining capacity of batteries. Aiming at the problems of low precision, long time, and strongly nonlinear system estimation of state of charge and state of energy of lithium-ion batteries based on traditional algorithm under complex working conditions, this paper proposes a hybrid method consisting of the long short-term memory neural network and square root extended Kalman smoothing. The long short-term memory neural network can enhance the memory ability of the previous time data. The sliding window technology is introduced into the network to improve the correlation between the last time and the subsequent time estimation. Based on the traditional Kalman filtering algorithm, the square root and reverse smoothing algorithms are introduced to solve the risk of the negative covariance matrix and the problems of slow convergence and significant estimation deviation caused by a strongly nonlinear system. According to experiments, under the hybrid pulse power characterization working condition at 25°C, the maximum absolute errors of state of charge and state of energy are 1.779% and 1.487%, and the mean absolute errors are 0.352% and 0.894%, respectively. Under the Beijing bus dynamic stress test working condition at 25°C, the maximum absolute errors of state of charge and state of energy are 2.703% and 2.369%, and the mean absolute errors are 0.462% and 0.621%, respectively. The experimental results show that this algorithm can obtain reliable state of charge and state of energy under different complex working conditions with high accuracy, convergence, and robustness.

KEYWORDS

Collaborative estimation; Long short-term memory; Square root extended Kalman smoothing; State of charge; State of energy

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1 INTRODUCTION

With the continuous development of Chinese industry, although it has promoted the progress of human beings, human beings are faced with severe problems such as environmental pollution and the energy crisis. Thus, the emergence of new energy provides the direction for environmental protection and solving the problem of energy exhaustion. Lithium-ion batteries are widely used as the power source of new-energy vehicles because of their high energy and power density, long cycle life, and no memory effect.[1,2] The research on lithium-ion batteries is of great significance to the protection of natural resources and is widely used in real life. Although lithium-ion batteries have been used in various engineering fields in recent years, explosion incidents of lithium batteries have occurred frequently at home and abroad. Therefore, lithium-ion batteries' safe and reliable operation is the focus and a difficulty for the researchers in this field. The power system of new energy vehicles is generally composed of hundreds of individual batteries connected in parallel or in series, and with related electromechanical equipment, the battery management system (BMS) can be used to detect the usage status of the batteries.[3] Under the overall planning of BMS, the power system can efficiently cooperate with the whole vehicle. Therefore, a sound BMS system is of great significance to batteries' efficiency, safety, reliability, and low-cost operation.[4] In BMS function management, the reliability analysis of the battery's state of charge (SOC), state of energy (SOE), state of health (SOH), state of power (SOP), state of temperature (SOT), state of safety (SOS), residual service life (RUL), and residual discharge time (RDT) is a prerequisite for the management of battery charge–discharge efficiency, safety management, thermal management and health.[5,6] Accurate and effective estimation of SOC and SOE is the basis of the battery management system and also the key to the estimation of battery remaining capacity. SOC stands for the state of the battery's remaining charge, and SOE stands for the state of the battery's remaining energy.[7,8] The accurate and effective estimation of the two factors plays a vital role in the rational allocation of power energy and the extension of battery range.

Standard algorithms for estimating SOC fall into four main categories, including characteristic parameter (CP),[9] ampere-hour integral (AHI) or Coulomb counting method, battery equivalent model,[10,11] and machine learning algorithm.[12–15] The open-circuit voltage and the internal resistance methods establish the corresponding relation between the relevant parameters and SOC to achieve the estimation purpose.[16] The Ampere integration method calculates SOC by integrating current in time, and an accurate initial estimate needs to be known or measured in advance.[17] The calculation formula of SOC for lithium-ion batteries based on amp-hour integration is shown in Equation (1).

$$SOC_k = SOC_{k-1} - \frac{\int_{k-1}^{k} \eta I_L dt}{Q_n}, \tag{1}$$

where η is the coulombic efficiency, I_L is the load current of the battery; Q_n is the total available capacity of the battery; SOC_k and SOC_{k-1} are the battery SOC value at time k and k-1, respectively. Typical battery equivalent circuit models include the Rint, Thevenin, and partnership for a new generation of vehicles (PNGV) models. The Rint model, also known as the internal resistance model, is the simplest equivalent circuit model, but it is not suitable for describing the nonlinear output characteristics of lithium batteries.[18] Thevenin model is also called the first-order resistor-capacitor (RC) model, which considers the polarization phenomenon of lithium-ion batteries. RC loop can describe the voltage stabilization characteristics of batteries after charging and discharging.[19] PNGV model describes the battery's open-circuit voltage change caused by the accumulated load current over time. This model can describe the battery's output characteristics, but the series capacitance will increase the cumulative error.[20] Most of the battery equivalent circuit models will combine algorithms to estimate the SOC of lithium batteries. Common algorithms include the Kalman filter (KF) algorithm and the synovial observer method. KF algorithm is mainly used to estimate the linear time-invariant systems. It adopts the recursive linear minimum variance estimation method and uses the observable output estimation error of the system to repair the unobservable state estimation error, thus significantly reducing the noise interference in the data stream and improving the estimation accuracy of the new system.[21] Researchers have recently developed a series of improved KF algorithms to obtain more accurate SOC estimates. For example, Ma et al, to enhance the volume of Kalman robustness and the estimation precision of the non-Gaussian noise environment, put forward a SOC estimation model based on cubature KF with generalized maximum correntropy criterion (GMCC- CKF).[22] Maheshwari and Nageswari use the sunflower optimization (SFO) algorithm to find the optimal values of the noise covariance matrices before applying extended KF (EKF) for online SOC estimation, the experimental results show that the algorithm has a high

SOC estimation accuracy and a relatively high convergence rate under static and dynamic working conditions.[23] Qi et al proposed the adaptive spherical unscented KF (AS-UKF) algorithm based on the equivalent circuit model of second-order RC cells, which improved accuracy and became more stable.[24] However, these algorithms generally require tedious parameter identification of the battery equivalent circuit model, which prolongs the prediction time. Machine learning algorithms mainly include long short-term memory (LSTM), support vector machines (SVMs), particle filter (PF), deep learning, and so forth. For example, Li et al adopted an online estimation method of lithium-ion health based on a particle swarm optimization-SVM algorithm (PSO-SVM), and the estimation results showed good adaptability and feasibility.[25] To obtain more accurate remaining useful life (RUL), Zhang et al proposed a Bayesian mixture neural network (BMNN) consisting of Bayesian convolutional neural network (BCNN) and Bayesian LSTM (B-LSTM).[26] This method does not need to establish a definite mathematical model, but it needs enough samples to estimate SOC. Compared with the method based on the battery equivalent circuit model, the machine learning algorithm reduces the prediction time.

SOE is a crucial fundamental state parameter for electric vehicles (EVs) remaining range estimation and charging remaining time prediction. Unlike SOC, SOE is not only an integral of battery current but also an integral of voltage. Standard SOE estimation algorithms are similar to the four types of algorithms for SOC estimation. SOE value can be obtained directly through SOC value and the mapping relationship between them, but the SOC estimation does not consider various energy loss factors, so the SOE estimation indirectly obtained from the mapping relationship is not accurate. As researchers pay more attention to SOE estimation, more and more accurate algorithms for SOE estimation have appeared in front of the public.[27,28] Traditional SOE estimation methods are divided into power integral and open circuit voltage (OCV) methods. The power integration method is mainly based on the definition of energy state. By integrating the input or output power of the lithium-ion battery at the current moment, the increased or consumed power of the battery is calculated, and the SOE value of the previous moment is added to obtain the remaining power information of the battery.[29] The SOE calculation formula of lithium-ion batteries based on the power integration method is shown in Equation (2).

$$SOE_k = SOE_{k-1} - \frac{\int_{k-1}^{k} U_k \cdot I_L dt}{E_n}, \tag{2}$$

where U_k is the terminal voltage of the battery, I_L is the load current of the battery, E_n is the total available energy of the battery, SOE_k and SOE_{k-1} are battery SOE values at time k and k-1, respectively. From the definition of the power integral method, it can be seen that the initial value of battery SOE determines whether the SOE estimation is accurate, so there is a significant error in using this method alone. The open-circuit voltage method is similar to the SOC estimation method, which uses the relationship between the battery energy state and the open-circuit voltage to obtain the SOE value at the current moment. Using the OCV method to measure SOE in real time is not feasible because the chemical reaction inside the battery needs to be stable when the energy state and OCV are obtained by experiment. To accurately construct and timely update the relationship between OCV and SOE, Zhang and Zhang proposed a non-experimental reconstruction method without additional battery testing.[30] In the method based on the equivalent circuit model, to improve the accuracy and robustness of SOE estimation, Lai et al proposes a method based on PF and EKF, which is insensitive to uncertain total available energy loss and ambient temperature.[31] Based on data, drive can better solve the nonlinear problems of the system. Mei et al proposed a joint prediction algorithm of SOE and SOH based on the combination of car-cloud collaborative LSTM, bidirectional LSTM (Bi-LSTM), and CNN. This method can not only use the historical battery data of the cloud platform to predict SOH but also correct SOE according to the predicted value of SOH. The error is kept within 3%.[32]

However, the single traditional SOC and SOE estimation methods cannot get more accurate and effective estimates. With the progress of science and technology, the emergence of machine learning has continuously solved this problem. Deep learning belongs to a special kind of machine learning. The deep learning model is a deep learning neural network model with multiple nonlinear mapping levels, which can abstract and extract features layer by layer from input signals to dig out more profound potential rules. Deep learning neural networks include convolutional neural network (CNN), generative adversarial network (GAN), and recurrent neural network (RNN).[33–35] Unlike the previous two types of neural networks, RNN considers the input of the previous moment and endows the network with a "memory" function of the previous content. To solve the problem of RNN gradient explosion and gradient disappearance, as well as the problem of long-term dependence, Hochreiter and Schmidhuber improved RNN in 1997 and proposed LSTM network. It was improved and popularized by Alex Graves.[36] As a new machine learning algorithm, LSTM neural network can realize the prediction of SOC and SOE of lithium-ion batteries through self-learning adjustment of network weight and bias parameters.[37–39] This algorithm can accurately map the measured information, such as voltage, current, battery surface temperature, and so forth, to SOC and effectively avoid the cumbersome parameter calculation process, such as the KF algorithm. In the training process, only the number of hidden layer neurons, batch size, the maximum number of iterations, and LSTM nuclei can be set to obtain the optimal model.[40] LSTM neural network can obtain different network parameter models under different working conditions, input parameter variables, and even different battery types. This has significant advantages compared with the traditional SOC and SOE estimation methods.[29,39] LSTM also has many variants, the most typical of which is the gated recurrent unit (GRU).[41–43] It has a recurrent gate that synthesizes the forgetting and input gates into a single updating gate and mixes cell and hidden states. However, compared with GRU, LSTM has more model parameters, is more powerful, and is more expressive, thus more suitable for large data sets such as battery data sets. Chen et al developed a hybrid data science model based on empirical mode decomposition (EMD), grey relation analysis (GRA), and deep RNNs, and the results show that compared with other variants of deep cyclic neural networks, LSTM network has the best effect.[44] Kim and Lee used the Gaussian process, multi-layer perceptron, and LSTM network to predict wind information. The results showed that the LSTM network had the best prediction effect on short-term time series wind data sets.[45]

Although the deep learning algorithm is not based on the lithium-ion battery model, it simplifies SOC and SOE joint estimation and has good generalization ability. However, there are still some problems, such as the need for a large number of sample data to train the network, the complexity of parameter adjustment, and the poor degree of fitting, leading to output instability and other problems.[46] In recent years, many researchers in lithium-ion batteries have combined deep learning algorithms with model-based algorithms.[47] Huang et al proposed a method based on EKF combined with LSTM to solve the shortcomings of continuity, cumulative error, divergence over time, and reliability of inertial navigation system (INS) and global positioning system (GPS). The paper pointed out that it is effective to combine EKF with GPS and INS data.[48] Yang et al proposed a method based on the combination of untracked KF and LSTM to accurately predict the SOC value of lithium-ion batteries.[49]

To accurately obtain the SOC and SOE, this paper proposes an improved LSTM neural network based on global optimization square root extended Kalman smoothing (SREKS) algorithm. Compared with UKF, EKF has higher computational efficiency and applicability. In the UKF algorithm, it takes a significant amount of time to process all sigma points for each sigma point sampling, whereas EKF only carries out one operation. In some research areas, such as location problems, the computational cost of UKF will be significantly greater than that of EKF. However, the EKF used in strong nonlinear systems has the problem of low accuracy and even divergence. The square root algorithm is introduced to ensure the positive quality of error covariance and solve the problems of non-convergence and significant estimation deviation of the strong nonlinear system like lithium battery estimated by the EKF algorithm. Compared with the literature[15,39] that only uses LSTM neural network, this algorithm has higher accuracy and better filtering effect. Meanwhile, compared with the correlation algorithm based on the battery equivalent circuit model in the literature,[50,51] it has less prediction time, more accessible model construction, and broader applicability. The main contributions of this paper are as follows.

1. An improved sliding window LSTM neural network model is constructed, which takes current, voltage, and temperature as inputs and SOC and SOE as outputs. The speed and accuracy of estimation are improved by fast learning of nonlinear relations.
2. The SREKS method is used to improve the SOC and SOE estimation results of LSTM network output to solve the problems of non-convergence and significant estimation deviation caused by a strong nonlinear system.
3. Hybrid pulse power characterization (HPPC) and Beijing bus dynamic stress test (BBDST) conditions are used at different temperatures (15 and 25°C) to verify the accuracy of the LSTM-SREKS algorithm. Besides this, the mean square error (MSE) and mean absolute error (MAE) values of the algorithm are significantly better than other algorithms (LSTM, LSTM-EKF, and LSTM-square root extended Kalman filter [LSTM-SREKF]).

2 MATHEMATICAL ANALYSIS

2.1 Improved LSTM neural network

The emergence of LSTM solves the problems of gradient disappearance, gradient explosion, and short-term memory of RNN. The gradient is used to update the weight value of the neural network. During the backpropagation of the network, the error is constantly reduced to update the weight, and RNN will face the problem of disappearing gradients. On the other hand, the complexity of calculation difficulty leads to gradient explosion.[52,53] RNN is specially used to process time series and can extract time series information. RNN has a cyclic structure inside, including the chain form of the repeating neural model. An RNN can be thought of as multiple copies of the same neural network, with each neural network module passing a message to the next. The RNN unfolded structure is shown in Figure 1, where A is each neural network module, x_t is the input data, h_t is the hidden layer output.

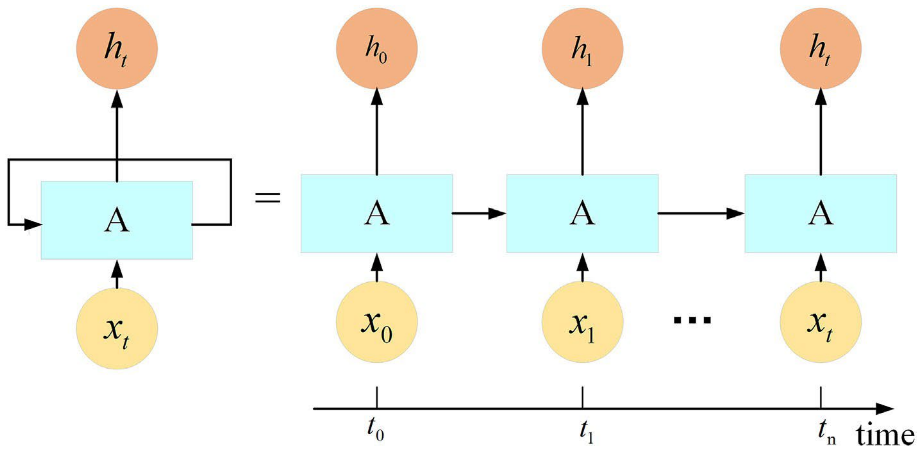


FIGURE 1 The expansion structure of the recurrent neural network.

LSTM is a modified RNN network designed specifically to solve the long-term dependency problem by designing a threshold structure: Keep relevant information to make predictions, and forget irrelevant data. LSTM has three types of gate structures: forget gate, input gate, and output gate to select the passage of information to protect and control the cell state. The structure of LSTM is shown in Figure 2.

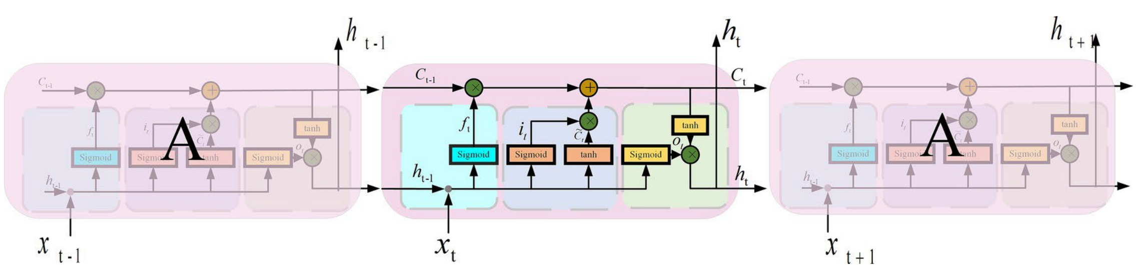


FIGURE 2 Long short-term memory network structure.

RNN is a repetitive single neural network layer, whereas the repetitive modules in LSTM contain three sigmoid and a tanh layer and interact in a particular way. A single cell of LSTM is shown in Figure 3.

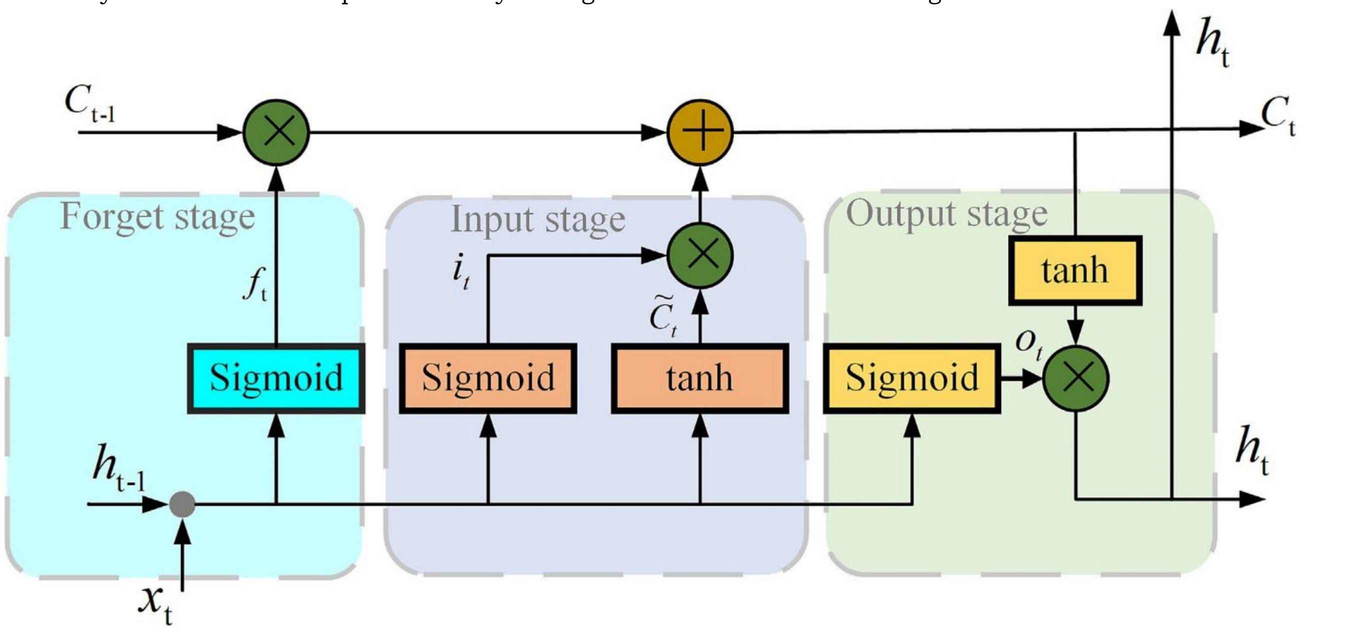


FIGURE 3 Long short-term memory single nuclear structure.

In Figure 3, the first step is the "forget gate," which reads the output h_{t-1} of the previous hidden layer and the current input x_t. The sigmoid neural network layer is used to discard and retain relevant information, and the information values are converted into numbers between zero and one. All information that is prohibited is represented by zero, and one means that all information is passed. The output vector f_t is multiplied by the previous cell state C_{t-1}. The second step is the "input gate." The sigmoid layer updates the relevant information, and the tanh layer creates a new candidate value $\tilde{C}_t$. The tanh layer converts all the information values to 0 and 1 and adds them to the cell state. The third step is updating the cell state from C_{t-1} to C_t state. Finally, there is the "output gate." The sigmoid layer determines the part of the cell state that needs to be outputted. The cell state is processed through the tanh layer and multiplied by the output of the sigmoid layer to output information. The parameter is calculated as Equation (3). W_f, W_i, W_C, and W_o represent the weight matrix, while b_f, b_i, b_C, and b_o represent the bias matrix.

$$\begin{cases} f_t = sigmoid(W_f \cdot [h_{t-1}, x_t] + b_f) \\ i_t = sigmoid(W_i \cdot [h_{t-1}, x_t] + b_i) \\ \tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C) \\ C_t = f_t * C_{t-1} + i_t * \tilde{C}_t \\ o_t = sigmoid(W_o [h_{t-1}, x_t] + b_o) \\ h_t = o_t * \tanh(C_t) \end{cases}. \tag{3}$$

2.2 | SREKS

The higher the precision of state estimation in the BMS system, the stronger the security and longer the service life of the lithium-ion battery. The KF algorithm based on the model can filter the noise from the observed signal. Because of the nonlinear nature of lithium-ion batteries, the extended Kalman algorithm was proposed to linearize the nonlinear state-space model, and the basic KF algorithm was used for filtering. To further improve the accuracy of SOC and SOE joint estimation, this paper proposes the SREKS algorithm. The square root algorithm is introduced to ensure the positive quality of the state covariance matrix.[54] Smoothing is introduced to smooth the results, thus significantly reducing the estimation error and improving the parameter estimation accuracy.

KF algorithm is mainly used to estimate linear time-invariant systems. The recursive linear minimum variance estimation method is used to repair the unobservable state estimation error with the observable output estimation error of the system, thus significantly reducing the noise interference in the data stream and improving the estimation accuracy of the new system. The equation of state and observation equation after discretization is shown in Equation (4).

$$\begin{cases} x_{k+1} = A_K x_k + B_k u_k + w_k \\ y_k = C_k x_k + D_k u_k + v_k \end{cases}. \tag{4}$$

x_k and y_k respectively represent the state variables and observation variables of the system. u_k represents the external excitation signal of the system; A_K is the state transition matrix; B_k is the control input matrix; C_k is the systematic observation matrix; D_k is the system feedforward matrix; w_k and v_k represent process noise and observation noise, respectively.

Each iteration of the KF algorithm includes two stages of prediction and update, and the optimal estimate in the sense of minimum variance is obtained through multiple iterations. For the discrete linear system expressed in Equation (2), the recursive formula of KF can be obtained, as shown in Equation (5).

$$\begin{cases} \widehat{x}_{k+1} = A_k x_k + B_k u_k \\ \widehat{P}_{k+1} = A_k P_k A_k^T + Q_k \\ K_{k+1} = \widehat{P}_{k+1} C_{k+1}^T \left(C_{k+1} \widehat{P}_{k+1} C_{k+1}^T + R_{k+1} \right)^{-1} \\ x_{k+1} = \widehat{x}_{k+1} + K_{k+1} (y_k - C_{k+1} \widehat{x}_{k+1} - D_{k+1} u_{k+1}) \\ P_{k+1} = (E - K_{k+1} C_{k+1}) \widehat{P}_{k+1} \end{cases}. \tag{5}$$

Q_{k-1} is the process noise covariance matrix; whereas, R_{k-1} is the observed noise covariance matrix.

The EKF algorithm solves the significant nonlinear problem of lithium-ion batteries during operation. The nonlinear system is linearized by using Taylor series expansion and ignoring higher-order terms to obtain a new state transition matrix and observation-driven matrix to update the linear space equation. The recursion of the EKF algorithm is shown in Equation (6).

$$\begin{cases} \widehat{x}_{k+1}^{-} = f(\widehat{x}_k) \\ \widehat{P}_{k+1}^{-} = A_k \widehat{P}_k A_k^T + Q_{k+1} \\ K_{k+1} = \widehat{P}_{k+1}^{-} C_{k+1}^T \left(C_{k+1} \widehat{P}_{k+1}^{-} C_{k+1}^T + R_{k+1} \right)^{-1} \\ \widehat{x}_{k+1} = \widehat{x}_{k+1}^{-} + K_{k+1} (y_k - C_{k+1} \widehat{x}_{k+1} - D_{k+1} u_{k+1}) \\ \widehat{P}_{k+1} = (E - K_{k+1} C_{k+1}) \widehat{P}_{k+1}^{-} \end{cases}. \tag{6}$$

The SREKS algorithm is divided into forward filtering and backward smoothing. Forward is the combination of the square root algorithm and Kalman filtering algorithm (SREKF), and backward is the smoothing of forward results. The forward square root algorithm decomposed the error covariance matrix P_k in the EKF algorithm into a product of the lower triangular matrix and its transpose, namely the Cholesky decomposition, which ensured the positive quality of the error covariance matrix. The backward smoothing of the forward results dramatically reduces the parameter estimation error.

2.2.1 Forward filtering

The state space equation of the SREKF algorithm is the same as that of the EKF algorithm. In the process of recursive state estimation, the minimum mean square error between the measured value and the estimated value is constantly minimized, and the state space equation established by the system itself is combined to obtain the optimal state value at the current time of the system. The state space equation of the nonlinear system of the SREKF algorithm is shown in Equation (7).

$$\begin{cases} x_{k+1} = f(x_k, u_k) + w_k = A_k x_k + B_k u_k + w_k \\ Z_k = g(x_k, u_k) + v_k = C_k x_k + D_k u_k + v_k \end{cases} \tag{7}$$

The SREKF algorithm could be derived from the EKF algorithm, and the derivation process is as follows: First, the error covariance matrix was decomposed by Cholesky into the lower triangular matrix S_k, and the transposition of the lower triangular matrix S_k^T, the following equation can be derived.

$$\begin{cases} P'_{k+1} = \tilde{S}'_{k+1}\left(\tilde{S}'_{k+1}\right)^T \\ \tilde{S}'_{k+1} = A_K S_K \\ P_{k+1} = \tilde{S}'_{k+1}\left(\tilde{S}'_{k+1}\right)^T - \tilde{S}'_{k+1}\left(\tilde{S}'_{k+1}\right)^T C_k^T C_k \tilde{S}'_{k+1}\left(\tilde{S}'_{k+1}\right)^T + R_k C_k \tilde{S}'_{k+1}\left(\tilde{S}'_{k+1}\right)^T \end{cases} \tag{8}$$

If $F_{k}\_ \left(\tilde{S}^{-}_{k+1}\right)^T C_k^T$ is satisfied, then it can be proven that:

$$\begin{cases} P_{k+1} = \tilde{S}'_{k+1}\left[E - \alpha_k F_k^T F_k \left(\tilde{S}'_{k+1}\right)^T\right]\left(\tilde{S}'_{k+1}\right)^T \\ \alpha_k = \left[F_k^T F_k + R_k\right]^{-1} \end{cases} \tag{9}$$

If $P_{k+1} = \tilde{S}_{k+1}\tilde{S}_{k+1}^T$ is satisfied, then it can be shown that:

$$\begin{cases} E - \alpha_k F_k^T F_k = E - \alpha_k F_k\left[2\gamma_k F_k^T - \alpha_k \gamma_k^2 F_k^T F_k\right] F_k^T \\ \tilde{S}_{k+1} = \tilde{S}'_{k+1}\left[E - \alpha_k \gamma_k F_k F_k^T\right] \end{cases} \tag{10}$$

According to Equation (6)–(8), the equation of the SREKF algorithm can be obtained as follow.

$$\begin{cases} x'_k = f(x_{k-1}, u_{k-1}) \\ \tilde{S}'_k = A_{k-1} S_{k-1} \\ K_{k-1} = \alpha_{k-1}\tilde{S}'_{k-1} F_{k-1} \\ \alpha_{k-1} = \left[F_{k-1}^T F_{k-1} + R_{k-1}\right]^{-1} \\ F_{k-1}\_ \left(\tilde{S}'_k\right)^T C_{k-1}^T \\ x_k = x'_k + K_{k-1}\left[Z_{k-1} - A_{k-1} C_{k-1} x'_k\right] \\ \tilde{S}_k = \tilde{S}'_k\left[E - \alpha_{k-1}\gamma_{k-1} F_{k-1} F_{k-1}^T\right] \\ \gamma_{k-1} = \dfrac{1 \pm \sqrt{\alpha_{k-1} R_{k-1}}}{1 - \alpha_{k-1} R_{k-1}} \end{cases} \tag{11}$$

2.2.2 Backward smoothing

$$\begin{cases} x'_{k+1} = A_k x_k \\ \tilde{S}'_{k+1} = A_k \tilde{S}_k A_k^T + Q_k \\ G_k = \tilde{S}_k A_k^T \left(S'_{k+1}\right)^{-1} \\ x_k^s = x_k + G_k\left(x_{k+1}^s - x'_{k+1}\right) \\ \tilde{S}_k^s = S_k + G_k\left(\tilde{S}_{k+1}^s - \tilde{S}'_{k+1}\right) G_k^T \end{cases} . \tag{12}$$

Based on the forward calculation of the N-dimensional state variable x_k and covariance matrix S_k, a new Kalman gain G_k, state variable x_k^s, and covariance matrix $\tilde{S}_k^s$ are obtained, so that the results of the LSTM-SREKF algorithm are smoothed backward, and the measurement accuracy of the whole system is improved.

2.3 LSTM based on global optimization SREKS algorithm

The traditional neural network or the algorithm based on the battery equivalent model cannot meet the expected estimation accuracy. This paper combines the neural network algorithm with the improved model-based algorithm and proposes an improved LSTM based on global optimization SREKS algorithm for the collaborative SOC and SOE estimation of lithium-ion batteries. The overall flowchart of the algorithm is shown in Figure 4.

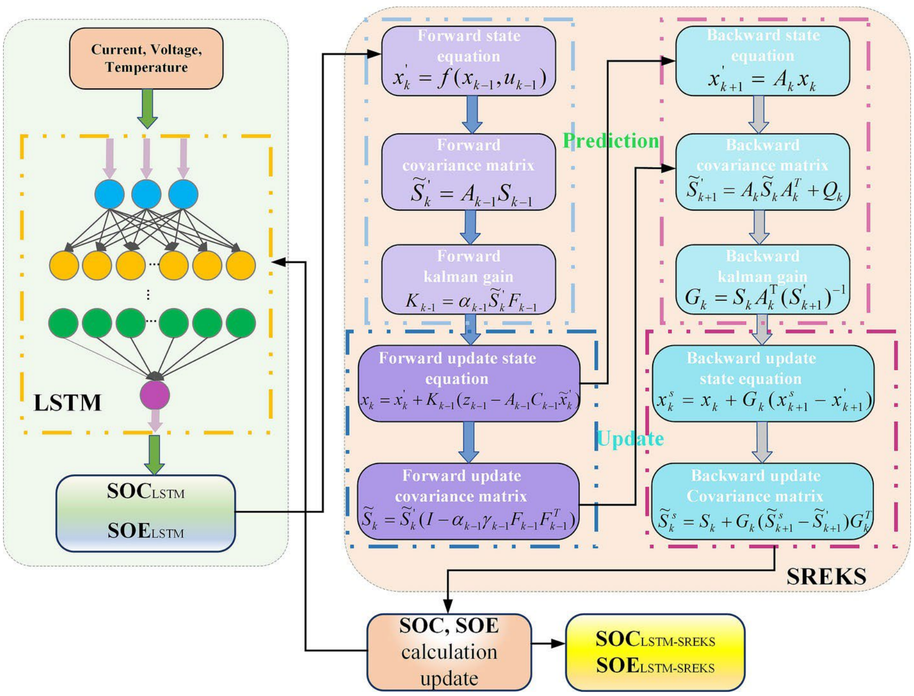


FIGURE 4 A joint state of charge (SOC) and state of energy (SOE) estimation method for lithium-ion batteries based on the long short-term memory-square root extended Kalman smoothing (LSTM-SREKS) model.

Figure 4 shows that the current, voltage, and temperature of lithium-ion batteries under certain conditions are taken as the input of the LSTM network to estimate and output the SOC and SOE. Then, SREKS algorithm is used to fit the output of the LSTM algorithm to get the final estimated SOC and SOE values.

3 EXPERIMENTAL ANALYSIS

3.1 Lithium-ion battery test platform design

Among standard lithium-ion batteries, terpolymer lithium-ion batteries are favored by researchers because of their excellent stability, durability, environmental friendliness, and low cost. This study was conducted based on a 72 Ah ternary lithium-ion battery. The experimental configuration is divided into five parts: temperature test chamber, charge– discharge test, lithium-ion battery, PC, and BMS. The lithium-ion battery test platform, as shown in Figure 5.

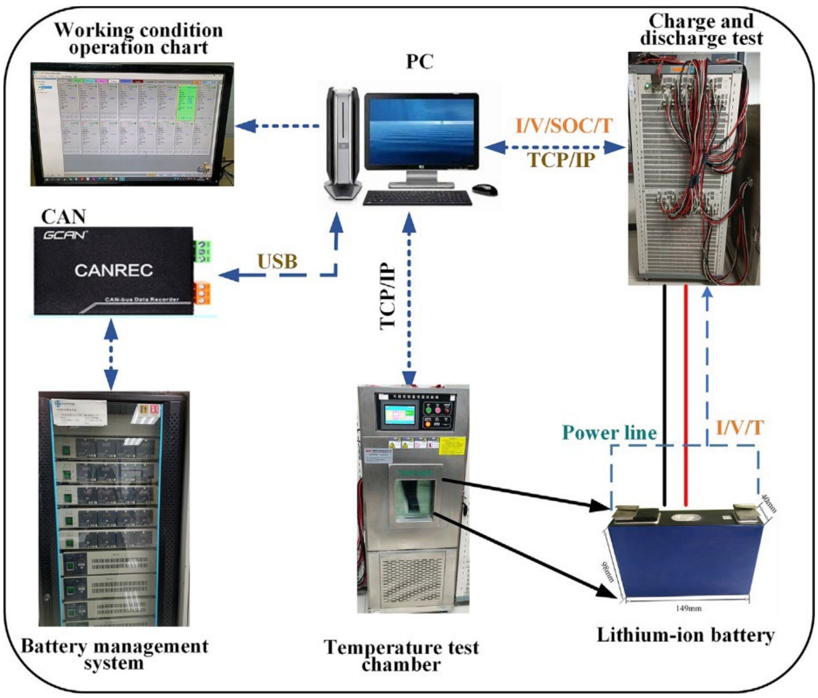


FIGURE 5 The constructed experimental test platform for the lithium-ion batteries test.

In this paper, the HPPC and BBDST tests were carried out on lithium-ion batteries under different temperature conditions. HPPC test data at 15°C were used as training data, and HPPC and BBDST test data at 25°C were used to verify the performance of the LSTM-SREKS algorithm. The test results are shown in Figure 6.

3.2 SOC estimation based on improved LSTM-SREKS algorithm

LSTM network uses HPPC test data at 15°C as the training data and uses HPPC and BBDST test data at 25°C to evaluate the performance of the network. The network takes the battery current (I), terminal voltage (U), and ambient temperature (T) as the characteristic inputs and the SOC and SOE as the outputs of the network. A “many to one” (m-1) input–output structure is formed, the sliding window technology is used to strengthen the data relationship, and the sliding window size is set as 3. Mean square error (MSE) was used as the loss function and Adam as the optimizer. To prevent the appearance of training overfitting, the dropout layer is added, and the ratio chosen in this paper is 0.3. The relevant parameter settings of the LSTM network are shown in Table 1.

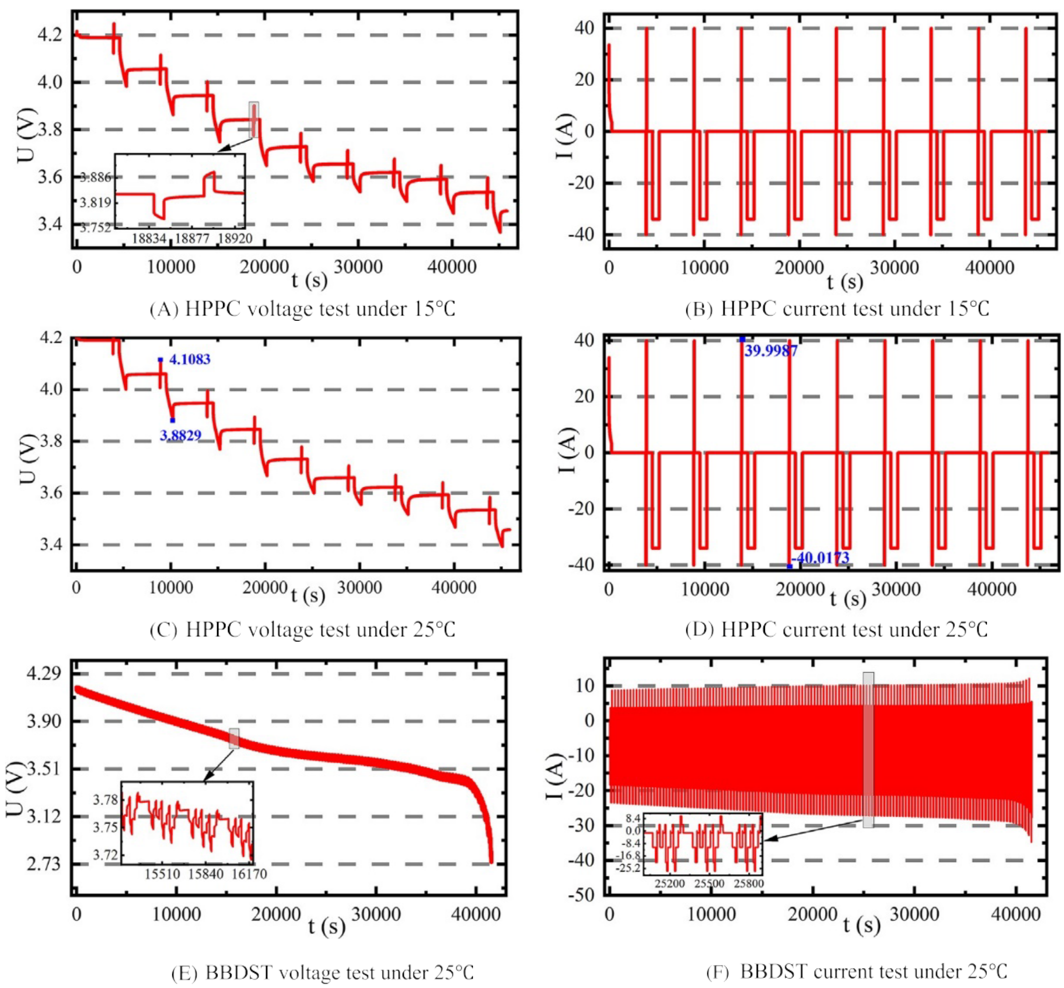


FIGURE 6 Hybrid pulse power characterization (HPPC) and Beijing bus dynamic stress (BBDST) voltage and current tests at ambient temperatures of 15°C and 25°C. (A) HPPC voltage test under 15°C. (B) HPPC current test under 15°C. (C) HPPC voltage test under 25°C. (D) HPPC current test under 25°C. (E) BBDST voltage test under 25°C. (F) BBDST current test under 25°C.

TABLE 1 Parameter setting of LSTM and LSTM-SREKS network models.

Parameter index	LSTM	LSTM-SREKS
Input layer node	3	3
Neural network layer node	32	32
Full connection layer node	1	1
Output layer node	1	1
Adaptive optimizer	Adam	Adam
Evaluation function	MSE/MAE	MSE/MAE
Batch size	100	100
Number of training iterations	20	20

Abbreviations: LSTM, long short-term memory; LSTM-SREKS, long short-term memory-square root extended Kalman smoothing; MAE, mean absolute error; MSE, mean square error.

To verify the accuracy of the improved LSTM-SREKS algorithm for SOC estimation, this paper also compares it with LSTM, LSTM-EKF, and LSTM-SREKF algorithms, and the results are shown in Figure 7.

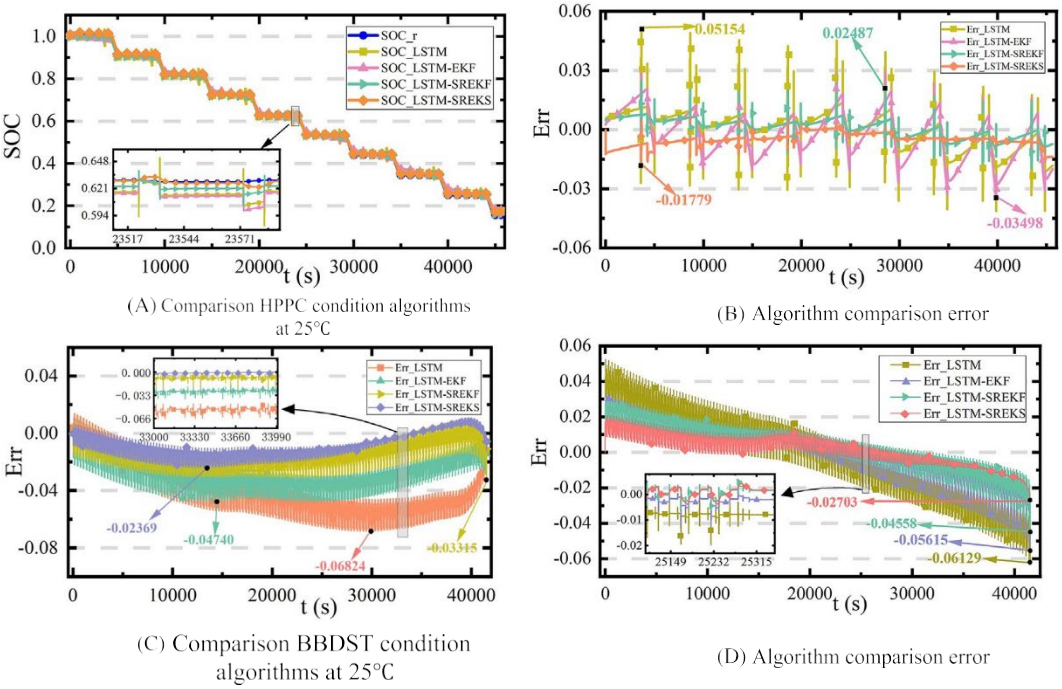


FIGURE 7 State of charge (SOC) estimation results of the long short-term memory (LSTM), LSTM-extended Kalman filter (LSTM-EKF), LSTM-square root extended Kalman filter (LSTM-SREKF), and LSTM-square root extended Kalman smoothing (LSTM-SREKS) algorithms under hybrid pulse power characterization (HPPC) and Beijing bus dynamic stress (BBDST) working condition. (A) Comparison HPPC condition algorithms at 25ºC. (B) Algorithm comparison error. (C) Comparison BBDST condition algorithms at 25ºC. (D) Algorithm comparison error.

Figure 7A,C shows the SOC estimation results of LSTM, LSTM-EKF, LSTM-SREKF, and LSTM-SREKS algorithms under HPPC and BBDST conditions at 25ºC. The initial estimated SOC value of the algorithm was set to 1, and the theoretical SOC value of the battery was calculated by the amp-hour integration method. It can be seen that the convergence and tracking effects of the LSTM-SREKS algorithm are superior to the other three algorithms. Figure 7B,D shows the comparison of the estimation errors of the four algorithms under HPPC and BBDST working conditions at 25ºC, respectively. The result shows that the maximum absolute errors of SOE estimation using LSTM, LSTM-EKF, and LSTM-SREKF algorithms are 5.154% and 6.129%, 3.498% and 5.615%, 2.486%, and 4.558%, respectively. As the improved LSTM-SREKS algorithm introduces the square root of the state error covariance matrix and smoothing algorithm for iterative operation, it solves the negative definite problem of the covariance matrix of the traditional algorithm and makes the estimation result more stable and close to the actual value. The maximum absolute errors of the LSTM-SREKS algorithm are 1.779% and 2.703%, respectively. Compared with the estimation errors of the other three algorithms, it can be proved that the improved LSTM-SREKS algorithm has a better estimation effect in the estimation of lithium-ion battery SOC. This paper also uses MSE and MAE to estimate the superiority of the algorithm, as shown in Table 2.

TABLE 2 MSE and MAE results of the algorithms.

	Estimation method	MSE	MAE
HPPC	LSTM	0.0045%	0.5491%
	LSTM-EKF	0.0034%	0.5059%
	LSTM-SREKF	0.0021%	0.4281%
	LSTM-SREKS	0.0012%	0.3516%
	Estimation method	MSE	MAE
BBDST	LSTM	0.0074%	0.7207%
	LSTM-EKF	0.0069%	0.6515%
	LSTM-SREKF	0.0061%	0.5912%
	LSTM-SREKS	0.0048%	0.4621%

Abbreviations: BBDST, Beijing bus dynamic stress; HPPC, hybrid pulse power characterization; LSTM, long short-term memory; LSTM-EKF, long short-term memory-extended Kalman filter; LSTM-SREKF, long short-term memory-square root extended Kalman filter; LSTM-SREKS, long short-term memory-square root extended Kalman smoothing; MAE, mean absolute error; MSE, mean square error.

According to Table 2, a bar chart of error distribution can be drawn, as shown in Figure 8. It can be seen that the MSE and MAE values of the LSTM-SREKS algorithm under HPPC and BBDST conditions are 0.0012%, 0.3516% and 0.0048%, 0.4621%, respectively. Compared with the other three algorithms, the LSTM-SREKS algorithm has the highest accuracy, the best tracking performance, and is closer to the actual value of SOC.

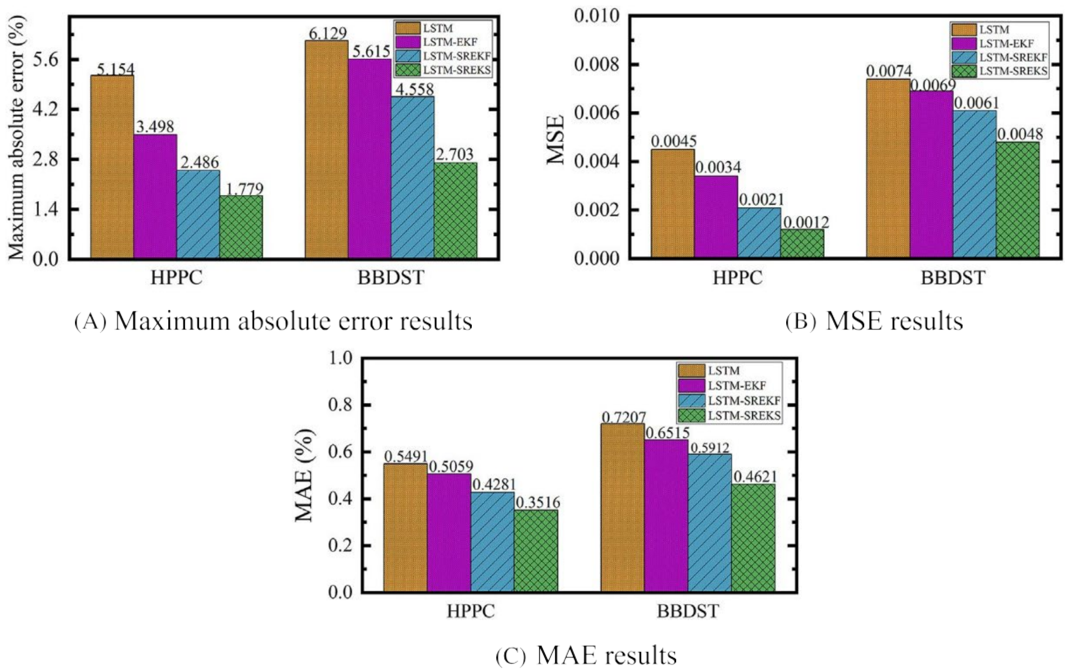
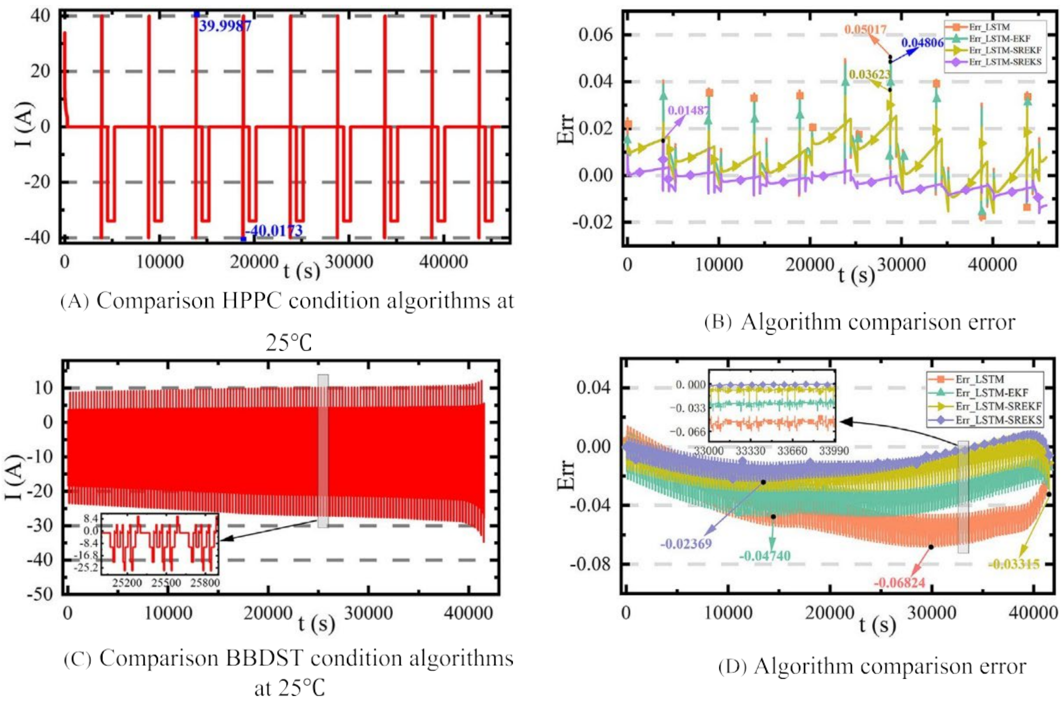


FIGURE 8 Maximum absolute error, mean square error (MSE), and mean absolute error (MAE) results of the four algorithms under hybrid pulse power characterization (HPPC) and Beijing bus dynamic stress (BBDST) working conditions. (A) Maximum absolute error results. (B) MSE results. (C) MAE results.

3.3 SOE estimation based on improved LSTM-SREKS algorithm

The accurate estimation of SOE is beneficial to improve the reliability of the prediction of battery remaining energy so that users can better understand the capacity status of the device battery. The estimation of SOE is the crucial index to ensure the rational distribution of power energy and extend the battery range. The initial estimated SOE value of the algorithm was set to 1, and the theoretical SOE value of the battery was calculated by the ampere integral method. LSTM network parameters are shown in Table 1. To verify the accuracy of the improved LSTM-SREKS algorithm for SOC estimation, this paper also compares it with LSTM, LSTM-EKF, and LSTM-SREKF algorithms, and the results are shown in Figure 9.



(A) Comparison HPPC condition algorithms at 25°C

(B) Algorithm comparison error

(C) Comparison BBDST condition algorithms at 25°C

(D) Algorithm comparison error

FIGURE 9 State of energy (SOE) estimation results of the long short-term memory (LSTM), LSTM-extended Kalman filter (LSTM-EKF), LSTM-square root extended Kalman filter (LSTM-SREKF), and LSTM-square root extended Kalman smoothing (LSTM-SREKS) algorithms under hybrid pulse power characterization (HPPC) and Beijing bus dynamic stress (BBDST) working conditions. (A) Comparison HPPC condition algorithms at 25°C. (B) Algorithm comparison error. (C) Comparison BBDST condition algorithms at 25°C. (D) Algorithm comparison error.

Figure 9A,C shows the SOC estimation results of LSTM, LSTM-EKF, LSTM-SREKF, and LSTM-SREKS algorithms under HPPC and BBDST conditions at 25°C. It can be seen that the improved LSTM-SREKS algorithm can track the change in the real SOE value. Figure 9B,D shows the comparison of the estimation errors of the four algorithms under HPPC and BBDST working conditions at 25°C. The maximum absolute errors of the LSTM-SREKS algorithm are 1.487% and 2.369%, respectively. By comparing with the estimation errors of the other three algorithms, it can be proved that the improved LSTM-SREKS algorithm has a better estimation effect in the estimation of lithium-ion battery SOE and dramatically improves the estimation accuracy, which can better improve the utilization rate of battery energy and increase the mileage of new energy vehicles. The estimated results of MSE and MAE are shown in Table 3.

According to Table 3, a bar chart of error distribution can be drawn, as shown in Figure 10. It can be seen that the MSE and MAE values of the LSTM-SREKS algorithm under HPPC and BBDST conditions are 0.0138% and 0.894% and 0.0097% and 0.621%, respectively. Therefore, the LSTM-SREKS algorithm has the highest accuracy and is closer to the actual value of SOE.

TABLE 3 MSE and MAE results of the algorithms.

	Estimation method	MSE	MAE
HPPC	LSTM	0.0347%	1.361%
	LSTM-EKF	0.0301%	1.268%
	LSTM-SREKF	0.0256%	1.015%
	LSTM-SREKS	0.0138%	0.894%
	Estimation method	MSE	MAE
BBDST	LSTM	0.0301%	1.423%
	LSTM-EKF	0.0247%	1.214%
	LSTM-SREKF	0.0152%	1.044%
	LSTM-SREKS	0.0097%	0.621%

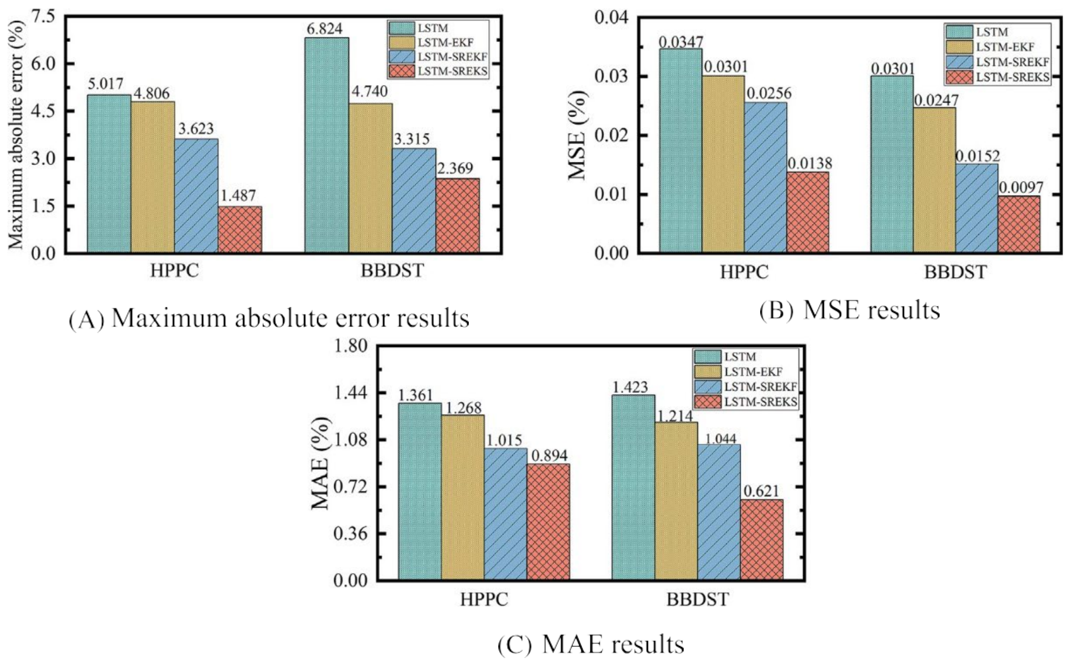


FIGURE 10 Maximum absolute error, mean square error (MSE) and mean absolute error (MAE) results of the four algorithms under hybrid pulse power characterization (HHPC) and Beijing bus dynamic stress (BBDST) working conditions. (A) Maximum absolute error results. (B) MSE results. (C) MAE results.

4 CONCLUSIONS

This paper proposes a hybrid method based on an improved LSTM network and SREKS. The LSTM network model is constructed, the sliding window technique is used to strengthen the relationship between data, and the speed and accuracy of estimation are improved through the fast learning of nonlinear relations. The square root and reverse smoothing algorithms are introduced to the traditional EKF algorithm. First, the square root algorithm is introduced to ensure the positive quality of the error covariance matrix and solve the problems of non-convergence and significant estimation deviation caused by the strong nonlinearity of the system. Then, the reverse smoothing algorithm is introduced to obtain a more accurate state estimate by taking full advantage of the measured values at all times in a fixed interval, which provides a higher precision than the unidirectional filtering and ensures the numerical stability of the filtering. The performance of the LSTM-SREKS algorithm is verified by using the data at different temperatures as the training set and the test set under the two complex conditions of HPPC and BBDST, and the results obtained are compared with the other three methods. Experimental results show that the algorithm has high accuracy and robustness. In this study, the deep learning algorithm was combined with the filter algorithm based on the battery equivalent model to solve the problems caused by the low accuracy, long time, and strong nonlinearity of the traditional algorithm in the estimation of the SOC and SOE of lithium-ion battery under complex working conditions, to improve the service life of the battery and ensure the safety of the battery system.

ACKNOWLEDGMENTS

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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