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The vaulting system of the Palatine Chapel: the Aachen Cathedral world heritage site documentation project.

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THE VAULTING SYSTEM OF THE PALATINE CHAPEL: THE AACHEN CATHEDRAL WORLD HERITAGE SITE DOCUMENTATION PROJECT

M. Attenni[1], R. Barni[1], C. Bianchini[1], M. Griffo[1*], C. Inglese[1], Y. Ley[2], D. Pritchard[3], G. Villa[1]

[1] Dipartimento di Storia, Disegno e Restauro dell'Architettura, SAPIENZA Università di Roma, Piazza Borghese 9, 00186 Roma Italy (martina.attenni, roberto.barni carlo.bianchini, marika.griffo, carlo.inglese, gugliemo.villa) @uniroma1.it
[2] RWTH Aachen University, Lehrstuhl für Architekturgeschichte, Chair of Architectural History, Schinkelstraße 1 D-52062 Aachen ley@ages.rwth-aachen.de
[3] Robert Gordon University, Aberdeen, Scotland – d.pritchard1@rgu.ac.uk

KEYWORDS: Aachen Palatine Chapel**,** integrated 3D survey, stereotomy, conic vaulting system, vault shape analysis.

ABSTRACT:
As part of a comprehensive survey and modelling project involving the Aachen Cathedral, this paper focuses on its oldest part, the Palatine Chapel, a domed octagonal hall supported by eight piers and enveloped by a sixteen-sided outer wall. Working on the data collected during an extensive 3D capturing campaign conducted between 2022 and 2023, this paper will focus on the conic vaults covering the ambulacrum of the 1st floor that represent quite a peculiar architectural and structural solution considering the VIII/IX century building know-how. In this framework, the Chapel's 3D point cloud has been analysed to extract the main 2D generative elements of the conic surfaces and then construct the corresponding 3D geometric models. These outputs have been compared against the captured point cloud to assess the differences between the actual vault data and the reconstructed ideal conic shapes. Finally, the method used to unfold the vaults' surfaces and create high-resolution ortho-images has been displayed.

1. INTRODUCTION

1.1 Aachen Cathedral

In the framework of the Aachen Cathedral Project (1), this paper focuses on its Carolingian core: the Palatine Chapel (Pieper and Schindler, 2017): the Palatine Chapel (Pieper & Schindler, 2017). The study aims to investigate the geometric components of the vaults covering the ambulacra and understand their configuration and constructive solutions.

The Palatine Chapel central hall is a triple-height octagonal space covered by a dome and encircled by an ambulacrum at the two accessible floors. The two ambulacra solve the transition between the octagonal space of the hall and the sixteen-sided shape of the external walls. This transition has been approached by experimenting with different solutions at the two storeys, resulting in various vaulting and connection typologies. Strong octagonal piers support the inner space on which lies the dome, covering the central hall. Groin vaults at the ground level and barrel and conic vaults at the double-height first floor compose the sixteen-sided circuits, the two storeys separated by a high cornice. Above the first-floor gallery's arches is an octagonal drum with window openings supporting the dome.

1.2 The Aachen Cathedral Project

Starting from an extensive 3D survey of the entire Cathedral combining digital photogrammetry and terrestrial laser scanning (2), the proposed workflow presents the geometric methods adopted for reconstructing the Palatine Chapel's conic vaults' shape (Zhang, 1994). The processing of the captured data entailed the 2D planimetric investigation of the first-floor ambulacrum, later focusing on the hi-res 3D modelling of the conic vaults themselves and, finally, on the 2D reprojection of these models onto a plane. This back-projection from 3D to 2D is the process on which hi-res ortho-image construction is based. High-resolution ortho-images have proved to be a valuable tool to support several types of investigation, spanning from the assessment of surfaces' conditions and state of conservation to the study of geometric decoration patterns (Bianchini, 2020; Emerson & Van Nice, 1943). In addition, a reliable interpretation of the geometric shape of the vaults can shed light on both their original design and the techniques adopted to transform that idea into matter (Priego et al., 2022).

1.3 Methodology

Following the objectives described in the previous paragraph, we have adopted the following workflow for the study of the ambulacra vaulting systems:

1. Analysis of the planimetric configuration of the vaulting system on the first floor
2. Analysis of the geometric components of one conic vault
3. 3D modelling of the ideal shape of its surface
4. Evaluation of standard deviation between the ideal model and the captured 3D point cloud
5. Discussion of the unfolding options of the conic surface given hi-res ortho-images generation.

2. PREDECESSORS OF AACHEN CATHEDRAL

To assess the historical significance of the vaults of Aachen Cathedral, it is first necessary to take a closer look at the predecessors of this important building. In this framework, it seems essential to mention that the architecture of the building deviates from the basilical form traditionally anchored in the West in favour of a central building, considered a typical element of Byzantine architecture (Figure 1).

* Corresponding author

1 Aachen Cathedral UNESCO World Heritage Site is a multi-phase collaborative project between the Sapienza University of Rome (Italy), Robert Gordon University, Aberdeen (Scotland), in partnership with RWTH Aachen University, and the Dombauhütte Aachen.

2 The documentation project has been carried out in two sessions, the first in October 2022 and the second in March 2023. The data-capturing process integrated a terrestrial laser scanner and digital photogrammetry, both terrestrial and aerial.

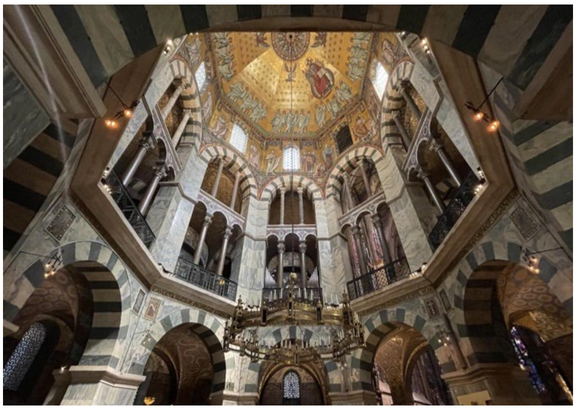

Figure 1. Palatine Chapel, Aachen Cathedral interior.

Figure 2. San Vitale in Ravenna, interior.

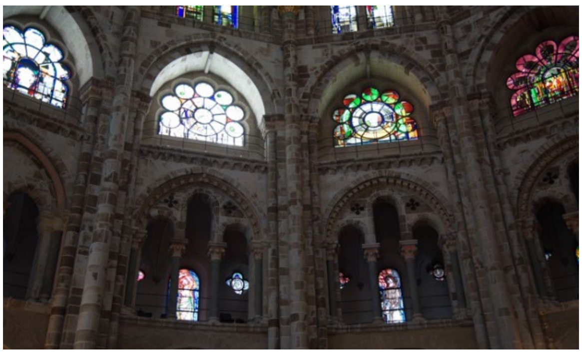

Figure 3. St. Gereon in Cologne, interior.

Figure 4. Santa Costanza in Rome, interior.

The church of San Vitale in Ravenna (Figure 2) is often described in the established literature as a predecessor of the Aachen Cathedral: It is also a central building on an octagonal ground plan, and in addition, its direct influence on the Palatine Chapel layout can be proven by the visits of Charlemagne to Ravenna between the years 786-787 (Pieper & Schindler, 2017).

However, apart from the apparent similarities, the church in Ravenna (537-547) shows far-reaching stylistic and constructive differences from Aachen Cathedral. On the one hand, the form of the octagon in Ravenna is dematerialised by the two-storey niches with the galleries set back. At the same time, these are spatially present and uninterrupted in Aachen until they reach the octagon. On the other hand, different construction methods were used to build the dome: in Ravenna, we found a lightweight construction method using *tubi fittili*, while in Aachen, the dome and all vaults are masonry (Raabe, Trautz and Di Pumpo, 2019). Finally, it should be mentioned that the ceilings in Ravenna were originally wood-covered (Pieper & Schindler, 2017) and thus of little comparable significance to the vaults in Aachen.

As a geographically close-by predecessor of Aachen Cathedral, the original oval-shaped building of the church of St. Gereon in Cologne (Figure 3) should be identified. According to archaeological finds, a central building was erected here in the IV century, probably between 350 and 365, which may have served as a mausoleum for the Frankish royal family (Steil, 2018). Conches, which may have been initially used to house the sarcophagi, adjoin the central space. However, the lack of vaulted structures like those in Aachen Cathedral limits the exemplary function of the church of St. Gereon to the formation of a monumental central space, identical to the previous remarks on the church of San Vitale in Ravenna.

The comparative remarks on the exemplary sacred buildings in Ravenna and Cologne can also be applied to many other sources of inspiration for the construction of the Aachen Cathedral discussed in the literature. However, as in the Mausoleum of Santa Constanza in Rome (Figure 4) or the Church of Saints Sergius and Bacchus in Istanbul (to mention two well-known cases), designing efforts seem to focus more on the refining of an exemplary central building idea than on the shaping of innovative vaulted structures.

In its magnificence, Hagia Sophia in Istanbul stands with its distinctive dome supported by four spherical pendentives, an actual compositional and structural invention by Anthemius of Tralles (Bianchini & Paolini, 2003). Quite apart from these incredible inventions, however, the central square space of Hagia Sophia appears typologically quite different from the buildings we mentioned in the previous lines and from the Palatine Chapel too.

Thus, the vaults of Aachen Cathedral represent a development that should be recognised as both an original compositional solution and a significant progress of construction technology in the IX century. Its documentation and analysis with the latest capturing and modelling technologies are critical.

3. GEOMETRIC DESCRIPTION OF THE PALATINE CHAPEL'S VAULTING SOLUTION

The complex space of the Palatine Chapel contains several design solutions at the two floors of ambulacra, resulting in various vaulting and connection typologies.

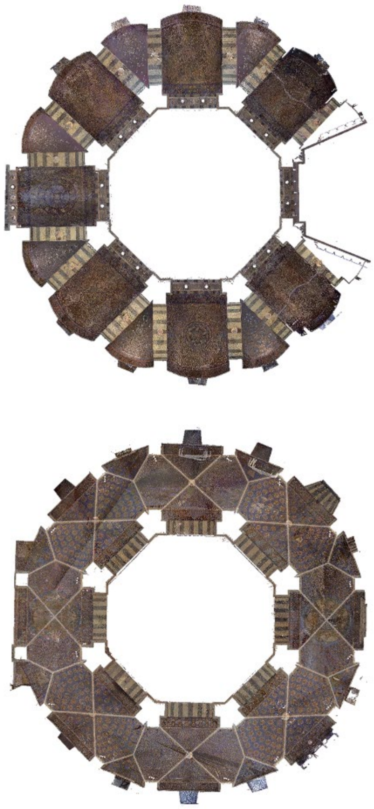

Figure 5. Vaulting typologies of Palatine Chapel ambulacra. Top: First-floor gallery. Bottom: ground entrance level.

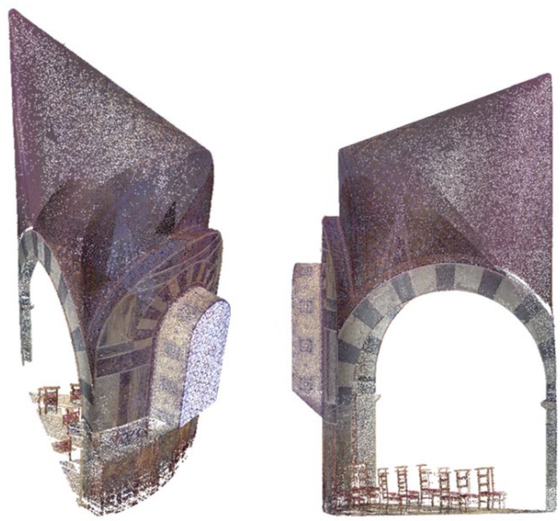

Figure 6. First-floor conic vault.

Their geometrical analysis starts from recognition and cataloguing.

The ambulacrum is covered with quadrangular and triangular groin vaults at ground level. These last display Y-shaped ribs and allow connecting the piers around the inner octagon to the sixteen-sided wall without any additional arch. On the first floor, each of these piers is connected by arches to two distinct half pilasters inserted in the outer wall. This solution divides the space into rampant barrel and conic vaults (Figure 5, Figure 6). These latter show a highly peculiar character in compositional and constructive terms, which is the core of this paper.

As well acknowledged in literature (Spallone, 2019; Cipriani, Fantini and Bertacchi, 2020), analysing and modelling complex structures like the Palatine Chapel must consider archival sources. As far as we could assess exploring archival architectural literature (treatises, manuals, sketchbooks, etc.), very little documentation (only some old photographs) refers to its vaulting system genesis and construction process. Thus, given this lack of information, the analysis of the conic vaults has been almost exclusively developed starting from their current state as captured by combining 3D laser scanning and digital photogrammetry during the 2022-23 integrated survey campaign (Bianchini & Russo, 2018; Vitali & Natta, 2019).

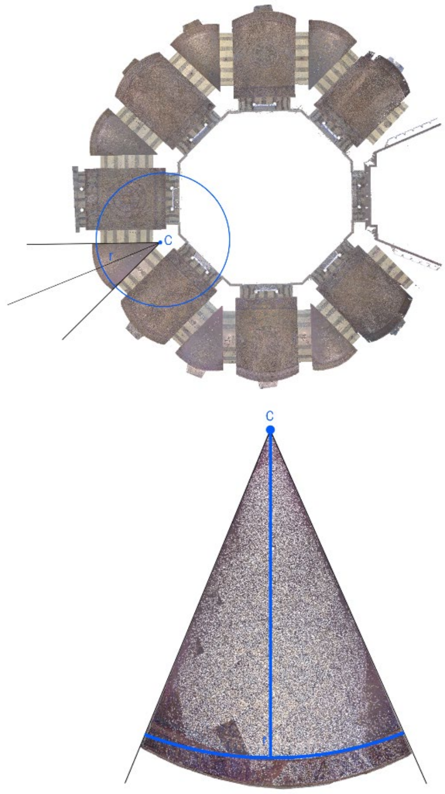

Figure 7. Construction of the ideal boundary curve of the cones.

The modelling software provides powerful tools for achieving reliable and consistent results and investigating their geometric properties. Using the survey data as a reference base, the approach adopted for the study of the vaults aimed at building the *ideal* models underlying the design of the conic vaults. This abstraction implies transforming the captured shape's distinctive features into the most straightforward and closest geometry.

This phase of the work started with the planimetric analysis using a hi-res ortho-image of the Chapel's point cloud. In this projection, we can recognise the seven cylindrical barrel vaults combined with the six conic ones.

Working on the third conic vault clockwise (Figure 8), the first step consisted in identifying the bisector of the angle that defines

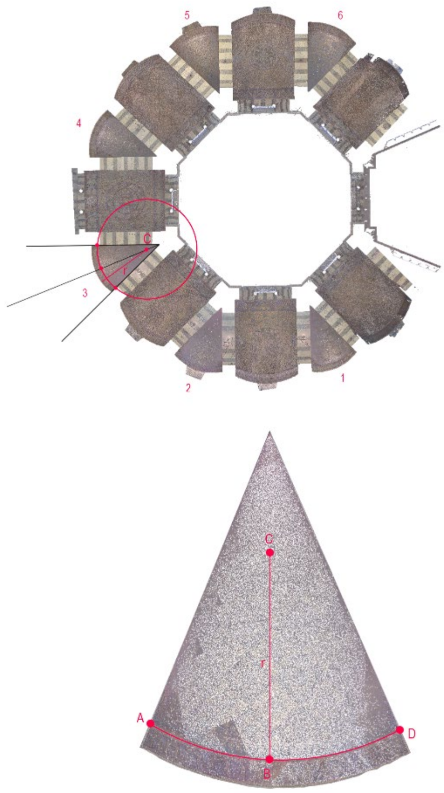

Figure 8. Construction of the real boundary curve of the cones.

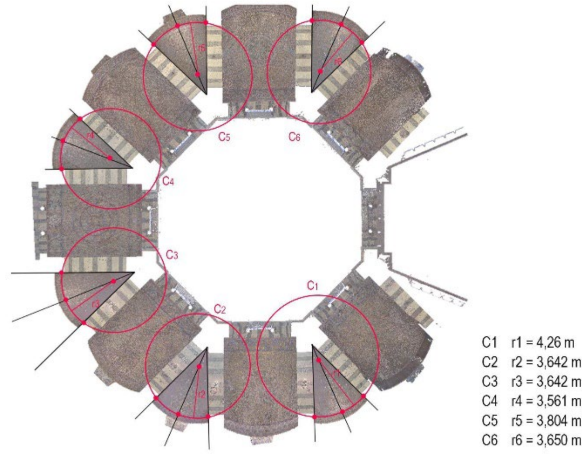

Figure 9. Analysis of the difference between the six curves

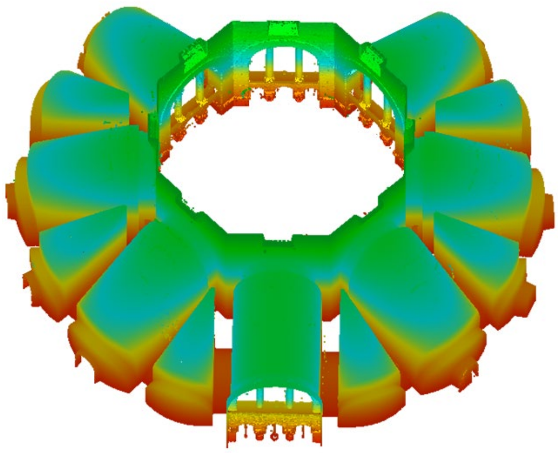

Figure 10. Point cloud elevation map.

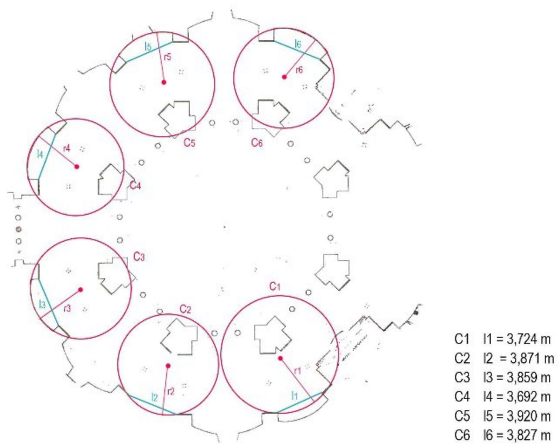

Figure 11. Conic vaults, pillars and construction of the circle chord

the width of the six cones, which were found to be incidents in the centre of the circle that circumscribes the inner octagon. At this point, we have drawn a circle using the projection of the crown line as a radius. This circular segment has been assumed as the boundary *ideal* curve of the conic vault in the outer direction. As expected, this construction does not match perfectly with the actual profile (Figure 7).
Constructing a circumference for three points (the two extremes and a point on the bisector), we obtained the result shown in Figure 8, which is slightly different from the ideal construction. Moreover, the same procedure, repeated for the other cones, shows a corresponding result for vaults 1 and 2: the angle width is 44°, and the two curves are almost coincident. For the others, this angle's width varies between 43° and 45°. The curves constructed using the same procedure are different (Figure 9). Circumferences 1 and 4 are the most discordant, as their deviation is around 60 cm. While circles 2 and 3 show the same radius, the difference for vaults 4, 5 and 6 is about 10 cm.

Considering the constructive function of these conic vaults, the found dimensional difference could be explained by comparing the array of barrel vaults to which they are connected. This comparison, however, shows that these vaults match both dimensionally and geometrically with a width that varies between 5,06 and 5,24 cm.

Another possible explanation for the conic vaults' differences can be found in structural deformations that could have affected the building. For this reason, an elevation map was generated to assess the height differences of all vaults' keystones. This analysis proved a significant homogeneity also for the keystones' level that led us to exclude structural deformation as a cause of the above-mentioned geometric differences (Figure 10).

Another group of analyses concerned the relationship with the vertical structural elements. The conic vaults connecting the barrel vaults lie on the sixteen outer rectangular pillars and the eight inner ones. The axes of the eight piers intersect in the very

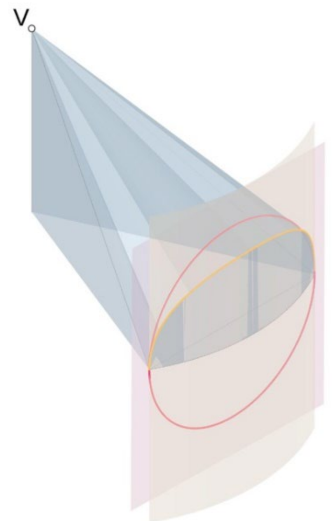

Figure 12. Circular conoidic surface with director plane.

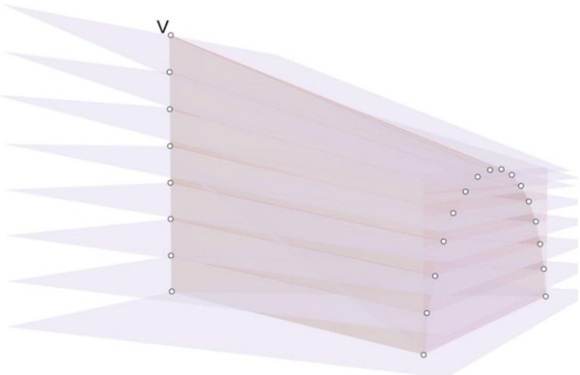

Figure 13. Generic circular conoidic surface without a director plane.

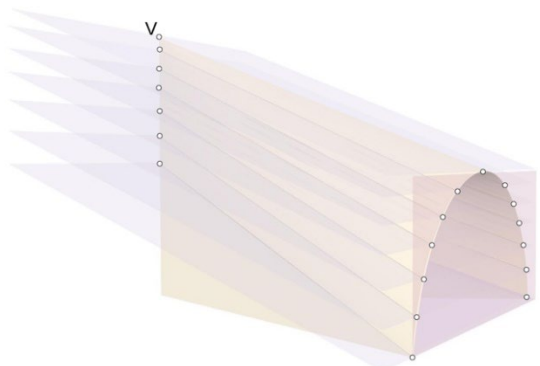

Figure 14. The conic surface intersects by a vertical plane to get a circumference (in red), and the quartic curve is generated from the intersection with the cylindrical surface (orange).

centre of the Palatine Chapel, accordingly with the bisectors of the angles that define the width of the conic vaults. The sixteen perimeter pillars appear to have the same width, between 1,13 and 1,16 m**.** Starting from this evidence, we passed at assessing the length of the circle chord for each conic vault (segment between the springing points of the outer arch – Figure 11, blue lines). They turned out to be relatively regular, with a length between 3,70 and 3,80 m, except for the chord of circumference 5, which measures 3,90 m (Figure 11). However, this only inconsistency could be explained considering its proximity to the barrel vault covering the landing area from the lower floor.

In this framework, we can conclude that the circles' chords (more than the outer curved profiles) could have represented the *baseline* traced at the springing level for building purposes. Their regularity confirms this hypothesis. The geometrical inconsistencies of the external shapes could also be explained, considering their tracing as a *secondary* construction connected to the adjoining conic vaults to the correspondent outer vertical walls.

Every geometric analysis of a building is deeply connected to the construction processes and design choices. Observing the object and analysing it from different points of view has made it possible to switch from the spatial to the planar configuration and then work on horizontal sections. It is thus possible to consider geometry as an expression of the designer's will and the representation as an instrument to control and verify the formal and constructive hypotheses. From this standpoint, the survey is necessary as a system of interpretation and knowledge, from design ideas to the current state of the architectural heritage (Vitali, 2018).

4. 3D GEOMETRIC MODELLING OF THE CONIC VAULT

After the general analysis discussed in the previous paragraph, the following step of our study focuses on the geometric elements guiding the construction of one of those conic vaults.

As any of the outer vertical walls of the Chapel is cylindrical (figure 8), the intersection between the vaults and these walls generates quartic non-planar curves. Quite apart from the novelty this solution represents in the framework of the history of architecture, these curves cannot be easily used to study the conic surface as a ruled one.

We had thus to identify a planar curve to be assumed as the surface directrix. Among the many possible ones, we tested as first the curve produced by the intersection with a vertical plane passing for the chords of the boundary arches discussed in the previous paragraph. As these sections well approximated a circle, we assumed them as directrices of the vaulted ruled surfaces. This choice had a pure geometric reason and a solid reference to a credible building workflow.

Although all elements suggest a cone as the guiding shape of the Chapel's vaults, nevertheless, we have tested this assumption against two other possibilities:

- a conoidic surface with circular vertical directrix and an oblique director plane parallel to the crown line (Figure 12);
- a generic conoidic character with circular vertical directrix without a director plan (Figure 13).

However, comparing these three surfaces immediately proved the conic one to be the most convincing concerning the captured data.

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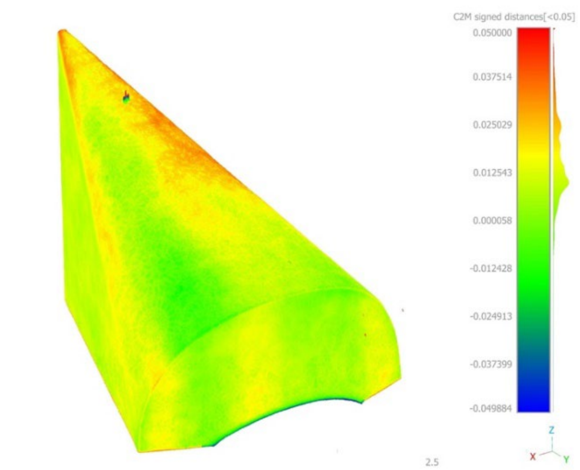

Figure 15. Cloud-to-mesh distance (in the range of -5cm to 5cm) between the captured point cloud and the reconstructed geometry of the cone.

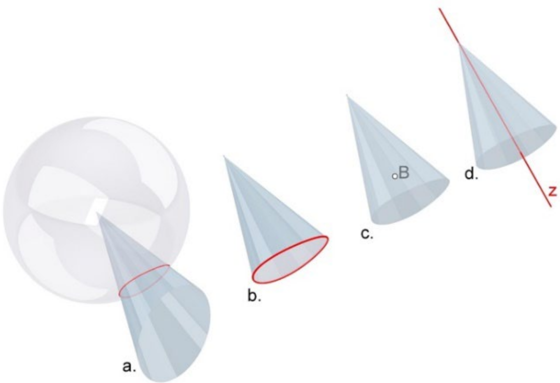

Figure 16. Construction of the axis of a quadric cone: a) Intersection between the cone and a generic sphere. b) The intersection curve (in red). c) The barycentre of the resulting cone (B). d) Construction of the axis of the cone (z).

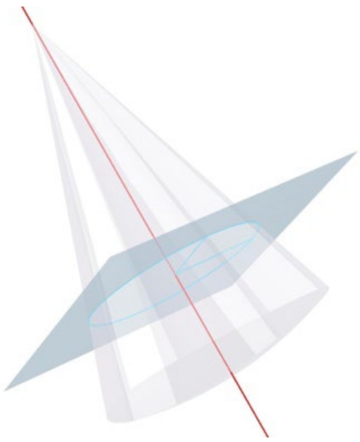

Figure 17. Construction of the ellipse as the directrix of the cone

We thus proceeded with the cone modelling using as generatrices the two inclined lines at the springing plane of the vault, as vertex V their intersection points and as directrix the vertical circle we discussed before. We then observed that this conic surface and the cylindrical one representing the outer wall intersected in a quartic curve well approximating the surveyed point cloud (Figure 14).

Furthermore, the resulting surface has been compared with the 3D point cloud using cloud-to-mesh distance software. The analysis showed a standard deviation of 1cm and a maximum distance of 4 cm on one of the two sides at the springer level (Figure 15).

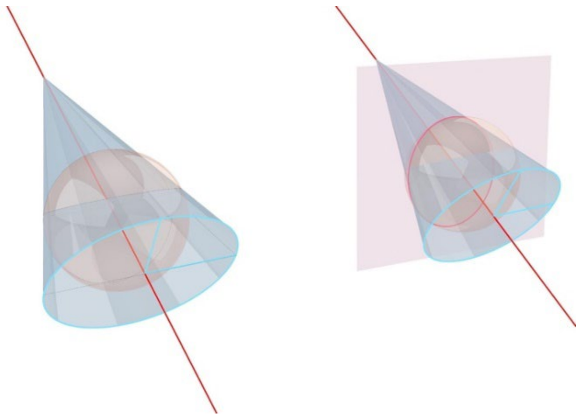

Figure 18. Construction of the two planes that cut the elliptic cone generating circumferences.

Figure 19. Final construction on the point cloud of the cone, the elliptical base and the vertical circle.

After assessing the high concordance between the conic surface and the captured point cloud, we passed at identifying the "standard" elements of this quadric cone: its axis and perpendicular base. As for the axis construction, we followed a mathematical modelling approach (Salvatore, 2012a). First, we built a generic sphere with its centre in the cone's vertex. The surfaces reciprocally intercepted by the two solids (Figure 16a) create a new solid (Figure 16b). Its barycentre B (Figure 16c), together with the vertex V, belongs to a straight line that represents the internal axis z of the cone (Figure 16d).

As the axis z is not perpendicular to the circular conic section lying on the vertical plane, we can affirm that the directrix of the cone will be an ellipse. This curve can be quickly built by cutting the cone with a plane perpendicular to its axis. The geometry of this ellipse is completed by determining its conjugate orthogonal axis (Figure 17).

Somehow reversing the workflow that given two generatrices, the vertex and a circular section of the cone, has led us to find its "standard" base and axis, and now we passed at building back its circular section. In any elliptic quadric cone, two circular sections always exist.

We found one of these sections following the procedure proposed by Théodore Olivier (Olivier 1852), later interpreted and presented in mathematical modelling by Marta Salvatore (Salvatore 2012b). It consists of constructing a tangent sphere to the sides of the triangle built using the elliptical base's central axis and the cone's vertex. The intersection between this sphere and the cone produces the searched circumference (Figure 18).

Finally, the comparison between this last curve and the one originally built on the point cloud has revealed a consistent overlapping only affected by a minor angular rotation.

Following the discussion developed so far, we can conclude that the considered Palatine Chapel's vaults are conic and that their construction on site has been reasonably guided by the three hypothesized elements: vertex, generatrices on the springing plane and vertical circular section (Fig. 19).

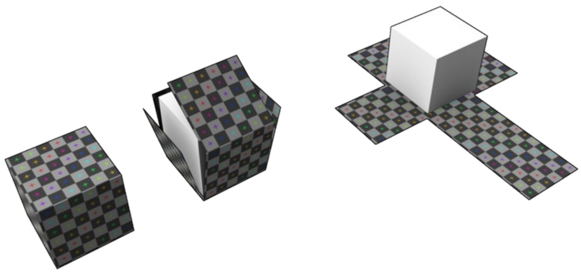

Figure 20. The unfolding process of a cube into a 2D representation for the U.V. map construction

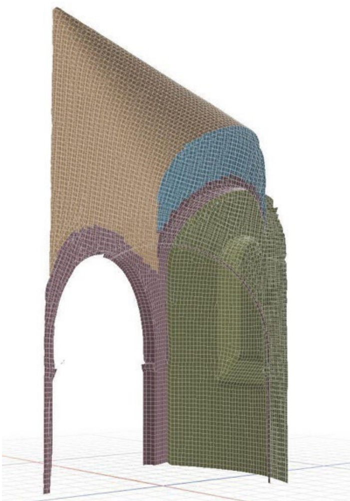

Figure 21. Sectors in which the 3d model has been divided.

5. BEST-FITTING UV MAP

The geometric modelling of the conic vaults has been coupled with their unfolding and mapping using the texture generated by Structure from Motion processes (D'Amelio S. and Lo Brutto M., 2009).

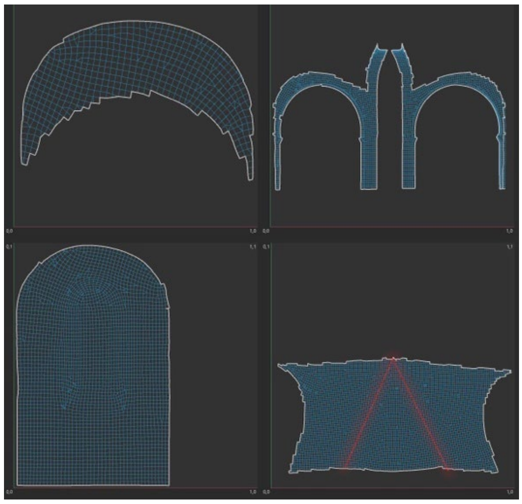

Figure 22. U.V. map of the four sectors.

Figure 23. The 3D model with the control texture was applied.

The elaboration of a 3D model continues after the geometrical and mathematical phases and involves crucial additional tasks, such as U.V. mapping, texturing, lighting and rendering.

Mapping, or better, U.V. mapping, represents a fundamental step to get to the final model. Hence, using unwrapping procedures, complex geometries, such as the Palatine Chapel's conic vaults, must be unfolded on a two-dimensional plane. As a result, each vertex of the three-dimensional polygonal model will share a set of two-dimensional coordinates with the texture and therefore be associated with its mesh faces and visualised in the 3D space (De Luca L., 2011).

The main goal of any unfolding projection is to create a flat and accurate mapping of the corresponding 3D surface, which can then be used to apply texture or visual effects.

Figure 24. The planar unfolding of the conic vault with its texture.

Figure 25. Rendering of the conic vault together with its mesh.

In most three-dimensional modelling software, interactive tools are provided for U.V. mapping. They allow for generating U.V. maps using standard projection methods (Maggiordomo A., Cignoni P. and Tarini M. 2021).

These standard maps work well when models display simple topologies made by basic primitives (cubes, polyhedra, cylinders). As cones do not belong to this class, for the Chapel's conic vaults, it was crucial to set a geometrically reliable unfolding that would allow working on a 2D image suitable for accurate reprojection onto the 3D model (Figure 20).
The main issue while unfolding a 3D model is to define a two-dimensional representation of as many correspondents as possible to the three-dimensional original. This result highly depends on the unwrapping phase that must consider the model's shape and topology to minimise pixel distortions and deformations. The goal of unwrapping is to produce a flat image respecting the original perception of depth and realism once applied as a texture to the 3D model.

After several tests, the default projection tools proved incompatible with our established standards, and thus we started working on a customised mapping solution.

In this framework, the model has been divided into several sectors: the area of the wall with the window, the two side arches up to their connection to the ground, the conic vault, and the intersection sector between the cone and the vertical cylinder of the outer wall (Figure 21).

Excluding the conic vault sector, we started from a parallel projection of the three other ones and passed then to the 2D unfolding of the surfaces.

Instead, we cut the mesh along the cone generatrices at the intersection with the vertical bearing walls for the conic vault. The resulting surface was finally unfolded. (Figure 22) (Pietroni N., Nuvoli S., Alderighi T., Cignoni P., Tarini M. 2021).

The quality of any created U.V. mapping can be assessed utilizing a control texture called a *checker* that, using a square grid made of numbers and colours, allows testing if the unfolding projection is correct (Figure 23).

Once obtained an acceptable projection, the next step is the unfolding relaxation. This task consists in "relaxing" the deformations generated by the projection. In practice, this means flattening the mapping, trying not to overlap the different parts of the model, and maintaining a good texture resolution by reducing map distortion.

Once the unfolding process is completed, the texture layout can be processed in a 2D graphics software where detailed and custom textures can be created for each model element.
The application of this workflow to the Palatine Chapels conic vaults was quite successful, as shown in Figures 24 and 25.

6. CONCLUSIONS

The Aachen Cathedral Project is a comprehensive set of activities that provides an integrated 3D survey of almost all the visible surfaces of the complex, the 3D modelling of all buildings and a critical analysis of their architectural, structural and evolutionary components.

After the capturing task and a preliminary 3D modelling phase, the first problem we decided to address refers to the shape of the conic vaults covering the ambulacrum on the first floor. The reason for this choice comes from the historical evidence that this compositional and structural solution represents quite an original and unexpected one in the context of the VIII/IX century building know-how.

Unlike the more conventional layout of the ground floor ambulacrum (triangular and quadrangular groin vaults), the designer's choice has radically changed on the first floor. The *centripetal* character of the former, in which the vaults' disposition emphasises the annular circulation around the central triple-height hall, becomes in fact *centrifugal* on the latter, where all the vaults originating from the arches facing the entrance are rampant barrel vaults radially pointing to the centre of the inner octagon.

While this compositional solution creates a fascinating dialectic between the lower and the upper ambulacra, it obliges the designer to figure out a solution for the "left-over" triangular portions between one barrel vault and the other.
Discussing this topic involves many layers of knowledge (architectural, historical, structural, etc.). In the Aachen Cathedral Project framework, we are very early in this process. Nevertheless, we have shown that those mentioned above in 2D triangular portions correspond to rampant conic surfaces in the 3D architectural space. The geometry of these cones has been identified and assessed against the captured data.

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These are the resulting conclusions:

- the vertexes lie on the concave corner of the piers facing the central hall.
- two generatrices lie on the vertical walls delimiting the triangular portions.
- the vertical plane built on the bases of those triangular portions cuts the cones creating a circle.
- The intersection between the cones and the outer cylindrical vertical surfaces of the Chapel is geometrically correct and produces a quartic non-planar curve.

This evidence is exciting, and very little expected for a VIII/IX-century building. We must consider that in that period, being Euclid's work became practically unknown, the level of geometrical knowledge in Western Europe was commonly judged as extremely poor and mainly limited to the very few collections of problems belonging to the so-called *Geometria Practica* (Bianchini 1995). From this standpoint, the Palatine Chapel's conic vaults could represent a *material document* contradicting this consolidated conclusion, although the sources of this document are still to be identified.

From a more constructive standpoint, we must stress that those conic vaults represent an absolute novelty when analysed in the framework of stereotomy. The first documents that show some similarity with this class of problems can be found some four centuries later in the Llivre de Portraiture by Villard de Honnecourt and in some constructive tracing on walls or floorings of medieval buildings (Bianchini et al. 2019). The first treatises tackling the complex intersections between cylinders and (partially) cones will instead appear in the early XVI century, reaching a good level of rigour only at the end of the XVII. Furthermore, as far as we could assess at this early stage of our research, similar structures have yet to be identified, neither in ancient buildings nor in others of the period. As with any masterpiece, the Palatine Chapel still deceives more than what we have been able to enlighten so far.

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