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Developing a Market-Based Approach to Managing the U.S. Strategic Petroleum Reserve

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Abstract

The Strategic Petroleum Reserve has not been used effectively to manage the consequences of oil shocks in the United States. The main reason is that political decision makers tend to hoard the reserves during crises and bureaucratic processes delay the sale of the reserves. Also, the enabling legislation focused on ameliorating shortages whereas disruptions result price spikes rather than shortages. We develop a Markov game of the build-up and drawdown of the reserve in which a public player aims to maximize consumer welfare at the same time private holders of inventory maximize their profit. The methodological contribution in this paper is the development of financial options to implement the public player's optimal policy. We use the solution of this game to calculate the number and value of options necessary for the private marketplace to trigger the optimal buildup and drawdown of the reserve.

1. Introduction

The recent 2008 run up in oil prices and increased price volatility, along with the U.S. government's continuing additions to the U.S. Strategic Petroleum Reserve (SPR) during a period of high prices have led to new debates on SPR policy. In early 1975 the U.S. Congress authorized the SPR in response to an attempt by Arab OPEC members to impose a directed embargo against the United States. Because this embargo failed to cut off US imports, the research and planning for the SPR focused on using the reserve to dampen price spikes and associated macroeconomic impacts due to losses in supply. The SPR holds 724 million barrels of oil and private crude stocks have ranged between 280 and 375 million barrels in 2008 and 2009. Thus, the SPR constitutes 2/3 of US crude stocks.

The first papers on the optimal level of the SPR drew from early work on storing grain for famine protection, Gustafson (1958). The first storage models by Balas (1979) and Teisberg (1982) optimized the buildup and drawdown of a reserve using dynamic programming. There is a free-rider problem in building the SPR because crude prices move in tandem around the world and others benefit from the US inventory withdrawals without having to pay any of the costs of building and storing inventory. Hogan (1983) looked at the free-rider problem of one country benefiting from another country's expenditure on a strategic reserve. He used a Stackelberg game with the US the leader in making decisions and other countries lumped into a single player that follows the leader.

Murphy et al. (1986, 1987, 1989), as part of a project for the Office of Policy of the Department of Energy, examined international free riding on a US reserve and the interaction of public reserves and private crude-oil inventories. They formulated a Nash game and developed an algorithm to find an equilibrium in an infinite-horizon game where the market states were described by a Markov process. See Chao and Manne (1983), Samouilidis and Magirou (1985), and Oren and Wan (1986) for other models of the reserve. All of these models were built before forward markets for crude and petroleum products were developed. Companies now buy and sell in futures and options markets to manage their risks. An oil producer can guarantee revenues by selling in forward markets. A refiner can buy crude oil futures and sell product futures to lock in a significant portion of its margins. Companies that consume large amounts of oil such as airlines can protect themselves from market volatility using forward markets.

The use of financial tools and improved business processes has meant that the ratio of the volume of inventory to the volume of sales has been in long-term decline. A byproduct of the inventory reduction is that there is less of a physical cushion, adding to the increased volatility in spot markets when disruptions in the supply chain occur.

Financial markets have directly affected the valuation of private stocks. One measure that is used to understand the marginal value of inventory in company operations is convenience value, an estimate of the value of the last barrel in inventory in facilitating operations. Convenience value is discussed in Pindyck (2001) and Considine and Larson (2001), where they estimate the value using financial markets, based on the relationship of spot and futures prices. Milonas and Henker (2001) also estimate the impact of price spreads on convenience values.

The contributions of this paper are the following. From a policy point of view, we explore an alternative way to manage the strategic reserve based on financial options. We develop a model that computes the number of options to be issued in order to replicate the optimal welfare policy. Next, we construct an example, illustrating the size, drawdown, and buildup policies and the social welfare benefits of the SPR and estimate the size of the options market required to support government policy. Finally, we develop an improved solution methodology for the Markov game that allows us to use standard optimization software, unlike earlier approaches, e.g., Murphy et al. (1986, 1989).

2. Reconsidering the SPR

A reserve makes sense only if there are losses that can be ameliorated through a government-held stock versus private inventories that are externalities to the internal costs of disruptions to the industry. The chief externality is macroeconomic impacts. Hamilton (2003) finds that oil-price increases dampen economic activity, decreases do not have the mirror-image benefit to the macro economy, and increases that are price recoveries from recent falls do not have as strong a predictive effect on the macro economy as an increase from a baseline price. Huntington (2007) shows that real incomes decline immediately, followed by the lagged effects on GDP.

The measured relationship between oil prices and economic activity has weakened since 1985, according to Hooker (1999) due to the activity of governments that have learned how to better manage the impacts of commodity shocks (see Bernanke et al., 1997), and because the recent studies were done during an era of low oil prices, see Jones et al. (2004) for an extensive review of this literature. From simulations done in 1992 at the Energy Information Administration, volatility was more important than price levels (Energy Information Administration, 1992, Appendix F). Since average monthly oil prices increased from $15 to $95 between 1998 and 2008, in real 2006 dollars, the oil share of domestic expenditures has increased several fold and oil-product costs are now crowding out other purchases by consumers.

Gordon (1992) was one of the first to argue that the government should not hold a reserve. Considine and Dowd (2005) show that there have been problems with the timing and amounts of sales from the reserve. Taylor and Van Doren (2005) state that "the government ought to cut its losses by selling the oil and shutting the program down." Considine (2006) develops a model of crude markets with Saudi Arabia acting as a constrained monopolist that undertakes actions to negate the value of a strategic reserve. Williams and Wright (1991, ch. 15) point out that most arguments for storage beyond the macroeconomic argument do not hold up to scrutiny. They note that the reserve can provide a strategic advantage over an intentional embargo by taking away the economic value of the embargo. However, this argument does not apply to the kinds of disruptions that historically have affected oil markets, which have been collateral damage from events such as the Iraqi invasion of Kuwait or the overthrowing of the Shah of Iran.

The main reason for reconsidering the SPR is that the US government has not proved to be adept in managing the reserve. Figure 1 presents real yearly average crude oil prices (in 2006 Dollars, source British Petroleum, 2009) versus the SPR size in millions (source Energy Information Administration, 2009), from 1977 to 2008. A DoE (2009) website summarizes the draw downs, and highlights the problem. The two main draw downs occurred around Iraq's invasion of Kuwait and Desert Storm in 1990-1991 and to replace domestic production and imports after hurricane Katrina in 1995.

The 1990-1991 events illustrate the weakness of DoE procedures. Iraq invaded August 2, 1990. Crude from the first sale of 4 million barrels flowed on October 19th, a two and one half month delay to replace less than three days of Kuwait's exports. Then 17.3 million barrels were released between February 5 and April 3 1991. Again too little too late to lessen the subsequent recession.

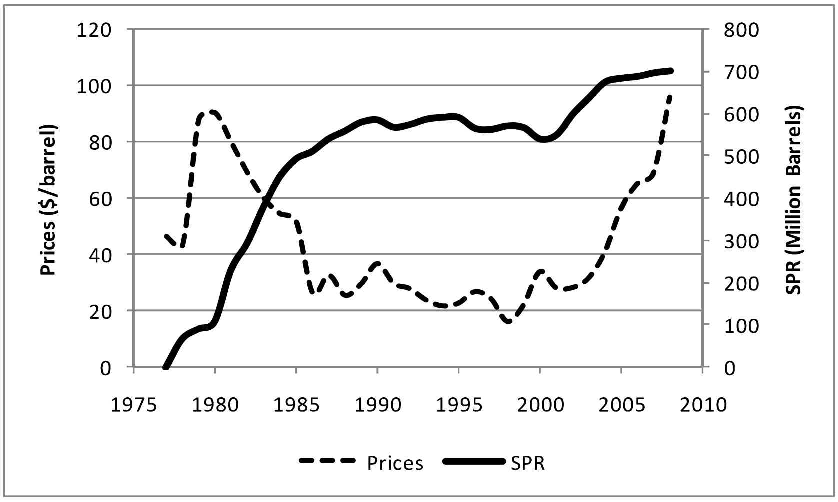

Figure 1: SPR Levels and Crude Oil Prices over Time (1977 to 2008).

The government responded better to Hurricane Katrina in 2005 with the hurricane hitting in late August and oil beginning to flow in late September. In 2008, after hurricanes Gustav and Ike, the SPR delivered over 5 million barrels of oil to companies that had lost supply in exchange for barrels delivered between January and May of 2009. So, although a portion of the high prices in 2008 was due to disruption threats in countries like Nigeria, the US government made no guarantees that the SPR would cover potential shortfalls, which it could not do because of the cumbersome sales process.

In 1996 and 1997 the SPR sold 28 million barrels at an average price below $20, nominal, to reduce the federal deficit and cover SPR expenses. On the purchase side, the highest fill rates were, understandably, during a period of high oil prices at the beginning of the reserve program. The high fill rate continued into the mid 1980's, a time OPEC's share of the market was falling and Saudi Arabia was losing its capacity to support prices by cutting production, signaling that prices were headed down. Once the price dropped precipitously, the fill rate declined.

Government policy can be described as skipping purchases and selling during periods of low prices because there is no immediate threat, being late on using the reserve in response to world issues, and acting quickly with exchanges of oil to address domestic disruptions because of a standardized contract structure for the exchange.

Several authors suggest using markets to determine the amounts to drawdown. Like much of the literature, these papers are old and focus on the issues of hoarding and panic buying. Devarajan and Hubbard (1984) note that a reserve can forestall hoarding by private inventory holders. Adelman (1982) proposes a preset price at the highest price of the day plus a year's storage cost. This restricts purchases to firms that have higher internal shortage costs than the market price and do not have access to oil

Historically, hoarding has been more of a problem with governments than with private players: purchases by the French and Japanese governments during the height of the Iranian crisis in 1979 drove prices much higher. Governments are more problematical than the private sector because the public decision makers do not face the financial losses from buying at the peak and score political points by showing that they are doing something to address anxieties with their purchases and hoarding of oil.

Hoarding by private players is less of a problem because the private players risk their own money. They also can stick to financial markets, if they are speculators, and not carry physical oil. If speculators hold physical oil because of a heightened perception of disruption risk, they profit only if on net they sell at high prices and buy at low prices, consonant with government goals of stabilizing prices.

Almost all consumers outside the largest users that hedge their consumption face the consequences of high prices during a disruption and the aggregate cash flows based on the market price dampen economic activity, as in 1991 and 2008. Economic costs are measured by physical transactions, as speculator profits or losses are included in social welfare. Financial transactions influence prices through changing the inventory behavior of commercial players. Inventories were low in mid 2008 because the spot price was above the forward price, and the cost of holding physical oil included the losses from the lower prices. In early 2009 inventories were so high that ships were chartered to sit fully loaded offshore. At that time the US forward price was above the spot price to the point where it was profitable with no risk to hold physical oil and sell it forward, rolling over the contracts as long as the price difference held. In the short run, psychology can move markets away from the fundamentals. We observed the fundamentals and the effect of psychology first hand in a shipping market with all deals taking place between vessel owners and shippers and no speculators, (see Mudrageda et al., 2004, and Mudrageda and Murphy, 2008). Thus, it is important to create certainty around the availability of SPR crude during times of crisis to dampen the fears of shortages that lead to higher prices. However, in this paper we consider only stochastic supply disruptions; for this reason the physical trades are very important determinant of price (as the physical inventory may be at risk).

If market signals can be harnessed to manage the reserve, the problem of suboptimal management of the reserve is ameliorated. Such an approach would take into account the actions of private players in the market and their inventory positions.

3. A Model of Public and Private Inventories and Purchase Decisions

The model consists of a public player that maximizes economic surplus and an aggregate private player that makes decisions on how much to store or sell based on the value of the oil held for running the business and current and expected market prices. The aggregate private player maximizes profits factoring in the value and cost of inventory, which reduces to an arbitrage condition where the discounted expected future price plus the value of the last barrel in inventory for running the production process equals the current price. Transitions among states are represented as a Markov chain that includes the market states and the levels of inventories of the public and private players.

We solve a Nash Markov game where the public player knows the decisions of the private player at each market and inventory state and optimizes presuming the private player does not change its decisions. The private player maximizes its profit assuming that the public player does not change its policy. Murphy et al. (1989) show that this is equivalent to solving a no-arbitrage condition. Let us define: *s,* the amount added or withdrawn from the public inventory; *x,* the amount added or withdrawn from the private inventory; *i* and *j*, indices for the market states 1,…,*I;* *g*, the discount factor; *S*, the level of public inventory; *X*, the level of private inventory; 0, a subscript indicating the current state; *p*, a subscript indicating potential future states; p_{ij}, the transition probability in the Markov chain; *q*, oil supply in a normal market; *w*(*i*), the level of supply reduction from a disruption in state *i* (*w*(1) = 0 in the normal-market state and reductions increase in *i*).

A state of the world is defined by the triple (*S,X,i*) in which $S \in \{S_1,...,S_{\bar{S}}\}$, $X \in \{X_1,...,X_{\bar{X}}\}$, and $i \in \{1,...,I\}$. The decisions *s* and *x* satisfy $S_p = s + S$, and $X_p = x + X$. We denote specific decisions by $s(S,X,i)$ and $x(S,X,i)$.

The transitions between states of the world are determined by the changes in public and private inventories and the transitions among the different levels of disruption. Let $pr(S_0, x_0, i, S_p, X_p, j)$ represent the transition probability from state (S_0, X_0, i) of the world to state (S_p, X_p, j). The only stochastic variable defining the state is the level of disruption. Thus, in any state (S_0, x_0, i), if $S_0 + s = S_p$ and $X_0 + x = X_p$, then the next state is (S_p, X_p, j) with probability $pr(S_0, X_0, i, S_p, X_p, j) = p_{ij}$ .This definition of state space is a simplification, assuming fixed prices for a given level of shortage, as we are concerned with supply disruptions only. A topic for further research is the modeling of price volatility within each state.

At each market state and possible combination of public and private inventory decisions, we compute an oil price. The oil supply is fixed at a level q_i determined by the market state i. We assume an isoelastic demand function, $D = kP^{-e}$, in which P is the oil price, e is the absolute value of the price elasticity of demand and k sets the level of demand given a price. In equilibrium, the price is such that the supply of oil in the market equals demand. At equilibrium we have (1), with $w(i)$ the reduction of supply in scenario i.

$$D = q - w(i) - s - x. \tag{1}$$

By manipulating the equilibrium condition, we derive the inverse demand function in which the equilibrium oil price is expressed as a function of demand and supply:

$$P(S, X, i, S_p, X_p) = \left[\frac{k}{q - w(i) - s - x}\right]^{\frac{1}{e}}. \tag{2}$$

The public player maximizes social welfare minus the cost of operating the SPR. Let h be the marginal holding cost of the SPR, and $l(X)$ and $b(X)$ stand for the total holding cost and total convenience value of private inventories in the current period, respectively. Social surplus is maximized by minimizing the cost function $c(S, X, i, S_p, X_p)$:

$$c(S, X, i, S_p, X_p) = \frac{k(P^{1-e} - P_n^{1-e})}{1-e} + Ps + Px + hS_p + l(X_p) - b(X_p) \tag{3}$$

This function has three main terms. The first term, $-\frac{k\left(P^{1-e}-P_n^{1-e}\right)}{1-e}$, represents the consumer surplus (which we truncate at a high price, P_n, for all states); the second term, $Ps + hS_p$, represents the direct cost (revenues) of purchases and sales from the SPR and the cost of storage for the SPR; the third term, $Px + l(X_p) - b(X_p)$, represents the costs and benefits of holding private reserves (this term is included because changes in private costs and benefits due to public actions should factor into government decisions).

Let $V\left(S, X, i, S_p, X_p\right)$ be the net present value of social costs, given the management of the SPR, starting in state (*S*, *X*, *i*). The public-player optimization is a dynamic program with the value of the optimal policy defined as $V^*(S, X, i)$,

$$V^*(S,X,i) = \min_{S_p, X_p} \left\{ c(S,X,i,S_p,X_p) + g\sum_{j=1}^{I} p_{ij} V^*(S_p, X_p, j) \right\} \tag{4}$$

We denote the optimal policies by $s^*(S,X,i)$ and $x^*(S,X,i)$.

The private sector exploits all arbitrage opportunities. By doing so, in equilibrium there are no arbitrage opportunities. For this reason we model the main task of the private sector to be reducing the available arbitrage opportunities in each market and inventory state. The private player's no-arbitrage condition is represented by equations (5) and (6), in which $l'(X_p)$ is the marginal holding cost and $b'(X_p)$ is the marginal convenience value. For all states, $S_p^* = s^*(S,X,i) + S$, we must satisfy (S, X, i, S_p, X_p) as computed by equation (5), in which S_{pp}^* and X_{pp}^* are the optimal levels of public and private inventory *two* periods away from the current state (*S*, *X*, *i*). We need to define a model two periods beyond the current state because the no-arbitrage constraint in (5) uses the prices of two sequential periods in the optimal policy.

$$\begin{aligned} arb(S,X,i,S_p^*,X_p) &= P(S,X,i,S_p^*,X_p) + l'(X_p) - b'(X_p) - \\ &- g \sum_{j,S_{PP}^*,X_{PP}^*} pr\left(S,X,i,S_p^*,X_p,j\right) P(S_p^*,X_p,j,S_{pp}^*,X_{pp}^*) \end{aligned} \tag{5}$$

and $X + x^* = X_p^*$ as computed by (6):

$$X_p^* = \arg\min_{Xp} \left| arb(S, X, i, S_p^*, X_p) \right| \quad (6)$$

When there are no arbitrage opportunities, $arb(S, X, i, S_p^*, X_p) = 0$. Given the optimal policy by the public player, $s^*(S, X, i)$, the private sector changes inventories by $x^*(S, X, i)$, resulting in X_p, exhausting all arbitrage opportunities and ensuring that

$$\begin{aligned} P(S, X, i, S_p^*, X_p) &= -l'(X_p) + b'(X_p) \\ &+ g \sum_{j, S_{pp}^*, X_{pp}^*} pr(S, X, i, S_p^*, X_p, j) P(S_p^*, X_p, j, S_{pp}^*, X_{pp}^*) \end{aligned} \quad (7)$$

That is, the current price equals the present value of expected future prices (adjusted by marginal holding costs and convenience benefits), taking into account the optimal policies by the public and private players in the future states of the world. As the model has a finite number of states, the no-arbitrage condition may not be met. For this reason, in (6) the model minimizes the absolute value of the arbitrage error.

In the public player's problem, let X_p^* represent the next state resulting from the optimal action by the private sector, $x^*(S, X, i)$, and a decision by the public sector, $s(S, X, i)$. The linear programming formulation of the dynamic program (see Wagner, 1975) is in (8a) and (8b) where, in the optimal solution, $u(S, X, i)$ is the value of the optimal policy.

$$Max \sum_{S,X,i} u(S, X, i) \quad (8a)$$

s.t. $\forall S, X, i, S_p:\ c(S, X, i, S_p, X_p^*) + g \sum_j pr(S, X, i, S_p, X_p^*, j) u(S_p, X_p^*, j) - u(S, X, i) \ge 0$ (8b)

Murphy et al. (1986, 1989) use the policy-iteration algorithm to solve the public problem and then iterate between the private arbitrage and public problem. We solve the government problem using the linear program and then iterate with the arbitrage problem, because the private player arbitrage links multiple time periods, making the decision space very large. A combined public/private model would no longer be an optimization. It would be a Mathematical Program with Equilibrium Constraints (MPEC). The algorithm is presented in Table 1. Because of the discretization of the state space, the solution can oscillate. We test for oscillations and stop when they are detected.

Table 1: Solution Procedure for the Markov Game

1. For all possible actions $s = S_p - S$ and $x = X_p - X$, initialize the equilibrium oil prices

$$P(S, X, i, S_p, X_p) = \left[\frac{k}{q - w(i) - (s) - (x)} \right]^{\frac{1}{e}}$$

2. For each state (*S,X,i*) and actions *s* and *x* initialize the public cost function

$$c(S, X, i, S_p, X_p) = \frac{k(P^{1-e} - P_n^{1-e})}{1-e} + Ps + Px + h.S_p + l(X_p) - b(X_p)$$

3. Initialize the probability matrix $pr(S_0, X_0, i, s, x, j) = p_{ij}$

4. Initialize the private player strategy: $X_p^* = X$

5. Select an initial solution, *s*, for the public player (e.g. $s = 0$).

6. At iteration k, solve the best response for the private arbitrage problem (5) and (6) for $X_p^k = X^k\{S, X, i)$.

7. At iteration k, solve for the best response using the public linear program (8) for S_p^k and $s^k(S, X, i)$ with $X_p = X_p^k$.

8. If $X_p^k = X_p^{k-1}$ and $S_p^k = S_p^{k-1}$, stop. Otherwise, return to step 6.

4. Designing Financial Options for Managing the SPR

For options to reproduce the optimal buildup and drawdown of the reserve, we need a method to determine how many options to issue and we need to define the properties of the options so that there is a one-to-one map between the number of options exercised and the optimal changes in the reserve for every state. We assume that auctions are used by the government for the periodic issuance of options to replace expiring options, auctions similar to those for Treasury securities. The new options can have their strike prices adjusted to match the evolving market for crude oil. This price adjustment ensures that the exercised options reproduce the optimal SPR build up and drawdown. The no-arbitrage condition ensures that the government does not lose money on average with a given strike price because the expected value of an option is its bid price.

We define two monotonicity properties of the solution that hold in our numerical examples. Essentially, these properties state that the more total inventory on hand the slower the total build up and the faster the drawdown. At the same time, when there is more starting inventory, given the same market state, the solution leads to more inventory in the next stage. These properties are important since they match the common sense of policy makers. This first property comes from Murphy et al. (1989).

Definition (Property M): A pair of policies $s(S,X,i) = S_p - S$ and $x(S,X,i) = X_p - X$ satisfies the property M if $S' + X' > S + X$, implies

$$s(S',X',i) + x(S',X',i) < s(S,X,i) + x(S,X,i)$$

$$S' + X' + s(S',X',i) + x(S',X',i) > S + X + s(S,X,i) + x(S,X,i)$$

This property states that changes in total inventory are monotonically decreasing in total inventory and total inventory in the next period is monotonically increasing in total current-period inventory. We add another property on prices.

Definition (Property M2): Let disruption severity increase with *i*. A pair of policies $s(S,X,i)$ and $x(S,X,i)$ satisfies the property M2 if $i < i'$,

$s(S,X,i') + x(S,X,i') \le s(S,X,i) + x(S,X,i)$, when $s(S,X,i') \le s(S,X,i)$

and $P(S,X,i,S_p,X_p) \le P(S,X,i',S_p,X_p)$.

This property states that the equilibrium price is monotonically nondecreasing in the level of disruption and the change in inventory is monotonically nonincreasing in *i*. As in Murphy et al. (1989) we cannot guarantee that these properties hold. Yet, the solutions we generate satisfy these properties. As these properties match common sense, any implemented policy would likely satisfy these properties.

We first construct a set of one-period financial options (e.g., Hull, 2007) that, when exercised by private players, replicate the optimal public inventory policy. A financial call (put) option gives its holder the right (but not the obligation) to buy (sell) a given quantity of crude oil at the exercise price stipulated in the option. We allow the government to buy and sell puts and calls. Since *X* and *S* are fixed in the one-period problem, we need to specify options for each market state that replicate the optimal decision, $s^*(S, X, i)$. With i=1 the normal state and $s^*(S, X, 1) > 0$, we want the total of puts minus calls to equal $s^*(S, X, 1)$, and we want the market-clearing price to be the resulting price from the solution to the model, which we denote by $P(S, X, 1, S_p^*, X_p^*)$. If the strike price of the puts is $P(S, X, 1, S_p^*, X_p^*)$ and players exercise less than $s^*(S, X, 1)$ puts, then the market price is below $P(S, X, 1, S_p^*, X_p^*)$. Exercising the remaining puts is profitable and all puts would be exercised. Using $P(S, X, 1, S_p^*, X_p^*) + \varepsilon$ can cover any transaction costs ensuring the puts are exercised. The same argument applies to setting the strike price for puts with $i > 1$ and any calls. The only difference with calls is that one could use $P(S, X, 1, S_p^*, X_p^*) - \varepsilon$ as the strike price.

Let us describe the variables used in the model: u_j^+, the amount of puts sold with a strike price of the price from the equilibrium solution for state *j*; u_j^-, the amount of puts bought with a strike price of the price from the equilibrium solution for state *j*; v_j^+, the amount of calls sold with a strike price of the price from the equilibrium solution for state *j*; v_j^-, the amount of calls bought with a strike price from the equilibrium solution for state *j*.

The following linear program provides the minimum number of options that have to be sold to match the optimal SPR policy. In the linear program (9a, 9b, 9c) we order the states by increasing equilibrium prices.

$$\min \sum_{j=1}^{I} (u_j^+ + u_j^- + v_j^+ + v_j^-) \tag{9a}$$

s.t.
$$\sum_{j \le i} u_j^+ - \sum_{j \le i} u_j^- - \sum_{j \ge i} v_j^+ + \sum_{j \ge i} v_j^- = s^*(S, X, i) \quad \forall i \tag{9b}$$

$$u_j^+ \ge 0,\ u_j^- \ge 0,\ v_j^+ \ge 0,\ v_j^- \ge 0 \tag{9c}$$

For each market state *i*, all puts sold, u_j^+, $j \le i$, are exercised at *i*. Also, all puts bought with strike prices associated with states $j \ge i$ are profitable to exercise. Analogous arguments explain the calls. From (9) we get the following result.

Theorem 1: If the government can buy and sell both puts and calls, there is a unique one-period set of options that replicates the optimal government strategy.

Proof: The linear program (9) can be modified as follows. For $i = 1$ in (9b) we have $u_1^+ - u_1^- - \sum_{j \ge 1} v_j^+ + \sum_{j \ge 1} v_j^- = s^*(S, X, 1)$. Subtracting the constraint for *i*-1 from *i*, we get

$u_i^+ - u_i^- + v_{i-1}^+ - v_{i-1}^+ = s^*(S, X, i) - s^*(S, X, i-1)$ $i = 2, \ldots, I$. Repeating (9c) $u_j^+ \ge 0$, $u_j^- \ge 0$, $v_j^+ \ge 0$, $v_j^- \ge 0$ we have the constraints for the same linear program as (9) with the feature that the coefficients of u_i^+ and u_i^- form identity matrices with opposite signs. Letting $u_i = u_i^+ - u_i^-$, we have another identity matrix with the variables unrestricted in sign, which guarantees a feasible solution that is unique in u_i. Setting $u_i^+ = u_i$ when $u_i \ge 0$ and $u_i^- = u_i$ when $u_i \le 0$, we minimize the objective function of the linear program, making the solution unique, and the result holds. ■

Proposition. With *q* the baseline world supply and *w*(*i*) the supply reduction in market state *i*, assume without loss of generality that *w*(*i*) is monotonically increasing in *i*. Assume the government never buys options on the reserve. If for all *S* and *X*, M2 holds, then a one-period set of options consisting of selling only can reproduce the optimal additions and withdrawals to the public inventory.

Proof. Let $P^*(S, X, i, S_p^*, X_p^*)$ be the equilibrium price resulting from the optimal SPR policy. Starting at $i = 1$, if $s^*(S, X, 1) \ge 0$, sell put options in this amount with a strike price of $P^*(S, X, 1, S_p^*, X_p^*)$. For $i = 2, \ldots, I$, sell call options in the amount of $\Delta(S, X, i) = s^*(S, X, 1) - s^*(S, X, i)$, with the strike price of $P^*(S, X, i, S_p^*, X_p^*)$. Now, $\Delta(S, X, i) \ge 0$ by assumption. Thus, we have constructed a set of options that reproduces the optimal inventory changes. The pattern is at most one set of puts at $i = 1$ and a set of calls for $i = 2,\ldots,I$. ■

Note that if $s^*(S, X, 1)$ is not monotonically decreasing, then a put has to be sold. In the 1-period problem, we know how many options to issue with each state because we know the optimal quantity decisions for the public and private players associated with each state. The linear program is solved for the number of options needed to match the public decision associated with the equilibrium price. With an option designed for multiple periods, we do not know the future state in which the options will be exercised. For an option defined solely by price to match the optimal policy, each equilibrium price has to be associated with a unique public decision. Otherwise, the quantity of options bought and sold will not match one of the possible decisions. Thus, by finding two decisions that result in the same price, no option can be designed to be exercised in a future period based solely on price that can match the optimal policy.

Theorem 2: It is not possible to match the optimal policy using multi-period options that are based solely on price without any other restrictions based on the state.

Proof: From Murphy et al. (1989) we know that if $X + S = X' + S'$ then $s(S, X, i) + x(S, X, i) = s(S', X', i) + x(S', X', i) \ \ \forall i$ and $S + s(S, X, i) = S'' + s(S, X, i)$. Consequently, we have multiple public (but unique total) inventory decisions associated with the same price outcome and the result holds. ■

Since single-period options are almost identical to political leaders making the decision, we need multi-period options for the market to trust that government will take the right actions. Thus, we need to design a more complicated triggering mechanism.

The design of the triggering mechanism of the option has to be as simple as possible to minimize disputes on whether it can be exercised. An obvious feature to include would be the size of the disruption, the state *i*. However, the proof of Theorem 2 shows that within a market state the public drawdown is not uniquely related to price. Thus, the triggering mechanism has to account for the size of the inventories.

We see multi-period options taking the following form. At the start of the policy, the government auctions a set of options with exercise prices and categorizations specifying when they can be used. The options have a reasonably long exercise period and the government auctions new options as these expire. The replacements are auctioned far enough ahead so that the market believes that the government is continuing to use market mechanisms for managing the reserve. The new options have the same start date and time as the expiration date and time of the existing options. Soon after a set of options is exercised, the government issues replacement options. Thus, the government keeps the right number of options in the market at all times to replicate the optimal policy. As the optimal policy changes, the new options are issued based on the new optimal policy. Since the value of the reserves is dependent on the level of imports and baseline world oil prices change, the optimal policy should change gradually. How to change the strike prices and number of options as markets evolve is a question for further research.

We now establish conditions that link a unique policy to each market-clearing price resulting from the optimal policy.

Theorem 3: If the optimal policy satisfies property M, then having the market clearing price from the optimal decision, the public inventory level, and market state uniquely identifies the full state (S, X, i) and the associated public decision.

Proof: From Murphy et al. (1989) we know that for a given market state, total inventory change, which determines the price, is a function of total inventory, not the inventories of each player. From property M total inventory change is different for each level of total inventory. Thus, there is a unique price associated with each level of total inventory for a given market state. Given the public inventory, the optimal market-clearing price, $P^*(S, X, i)$, is different for each level of private inventory, as total inventory is different. Thus, given a market state and the level of public inventory, there is a one-to-one map between private inventory and price, and we can determine the full state (S, X, i) and its associated decision. ■

We now present a linear program that provides a set of options for a given market state assuming that the prices associated with the different levels of public inventory are unique. We remove the market-state index, noting that the linear program has to be solved for each state. Let us define the variables. $s^*(P)$, the optimal change in public inventory associated with the resulting market price *P*; u_P^+, the amount of puts sold with a strike price *P*, which is the market clearing price for state $s^*(P)$; u_P^-, the amount of puts bought with a strike price *P*, which is the market clearing price for state $s^*(P)$; v_P^+, the amount of calls sold with a strike price *P*, which is the market clearing price for state $s^*(P)$; v_P^-, the amount of calls bought with a strike price *P*, which is the market clearing price for state $s^*(P)$. As in the one-period model, the linear program (10) provides the minimum number of options that have to be sold to match the optimal SPR policy, *N*-periods ahead.

$$\min \sum_{P\in\Pi}(u_P^+ + u_P^- + v_P^+ + v_P^-) \tag{10a}$$

subject to

$$\sum_{P'\le P} u_{P'}^+ - \sum_{P'\le P} u_{P'}^- - \sum_{P'\ge P} v_{P'}^+ + \sum_{P'\ge P} v_{P'}^- = s^*(P) \qquad \forall i \tag{10b}$$

$$u_P^+ \ge 0,\ u_P^- \ge 0,\ v_P^+ \ge 0,\ v_P^- \ge 0 \tag{10c}$$

Theorem 4: Assume property M holds. Solving (10) for each combination of public-inventory level and market state replicates the optimal SPR policy. The government need only sell both puts and calls that replicate the optimal government policy.

Proof: That (10) has a feasible solution follows from the proof of Theorem 1. The optimal policy is replicated because the full state is specified by the price from Theorem 3. That the government does not need to buy as well as sell comes from property M and the Proposition. Thus, the result holds. ■

5. Sample Results

We discretize the possible levels of crude-oil inventory in 25 MMBBL increments. The maximum public inventory is 1.4 billion BBL and private crude-oil inventories can to range between 200 and 500 MMBBL. We include three states of for supply in the oil market: normal, disrupted, and very disrupted. We use two sets of equilibrium prices assuming no inventory change: low prices ($35, $50 and $70 per barrel) and high prices ($60, $90, and $120 per barrel) for each state. Since the model deals with short-run disruptions, we use a short-run demand elasticity of 5%. The per-barrel, per-quarter holding cost is set at $1.20 for the public player and at 1% of the level of inventory for the private player, on average about $3.50 per quarter/barrel.

We assume that the convenience value of inventory is inversely related to the size of the private inventory with a value per quarter of $b(X) = 1000 / X$.

The transition probabilities are presented in Table 2 with states normal (N), disrupted (D) and very disrupted (V). They were computed using the real yearly average crude oil prices (source British Petroleum).

Table 2: Transition probabilities between states (%)

		Disruption Level		
		N	D	V
Disruption Level	N	90.1	7.9	2.0
	D	24.6	53.8	11.5
	V	0.0	22.2	77.8

For the results in Figures 2 and 3 we use a discount factor of 0.99 per quarter for the public player and two different discount factors for the private players of 0.98 and 0.97 per quarter. We use two different oil-price scenarios, one with low and another one with high reference prices. Figures 2 and 3 depict the results of four different scenarios. A: (0.98/quarter private discount factor, and high reference prices); B: (0.97/quarter private discount factor, and high reference prices); C: (0.98/quarter private discount factor, and low reference prices); D: (0.97/quarter private discount factor, and low reference prices). In all these scenarios we have used a q equal to 12800. In the scenarios with high reference prices we have a k equal to 15700 and a $w(i)$ equal to 270 and 440 for, respectively, disrupted and very disrupted states. In the scenarios with low reference prices we have a k equal to 15300 and a $w(i)$ equal to 220 and 430 for disrupted and very disrupted states, respectively.

The graphs illustrate the optimal policies by showing the build up starting from no public inventory with no disruption and showing the drawdown assuming the disruption persists. The reason for choosing persistent market conditions is to illustrate the general properties that the total inventory during build up (drawdown) is generally concave (convex) over time and the monotonicity properties hold. Also, the additions and withdrawals for any inventory levels at the start of normal or very disrupted periods can be read from these graphs.

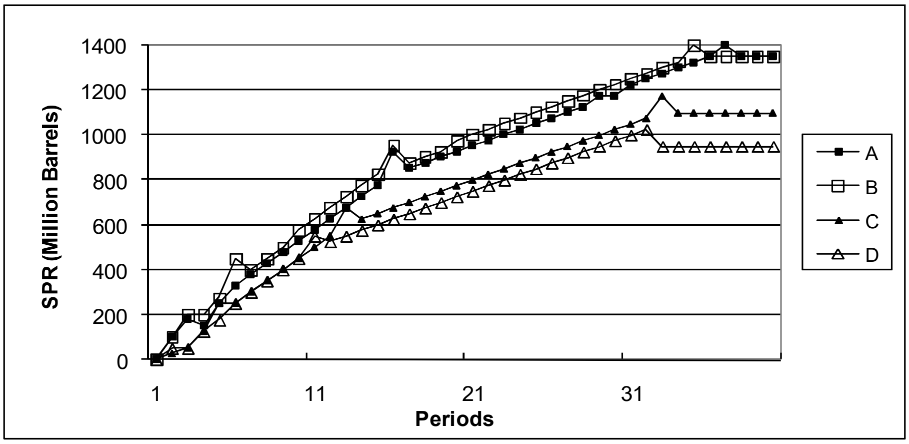

Figure 2: The buildup pattern for the SPR under normal market conditions starting from the lowest possible inventories.

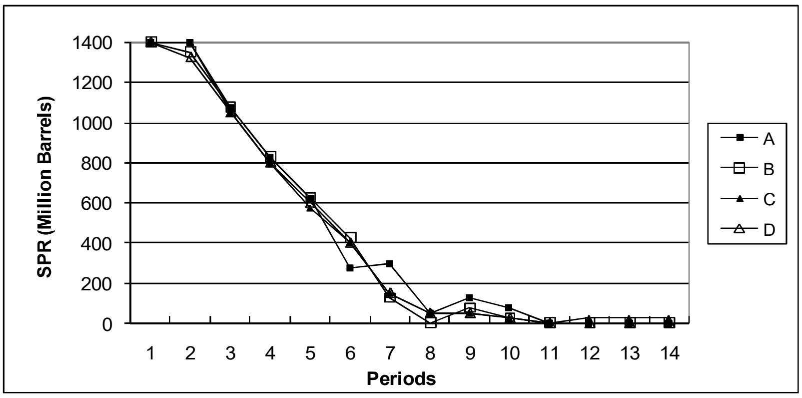

Figure 3: The drawdown pattern for the SPR under very disrupted market conditions starting from the highest possible inventories.

The results from experiments A and B suggest that under high prices the private players' discount factor is not important since SPR stocks tend to be close to the maximum (which is almost double of its current level and more than double of the its average value of 680 million barrels in these simulations). Moreover, by comparing the results from A & B with the results from C & D it is evident that under high prices the optimal level of the SPR is higher. Finally, experiments C and D show that interest rates (i.e., private discount factors) have an impact on the optimal value of the public reserves. In the presence of higher private interest rates, the value of the SPR is larger to compensate for lower reserves in the private sector.

Figure 3 presents the complete withdrawals from a full SPR with a very disrupted market that persists. From the limited data, the probability of a very severe disruption in oil markets lasting eight quarters is about 13%. The initial drawdown is less than the drawdowns in the immediately following periods because of a large initial reduction in private inventories.

In Figures 4 and 5 several patterns emerge. First, total inventory during the build-up is a concave function of time and a monotonically nondecreasing function of inventory levels. Similarly, the total drawdown is a monotonically nonincreasing function of inventory levels. The solutions satisfy properties M and M2. From Murphy et al. (1989), since M holds, the solution is unique. We see some swapping of inventory between the public and private players. This is most likely due to the inability to solve the arbitrage exactly because of the discretization of the state space.

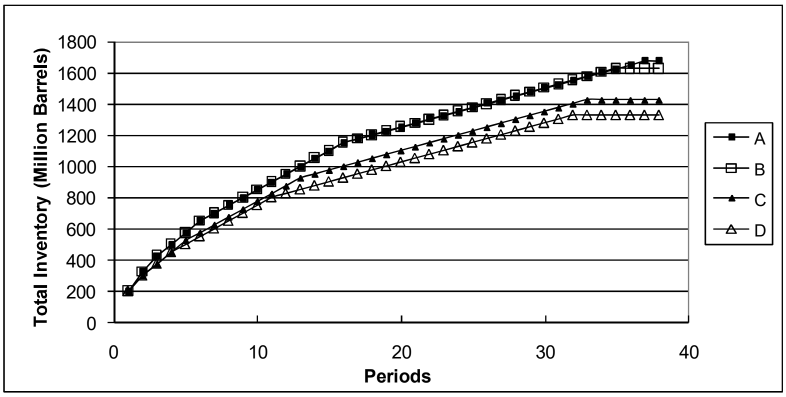

Figure 4: The buildup pattern for total inventories under normal market conditions starting from the lowest possible inventories.

In Table 2 we can compare the impact on expected prices of the optimal policy, comparing the prices with and without SPR. Here the expected prices increase about 4% in the normal state, decreasing about 8% in the disrupted state, and 9% in the very disrupted state. Overall, expected prices decrease about 1% in all the scenarios, suggesting that the active management of the SPR would lead to lower price differences between states and lower average prices. For example, the lowest average price increased from $36 to $37.5 per barrel in scenario D, in the normal state, and the highest average price decreased from \$116.1 to \$105.9 per barrel in scenario B, in the very disrupted state.

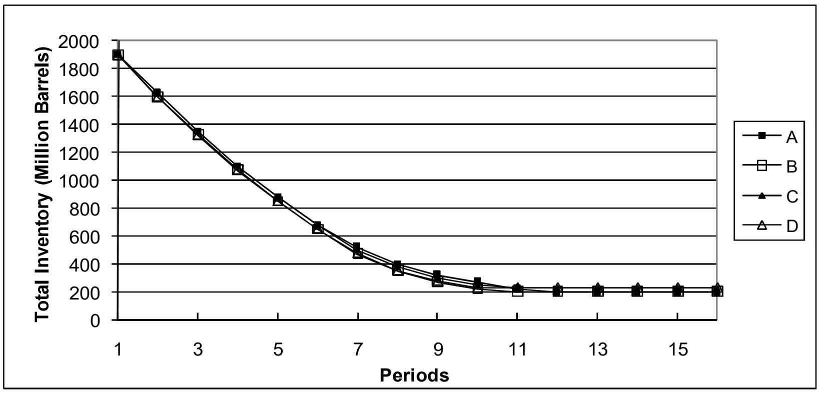

Figure 5: The drawdown pattern for total inventories under very disrupted market conditions starting from the highest possible inventories.

Table 2: Percentage expected price change as a result of introducing the SPR.

Price Change (%)				
	Normal	Disrupted	Very Disrupted	Global
A	4.3	-8.0	-8.9	-1.1
B	4.7	-8.8	-9.6	-1.3
C	4.2	-7.6	-8.8	-1.1
D	4.1	-7.9	-8.1	-1.1

In Table 3 we present the summary results for the number of contracts for each scenario (A to D) and by type of contract issued by the public player. These contracts are the total number of calls and puts bought and written by the public player at different exercise prices, and under different levels of SPR inventory, as required to implement the optimal levels of the SPR.

Table 3: The number of option contracts auctioned by the public player by scenario.

Number	u^+	u^-	v^+	v^-	Total
Contracts A	8325	1775	17525	1900	29525
Contracts B	6775	575	20200	1050	28600
Contracts C	4400	200	14150	1025	19775
Contracts D	4300	325	13675	650	18950

The total number of contracts for each scenario are between 17.5 times (C and D) and 20.5 times (A and B) the maximum level of the SPR in the corresponding scenario. From Table 3 we see that that the number of contracts written in the scenarios with higher reference prices (A and B) is higher. The majority of the contracts are calls sold by the public player (v+), between 59% and 72% of the total, followed by puts sold by the public player (u+), between 22% and 28% of the total. This result indicates that the private players can use the SPR for risk management, buying calls to cover what they use in disrupted states.

Finally, overall the management of the SPR would be profitable, as the private sector would pay for the financial contracts and would buy crude when the price is high and sell when the price is low.

6. Conclusions

We have made the first steps in designing a set of financial options that reproduce the optimal build up and drawdown policy for the US Strategic Petroleum Reserve. We show the potential for using markets to better manage the Strategic Petroleum Reserve in order to provide the certainty of action necessary for government SPR policy to be credible and implement government plans without the tensions associated with an immediate crisis shaping the decisions. We have been able to show that standard options based solely on price cannot replicate the optimal policy and the options have to be defined in terms of the state space. We present one set of such options.

Finally, we look at factors influencing the size of the SPR and the social benefits of a more active policy in the management of the reserve by using a numerical example. Our results suggest that market-based management of the SPR would reduce price differences across market states, decrease the expected level of prices, and would be able to be self sustaining. Furthermore, the SPR policy would be credible, dampening the price effect of fear of disruption, as was present in 2008.

More research is necessary before options can be considered for managing the reserve. For example, in 2008 oil prices were highly volatile due to rapid changes in the world macro-economy. How the options would affect SPR policy in the face of this volatility needs to be examined. As one of the referees pointed out, the Markov chain has only one price associated with a market state. However, market prices are clearly volatile given a market state. An important research question is whether this volatility can trigger inventory builds and withdrawals that deviate from the optimal policy. Note that since the real function of the SPR is to dampen price spikes, adding this representation of volatility could change the optimal policy.

A larger question is whether political leaders would surrender control over the build up and withdrawal decisions to something mechanical like an options policy. Currently, that is unlikely, especially after the financial meltdown of 2008. Nevertheless, in the 1970's, when one of the authors was engaged in policy analysis at the Federal Energy Administration and its successor, the Energy Information Administration, deregulation of energy markets looked very unlikely because the congressional leaders had their formative experiences in the Depression and WWII, where markets either failed or were irrelevant and government action was successful. Yet, the Carter administration succeeded in changing the laws to phase in deregulation in oil and natural gas markets as well as non-energy sectors of the economy. Eventually, an inability to sell oil quickly will lead to policy changes for expediting sales and creating certainty on the availability of SPR oil in the event of a disruption.

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