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GÜLPıNAR, N. and OLIVEIRA, F.S.

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Robust Trading in Spot and Forward Oligopolistic Markets

N. Gulpinar: Warwick Business School, University of Warwick, Coventry, CV4 7AL, UK.

E-mail: Nalan.Gulpinar@wbs.ac.uk

F. S. Oliveira*: ESSEC Business School, 100 Victoria Street, National Library Building, #13-02, Singapore 188064. Email: Oliveira@essec.edu

Abstract: In this article we consider the interaction between forward and spot prices and analyse trading in oligopolistic markets under uncertainty. We extend the two-stage risk-neutral stochastic model to worst-case analysis with rival demand scenarios. At the methodological level we develop a robust analysis in oligopolies and derive analytical results on the impact of demand uncertainty on the oligopolies' behaviour, when using robust optimal strategies. We compare the performance of robust optimization with the cases of no uncertainty and risk-neutral uncertainty. We show that under robust oligopolies the firms tend to produce more and to trade more in the forward markets, moreover, under robust optimization market prices are lower.

1. Introduction

Procurement (e.g., Virolainen, 1998) and trading strategies have been the topic of extensive research, including trading that consider electronic exchanges (e.g., Teich et al., 2006), procurement in electronic markets (e.g., Chen & Liu, 2007; Ganeshan et al., 2009; Gunasekaran et al., 2009), trading of production technologies in second-hand markets (Bunn & Oliveira, 2007) considering regulatory intervention (Bunn & Oliveira, 2008), the relationship between spot markets and bilateral contracts (e.g., Seifert et al., 2004; Agrawal & Ganeshan, 2007), and between forward contracts and spot markets (e.g., Wu et al., 2002; Wu & Kleindorfer, 2005; Anderson et al., 2007; Dong & Liu, 2007; and Mendelson &Tunca, 2007).

On the relationship between forward contracts and spot markets, Allaz & Vila (1993) have shown that the introduction of futures markets would improve competition in any oligopolistic market with Cournot players, reducing the ability of firms to increase prices. This hypothesis was empirically tested by Herguera (2000) in the context of bilateral electricity markets, who found evidence supporting it. Le Coq & Orzen (2006) have also confirmed this hypothesis by using laboratory experiments. However, it has also been argued that, in an oligopoly, firms buy their own production in the forward markets, increasing equilibrium prices, when compared with the scenario without forward trading, Mahenc & Salanie (2004) and it has been shown that speculation in a commodity, in oligopolistic markets, leads to lower level of inventory and higher price volatility than under perfect competition, McLaren (2003).

The conventional approach to decision making under uncertainty is based on the optimization of expected value. The main concern with this approach is that it avoids the worst-case effect of uncertainty in favor of expected values, Rustem & Howe (2002). While expected optimization is an acceptable tool for certain instances, decision making on stochastic programming needs to be justified in view of the worst-case scenario. This is also important if the decision to be made can be influenced by the uncertainty in such a way that, in the worst-case, it has critical consequences on the underlying system. Furthermore, it is known that the forecasts and estimations of uncertain parameters are inherently inaccurate as there are different rival estimates, or scenarios, e.g., Fair (1984). When predicting the future, it is often difficult or impossible to settle on a single forecast or an arbitrary pooling of scenarios, Gulpinar & Rustem (2007). Additionally, model-based policy design entails a reasonable specification of the underlying model and an appropriate characterization of the uncertainties, Becker et al. (2000). The latter can be due to an exogenous effect, uncertainty on the

parameters, or on the structure of the model (which requires a setting that admits rival structures), Hall & Stephenson (1990). The significance of robust strategies is increasingly recognized as attitudes towards risk evolve in diverse areas such as economics, financial markets and engineering, e.g., Rustem & Howe (2002).

The risk of making incorrect decisions in an uncertain environment can be managed using robust optimization. The minimax algorithms with several applications to a number of problems in diverse areas are presented in Rustem & Howe (2002). In general, the parameter uncertainty is characterized either in terms of a number of rival scenarios or ranges in which the uncertain parameters, or exogenous effects, may vary. Such characterization of uncertainty leads to discrete and continuous minimax models. The discrete minimax approach enables the use of point forecasts or specifications such as rival models. The optimal strategy is determined taking into account all specified rival scenarios simultaneously, rather than any single scenario. The continuous minimax strategy provides a guaranteed optimal performance in view of continuum of scenarios varying between upper and lower bounds. Thus, in this case, there are an infinite number of future scenarios in ranges arising from statistical properties associated with the uncertainty, Gulpinar & Rustem (2007). Another alternative is to define the worst-case over uncertainty sets, such as an ellipsoid, that offers significant advantages over the more traditional approaches with probabilistic representation of noise.

Following Allaz (1992), Dong & Liu (2007) and Mendelson & Tunca (2007), we consider a two-stage *n*-player dynamic game with uncertainty. We assume that in the spot market the players have perfect information regarding the level of demand and forward positions of all the players in the market. In the forward market there is uncertainty regarding the level of demand. Our contributions can be briefly summarized as follows. First, we model inter-temporal decision making in oligopolistic markets using a scenario based stochastic program, e.g., Fair (1984), Gulpinar & Rustem (2007). Demand uncertainty is represented by discrete scenarios for the intercept of the inverse demand function. Second, we extend this model to worst-case design with rival demand scenarios. Therefore, the imprecise nature of parameter forecasts for dynamic Cournot games and the impact of demand uncertainty on the relationship between forward and spot trading are addressed. Moreover, we analyze the relationship between forward contracts and spot markets using scenario based stochastic program (after adapting it to the discrete scenario framework), and compare their performance with a robust game in which each player computes his worst-case profit given the strategies of his opponents (worst-case profit at the player level).

At the methodological level, the contribution of this paper is to develop worst-case design approach at the player level. In this model, each player maximizes his worst-case profit in view of all rival scenarios of the uncertain demand parameter, taking into account that all the other players in the game are simultaneously maximizing their worst-case profit. Our analysis shows that, in this case, the worst-case profit of the different players is not higher than the worst-case profit in the risk-neutral stochastic model. This observation suggests that when firms are conservative the prisoners' dilemma effect is even stronger as firms tend to base their choices on the worst possible scenarios. This leads to an industry that is more competitive than in the risk-neutral case.

The rest of the paper is organized as follows. In Section 2 we present the deterministic and stochastic two-stage oligopoly market models, in which stochastic programming maximizes expected total profit under demand uncertainty. Section 3 summarizes the minimax problems in general. In Section 4, we develop robust trading approach for two-stage oligopoly model. Sections 5 and 6 present the analytical and computational results, respectively. Section 7 gives a short summary and our conclusions.

2. Two-stage Oligopoly Models

The two-stage oligopoly model involves trading in forward and spot markets. It is assumed that forward trading occurs before production. Thus, forward contracts are traded in the first stage, in the oligopolistic industry, under price uncertainty. In other words, the spot price is still unknown when trading takes place in the forwards market. In the second stage, the price uncertainty is resolved, the players trade in the spot market, and the actual service (or product) is delivered. Taking into account the forward position taken in the first stage each producer makes a decision on the amount of production. Therefore, an important factor determining spot prices is the demand for the commodity. In this context, Allaz (1992) developed a stochastic model where demand is assumed to be uncertain and normally distributed and Allaz & Vila (1993) introduced the deterministic model in which average demand was considered. In this section, we summarise the two-stage deterministic and risk-neutral stochastic oligopoly market models, Allaz (1992) and Allaz & Vila (1993).

We use d, s and w superscripts to distinguish the notation used for deterministic, stochastic and worst-case models, respectively. Expected value of random parameter $\widetilde{\omega}$ is denoted by $E[\widetilde{\omega}]$. We introduce

deterministic and stochastic trading models in this section and extend it to the worst-case analysis in the next section.

2.1 Deterministic Model

We consider n players that are represented by index of i where $i = 1, 2, \cdots, n$. Total profit of a player i during entire planning horizon is denoted by π_i^d for $i = 1, 2, \cdots, n$. Let Q_i^d be total production of the player i for $i = 1, 2, \cdots, n$. The oligopoly's total output Q^d, is the sum of outputs of all players in the spot market and formulated as $Q^d = \sum_{i=1}^{n} Q_i^d$.

The deterministic model possesses a two-stage decision process where forward and spot markets have its own characteristics. The forward position of a firm at the first period has an influence on the spot price at time zero. Let F_i^d denote trading made by firm i in forward market. It is called *forward sale* if $F_i^d > 0$, or *forward purchase* if $F_i^d < 0$. Let P^d and S^d represent forward and spot prices, respectively. Following Allaz (1992), we model consumer behaviour using a linear function, in which price is a function of production as $S^d = S^d(Q^d) = a - bQ^d$, where the level of demand is defined by the parameter a and b is constant. It is worthwhile to mention that the methodology developed in this paper can be applied to a nonlinear price function with certain complexities on derivation of equilibrium conditions.

The cost of production, C_i^d for player i, is assumed to be $C_i^d = c_i Q_i^d$ where the marginal cost of each player is $0 < c_i < a$. Profits earned in the spot and forward markets, so-called spot and forward profits are defined, respectively, as $U_{spot}^d = S^d(Q_i^d - F_i^d) - c_i Q_i^d$, and $U_{forward}^d = P^d F_i^d$. Hence, a firm's aim is to maximize its total profit, which is sum of the profits earned in the spot and forward markets:

$$\pi_i^d = U_{spot}^d + U_{forward}^d = S^d(Q_i^d - F_i^d) - c_i Q_i^d + P^d F_i^d.$$

The Cournot-Nash equilibrium in the spot market is a vector of outputs $(Q_1^d, Q_2^d, \cdots, Q_n^d)$ such that the first-order necessary and second-order sufficient conditions are satisfied for all producers $i = 1, 2, \cdots, n$.

$$\frac{\partial \pi_i^d}{\partial Q_i^d} = \frac{\partial S^d}{\partial Q_i^d} Q_i^d + S^d - \frac{\partial C_i^d}{\partial Q_i^d} - \frac{\partial S^d}{\partial Q_i^d} F_i^d \leq 0$$

$$\frac{\partial^2 \pi_i^d}{\partial (Q_i^d)^2} = \frac{\partial^2 S^d}{\partial (Q_i^d)^2} Q_i^d + 2\frac{\partial S^d}{\partial Q_i^d} - \frac{\partial^2 C_i^d}{\partial (Q_i^d)^2} - \frac{\partial^2 S^d}{\partial (Q_i^d)^2} F_i^d < 0$$

The solution of this equation system is obtained as $Q_i^d = \dfrac{a - nc_i + nbF_i^d + \sum\limits_{j=1,\ j\neq i}^{n} c_j - b \sum\limits_{j=1,\ j\neq i}^{n} F_j^d}{(n+1)b}$ and the

spot price is determined as $S^d = \dfrac{a + \sum\limits_{i=1}^{n} c_i - b\sum\limits_{i=1}^{n} F_i^d}{n+1}$.

2.2 Stochastic Model with Discrete Demand Scenarios

Stochastic decision models rely on future events. This is one way of characterising uncertainty in terms of future scenarios. The future is viewed in terms of scenarios that are essentially a discrete set of realisations of uncertainty given the probability domain. A method to obtain the discrete outcomes for the random variables is referred to as "scenario tree generation". There are several approaches to generate scenario trees such as simulation, clustering and optimisation techniques, e.g. Hoyland et al. (2003). The discretization of the random values and the probability space leads to a framework in which a random variable takes finitely many values. Thus, the factors driving the risky events are approximated by a discrete set of scenarios, or sequence of events. Given the event history up to a particular time, the uncertainty in the next period is characterized by finitely many possible outcomes for the next observation. This branching process is represented using a scenario tree. The root node in the scenario tree represents the "today" and is immediately observable from deterministic data. The nodes further down represent the events of the world which are conditional at later stages. The arcs linking the nodes represent various realizations of the uncertain variables. An ideal situation is that a generated set of scenarios represents the whole universe of possible outcomes of the random variable. Therefore, scenarios should include both optimistic and pessimistic projections.

Assume that demand uncertainty is described by discrete scenarios for the intercept of the inverse demand function. Let u denote a scenario that is defined as a possible realisation of the uncertain parameter $\tilde{a}$. Let p_u be the conditional probability of event or scenario u that must sum to one, i.e. $\sum\limits_{u=1}^{K} p_u = 1$ where K is

the number of finite realisations of $\widetilde{a}$. Hence, spot price at scenario $u = 1, 2, \cdots, K$ is described by a

discretized probabilistic model as $S_u^s = a_u - b \sum_{i=1}^{n} Q_{iu}^s$, where Q_{iu}^s denotes the output produced by player i at

scenario u. The production cost under scenario u for player i is computed as $C_{iu}^s = c_i Q_{iu}^s$ where

$0 < c_i < E[\widetilde{a}]$.

The expected forward price based on K number of scenarios is estimated as $E[P^s] = \sum_{u=1}^{K} p_u S_u^s$. The

Cournot Nash equilibrium in spot market in view of all rival scenarios is represented by a set of equations:

$$Q_{iu}^s = \frac{a_u - nc_i + nbF_i^s + \sum_{j=1, \ j \neq i}^{n} c_j - b \sum_{j=1, \ j \neq i}^{n} F_j^s}{(n+1)b}, \quad i = 1, 2, \cdots, n, \quad u = 1, 2, \cdots, K.$$

A stochastic model takes into account a firm's attitude towards risk due to uncertainty on the spot price.

Figure 1 represents the time path of the sequence of decisions and realizations of the stochastic parameters in

the model. At time one, the decisions on the quantity to trade forward and the equilibrium price in the forward

markets depends on the expected value of a. At time zero, the spot market takes place in which the quantities

produced and spot prices are decided already taking into account the specific realization of the scenario about

demand.
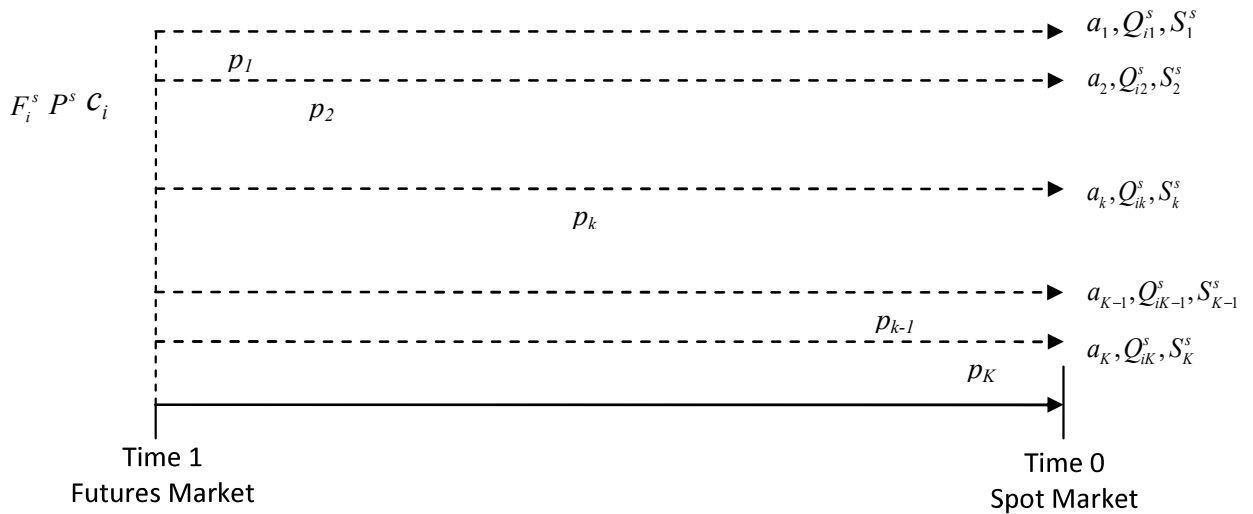
Figure 1: Time path of the sequence of decisions and stochastic parameters in the stochastic model.

The total profit of the producer i, for $i = 1, 2, \cdots, n$, at scenario u is expressed as

$\pi_{iu}^s = S_u^s \left(Q_{iu}^s - F_i^s \right) - c_i Q_{iu}^s + P^s F_i^s$. The player i's expected profit is computed as $E\left[\pi_i^s \right] = \sum_{u=1}^K p_u \pi_{iu}^s$. Each

producer aims to maximize its expected profit in view of all discrete demand scenarios. Therefore, the optimal

trading strategy is achieved by solving the scenario based profit maximization problem (1).

$$
\begin{aligned}
\max_{F_i^s} \quad & E[\pi_i^s] \\
s.t \quad & \\
\pi_{iu}^s = \quad & S_u^s \left(Q_{iu}^s - F_i^s \right) - c_i Q_{iu}^s + P^s F_i^s, && u = 1, 2, \cdots, K \\
S_u^s = \quad & a_u - b \sum_{i=1}^n Q_{iu}^s, && u = 1, 2, \cdots, K \\
P^s = \quad & \sum_{u=1}^K p_u S_u^s && (1) \\
Q_{iu}^s = \quad & \left[a_u - n c_i + n b F_i^s + \sum_{j=1,\ j \neq i}^n c_j - b \sum_{j=1,\ j \neq i}^n F_j^s \right] \frac{1}{(n+1)b}, && u = 1, 2, \cdots, K.
\end{aligned}
$$

This is a nonlinear program where c_i, a_u, b, and p_u are inputs and K and n are pre-specified parameters. The size of the stochastic model depends on the number of players in the market and the number of scenarios generated. Discrete scenarios can be generated by either using expert-knowledge or applying a scenario generation method or a forecasting technique, e.g. Gulpinar et al. (2004).

3. Robust Optimal Decisions under Uncertainty

The stochastic characterization of uncertainty relies on the average or expected performance of the system in the presence of uncertain effects. Although expected performance optimization is often adequate, it is the realization of the worst-case that causes the failure of the system, Rustem & Howe (2002). An important tool to address the inherent error for forecasting uncertainty is worst-case analysis. Worst-case analysis (min-max) provides robust optimal strategies that yield guaranteed performance. From the risk management point of view, minimax yields the best strategy determined simultaneously with the worst state of the underlying system and also analyses the effects of uncertain events. Min-max optimal strategy is determined in view of all the scenarios, rather than any single scenario. Thus, min-max optimisation is more robust to the realisation of worst-case scenarios than considering a single scenario or an arbitrary pooling of scenarios. It is therefore

suitable for situations which need protection against risk of adopting a strategy based on the wrong scenario, Gulpinar & Rustem (2007).

The worst-case approach minimizes the objective function with respect to the worst possible outcome of the uncertain variables. Let $G : R^{n+m} \rightarrow R$ be a function of a decision variable x and uncertain variable v for a stochastic system. The general minimax optimization problem can be stated as:

$$\min_{x \in X} \quad \max_{v \in V} \quad G(x,v)$$
$$s.t \qquad X \subset R^n, V \subset R^m \tag{2}$$

To be on the conservative side, the decision x is required to be optimal with respect to each observation of uncertain variable v. Therefore, x is chosen to minimize the objective function, where nature chooses v to maximize it. When the objective function is convex with respect to the uncertain variables the maximum will correspond to one or more vertices of the hypercube defined by the upper and lower bounds on the uncertain variables. If the objective function is concave with respect to the uncertainties the maximum may lie anywhere within the hypercube. The discrete minimax problem arises when the worst-case is to be determined over a discrete set. If V is a finite set, then (2) is called a discrete minimax problem and formulated as

$$\min_{x \in X} \quad \max_{v \in V} \quad G(x,v)$$
$$s.t \qquad V = \{v_1, v_2, \cdots, v_m\} \tag{3}$$

Introducing a more familiar notation, $G(x,v_j) = g_j(x)$ for $j = 1,2,\cdots,m$, we have $\min_{x \in X} \max_{j=1,2,\cdots,m} g_j(x)$. It can be shown that the minimax problem (3) is equivalent to the following optimization problem (4).

$$\min_{x \in X, z} \quad z$$
$$s.t \qquad g_j(x) \leq z, \qquad j = 1, 2, \cdots, m \tag{4}$$

If V has an infinite number of elements, then it is called continuous minimax. In this case, the minimax problem (2) can be reformulated as the semi-infinite optimization problem:

$$\min_{x \in X, z} \quad z$$
$$s.t \qquad G(x,v) \leq z, \qquad \forall v \in V = \{v_1, v_2, \cdots, v_m\} \tag{5}$$

with an infinite number of constraints corresponding to the elements in V. Worst-case analysis is a robust framework for decisions under uncertainty as the actual performance of the decision has a non-inferiority

property. Robustness is ensured by considering the optimal strategy in view of multiple rival scenarios generated and evaluating the expected profit corresponding to the best performance, simultaneously with the worst scenario. Therefore, the min-max strategy has the best lower bound performance which can only improve if any scenario, other than the worst-case, is realized, e.g. Gulpinar & Rustem (2007) and Rustem & Howe (2002).

Let x^* and v^* solve (2). Then, the following inequality is valid for all feasible $v \in V$, $G(x^*, v^*) \geq G(x^*, v)$. This inequality simply states the optimality of v^* for the corresponding problem (2). In addition, it signifies the robustness of minimax in that performance is assured to improve if the worst-case v^* does not happen. Similarly, under the same assumptions, for all feasible x, we have $G(x^*, v^*) \leq G(x, v^*)$.

As stated by Rustem & Howe (2002), in the presence of a discrete set of rival models, forecasts or scenarios purporting to describe the same system, the optimal decision needs to take into account all possible representations. The minimax problem arises when statistical or economic analysis cannot rule out all but one of the rival possibilities.

4. Discrete Minimax Approach for the Two-stage Oligopoly Model

In decision problems, when uncertainty is treated as a stochastic effect, the optimisation of the underlying system imposes a heavy burden on the policy modeller. The solution methodology for stochastic programming or expected value optimisation procedure requires the precise definition of the probability distributions. This can be relaxed by introducing ranges or rival scenarios for these probabilities and solving an optimisation problem robust to the imprecise nature of the probabilities. We are concerned with worst-case analysis for oligopoly markets in view of rival demand forecasts due to three main reasons. First, the optimal strategy based on expected value optimisation (stochastic program) needs to be justified when a probability distribution is associated with the demand uncertainty. Second, the imprecise nature of probability measurements leads to worst-case analysis in order to compute the best investment decision in view of rival demand specifications. Finally, it is not realistic to make a decision based on a single scenario (such as average value), but it is better to consider discrete rival scenarios describing different views on the future.

Given K rival scenarios, the discrete minimax problem, for two-stage oligopoly market at the player level, can be formulated by (6).

$$
\max_{F_i^w} \quad \min_{u=1,2,\cdots,K} \quad \pi_{iu}^w
$$

s.t

$$
\begin{aligned}
\pi_{iu}^w &= S_u^w\left(Q_{iu}^w - F_i^w\right) - c_i Q_{iu}^w + P^w F_i^w, & u &= 1,2,\cdots,K \\
S_u^w &= a_u - b\sum_{i=1}^n Q_{iu}^w, & u &= 1,2,\cdots,K \\
P^w &= \sum_{u=1}^K p_u S_u^w & & \qquad\qquad (6) \\
Q_{iu}^w &= \left[a_u - nc_i + nbF_i^w + \sum_{j=1,\ j\neq i}^n c_j - b\sum_{j=1,\ j\neq i}^n F_j^w \right]\frac{1}{(n+1)b}, & u &= 1,2,\cdots,K.
\end{aligned}
$$

There are several algorithms to solve the minimax problems; for instance, see Rustem & Howe (2002). We consider reformulation of the problem as a nonlinear program. Let μ_i denote the worst-case profit for player i. The worst outcome of player's profit in view of all rival scenarios is the minimum profit. This is described by a set of constraints. Then the minimax problem in (6) can be reformulated as the following nonlinear programming problem:

$$
\max_{F_i^w} \quad \mu_i
$$

s.t

$$
\begin{aligned}
\pi_{iu}^w &= S_u^w\left(Q_{iu}^w - F_i^w\right) - c_i Q_{iu}^w + P^w F_i^w, & u &= 1,2,\cdots,K \\
S_u^w &= a_u - b\sum_{i=1}^n Q_{iu}^w, & u &= 1,2,\cdots,K \\
P^w &= \sum_{u=1}^K p_u S_u^w & & \qquad\qquad (7) \\
Q_{iu}^w &= \left[a_u - nc_i + nbF_i^w + \sum_{j=1,\ j\neq i}^n c_j - b\sum_{j=1,\ j\neq i}^n F_j^w \right]\frac{1}{(n+1)b}, & u &= 1,2,\cdots,K \\
\pi_{iu}^w &\geq \mu_i, & u &= 1,2,\cdots,K.
\end{aligned}
$$

Since discrete minimax may yield pessimistic strategies, the rival scenarios need to be selected among likely values. The minimax strategy addresses the question of what the best strategy should be in view of the worst-case. The discrete minimax strategy ensures a guaranteed optimal performance in view of the worst-case and this is ensured for all rival scenarios.

5. Analytical Results

In this section, we derive analytical properties describing the relationships between deterministic, stochastic and discrete minimax models in terms of production level, forward trading and the player's profit.

Proposition 1. *When the expected value of the intercept of demand equals its value in the deterministic model, the following statements hold for any firm in oligopolistic markets:*

i.)   Spot trading in the deterministic model is equal to the expected spot trading in the risk-neutral model;

ii.)   Forward trading is the same in the deterministic and risk-neutral models.

Proof: From the equilibrium conditions in the deterministic and risk-neutral models, we obtain the spot trading and the expected spot trading for a firm i, respectively, as

$$E[F_i^s] = \left[(n-1)E[\tilde{a}] - (n-1)(n^2 + 1)c_i + (n-1)n\sum_{j=1}^{n} c_j \right] \frac{1}{(n^2 + 1)b}$$

and $Q_i^d = \left[na - n(n^2 + 1)c_i + n^2 \sum_{j=1}^{n} c_j \right] \frac{1}{(n^2 + 1)b}$. Therefore, comparison of these expressions leads to

$Q_i^d = E[Q_i^s]$ only when $E[\tilde{a}] = a$. Moreover, using the same, respective, equilibrium conditions we can calculate the forward trading for a firm i, in the deterministic model as follows

$$F_i^d = \left[(n-1)a - (n-1)(n^2 + 1)c_i + (n-1)n\sum_{j=1}^{n} c_j \right] \frac{1}{(n^2 + 1)b}.$$

Similarly, we can obtain the forward trading using the risk-neutral stochastic model as

$$E[F_i^s] = \left[(n-1)E[\tilde{a}] - (n-1)(n^2 + 1)c_i + (n-1)n\sum_{j=1}^{n} c_j \right] \frac{1}{(n^2 + 1)b}.$$ Hence, $F_i^d = E[F_i^s]$ if and only

if $E[\tilde{a}] = a$.   ∎

Proposition 1 suggests that even though, in general, the deterministic and the stochastic models consider different forward and spot prices, they produce the same expected quantities traded in the spot market as well as the same expected forward trading under the specific choice of demand. In those circumstances, a risk-neutral analysis of a decision problem may not provide additional information to the decision maker when compared to the deterministic approach.

Next, we establish a relationship between the weighted averages of spot prices in the risk-neutral model (in which the weights are the quantities produced in the spot market in all the possible scenarios) with the deterministic model such that the players' profits achieved by the deterministic and risk-neutral scenario based Cournot games are comparable. Those circumstances stated in Proposition 2 reinforce the idea that the risk-neutral analysis provides no additional information when compared to the deterministic analysis.

Proposition 2. *A player's expected profit in the risk-neutral stochastic model is equal to (higher or lower than) his expected profit in the deterministic model if and only if the spot price in the deterministic model is equal to (less or higher than) the weighted average of the prices under different scenarios in the stochastic model.*

Proof: Assume that the non-arbitrage condition holds. Let π_{iu}^s denote the profit of player i at scenario u under risk-neutral stochastic model. The expected profit of the player i is computed as

$$E[\pi_i^s] = \sum_{u=1}^{K} p_u \pi_{iu}^s = \sum_{u=1}^{K} p_u \left\{ \left(S_u^s - c_i\right)Q_{iu}^s + \left(P^s - S_u^s\right)F_i^s \right\}.$$ Since in equilibrium $P^s = E[S^s] = \sum_{u=1}^{K} p_u S_u^s$

holds, we obtain $E[\pi_i^s] = \sum_{u=1}^{K} p_u \left(S_u^s - c_i\right)Q_{iu}^s$. Recall that $\pi_i^d = S^d Q_i^d - c_i Q_i^d$. The difference between the expected profit earned by the stochastic model and the profit by the deterministic model becomes

$$E[\pi_i^s] - \pi_i^d = \sum_{u=1}^{K} p_u S_u^s Q_{iu}^s - c_i \sum_{u=1}^{K} p_u Q_{iu}^s - S^d Q_i^d + c_i Q_i^d.$$ Using $Q_i^d = \sum_{u=1}^{K} p_u Q_{iu}^s$ from Proposition 1, we

obtain $E[\pi_i^s] - \pi_i^d = \sum_{u=1}^{K} p_u S_u^s Q_{iu}^s - S^d \sum_{u=1}^{K} p_u Q_{iu}^s = \sum_{u=1}^{K} p_u Q_{iu}^s \left(S_u^s - S^d\right).$ Therefore, we can easily show

that the following properties exist:

i.) $E[\pi_i^s] \geq \pi_i^d$ if and only if $\displaystyle\sum_{u=1}^{K} \frac{p_u Q_{iu}^s}{Q_i^d} S_u^s \geq S^d$, and

ii.) $E[\pi_i^s] < \pi_i^d$ if and only if $\displaystyle\sum_{u=1}^{K} \frac{p_u Q_{iu}^s}{Q_i^d} S_u^s < S^d$. ∎

In the rest of this section, in order to simplify our calculations, we assume that all firms are homogeneous, i.e., they have the same production technology. Let $\bar{a}$ denote expected value of $\tilde{a}$ over K scenarios:

$\bar{a} = \sum_{u=1}^{K} p_u a_u$. The worst-case profit μ_i^* of player i is obtained by solving the following nonlinear program.

$$\max_{F_i^w, \mu_i} \left\{ \mu_i \mid \pi_{iu}^w \geq \mu_i, \quad u = 1, 2, \cdots, K \right\} \tag{8}$$

Using (8) we can derive the worst-case solution of the Cournot dynamic games including the forward trading position, as presented in Proposition 3.

Proposition 3. *The worst-case forward trading for homogenous firms is*

$$F^w = \left[(b(n-1) - n - 1)a_{u^*} + (n^2(b-1) + b - n)c + (n+1)(n+1 - bn)\bar{a} \right] \frac{1}{b(n^2 + 1)}$$

Proof: Let $\lambda_{iu} \geq 0$ for $u = 1, 2, \cdots, K$ be the Lagrangian multipliers associated with constraints $\pi_{iu}^w \geq \mu_i$. Lagrangian function associated with the worst-case profit optimization problem (8) is given by

$L_i(\mu_i, F_i^w, \lambda_i) = -\mu_i + \sum_{u=1}^{K} \lambda_{iu} \left(\mu_i - \pi_{iu}^w \right)$ The first order necessary optimality conditions for (8) can be derived as

$$\frac{\partial L_i}{\partial \mu_i} = -1 + \sum_{u=1}^{K} \lambda_{iu} = 0, \tag{9}$$

$$\frac{\partial L_i}{\partial F_i^w} = -\sum_{u=1}^{K} \lambda_{iu} \frac{\partial \pi_{iu}^w}{\partial F_i^w} = 0, \tag{10}$$

$$\frac{\partial L_i}{\partial \lambda_{iu}} = \mu_i - \pi_{iu}^w \leq 0, \qquad u = 1, 2, \cdots, K \tag{11}$$

$$\lambda_{iu} \left(\mu_i - \pi_{iu}^w \right) = 0 \qquad u = 1, 2, \cdots, K \tag{12}$$

$$\lambda_{iu} \geq 0 \qquad u = 1, 2, \cdots, K \tag{13}$$

Let (μ_i^*, λ_i^*) solve the problem (8). The case that $\lambda_{iu} = 0$ for all scenarios $u = 1, 2, \cdots, K$ is not possible since it contradicts (9). Therefore, suppose that there exists at least one scenario so that $\lambda_{iu} \neq 0$. Let u^* denote this scenario. In this case, $\lambda_{iu^*} = 1$ and for $u \neq u^*$ and $u = 1, 2, \cdots, K$ $\lambda_{iu^*} = 0$. The optimality conditions

(10) and (11) become $\lambda_{iu^*} \dfrac{\partial \pi_{iu^*}^w}{\partial F_i^w} = 0$ and $\mu_i - \pi_{iu^*}^w = 0$ for $u = u^*$. For $u \neq u^*$, $\mu_i - \pi_{iu^*}^w \leq 0$ holds.

Therefore, we obtain $\dfrac{\partial \pi_{iu^*}^w}{\partial F_i^w} = \dfrac{\partial S_{u^*}^w}{\partial F_i^w}\left(Q_{iu^*}^w - F_i^w\right) + S_{u^*}^w \left(\dfrac{\partial Q_{iu^*}^w}{\partial F_i^w} - 1\right) - c_i \dfrac{\partial Q_{iu^*}^d}{\partial F_i^w} + \dfrac{\partial P^w}{\partial F_i^w} F_i^w + P^w = 0.$

$$-b\left(\sum_{i=1}^n \dfrac{\partial Q_{iu^*}^w}{\partial F_i^w}\right)\left(Q_{iu^*}^w - F_i^w\right) + \left(a_{u^*} - b\sum_{i=1}^n Q_{iu^*}^w\right)\left(\dfrac{\partial Q_{iu^*}^w}{\partial F_i^w} - 1\right) - c\dfrac{\partial Q_{iu^*}^w}{\partial F_i^w} - b\left(\sum_{u=1}^K p_u\left[\sum_{i=1}^n \dfrac{\partial Q_{iu^*}^w}{\partial F_i^w}\right]\right)F_i^w + \sum_{u=1}^K p_u S_u^w = 0$$

where $\dfrac{\partial Q_{iu^*}^w}{\partial F_i^w} = \dfrac{n}{n+1}$ and $\dfrac{\partial S_{u^*}^w}{\partial F_i^w} = \dfrac{\partial P^w}{\partial F_i^w} = \dfrac{-b}{n+1}$. This can be further simplified as

$$-\dfrac{-bn}{n+1}\left(Q_{iu^*}^w - F_i^w\right) - \dfrac{1}{n+1}\left(a_{u^*} - b\sum_{i=1}^n Q_{iu^*}^w\right) - \dfrac{cn}{n+1} - \dfrac{bF_i^w}{n+1} + \sum_{u=1}^K p_u S_u^w = 0 \qquad (14)$$

As all players are homogenous, that is $F_1^w = F_1^w = \cdots = F_n^w = F^w$ and $Q_{1u^*}^w = Q_{2u^*}^w = \cdots = Q_{nu^*}^w = Q_{u^*}^w$ and

have the same marginal cost $c_1 = c_2 = \cdots = c_n = c$, the equation (14) becomes

$$\left(b(n-1) - n - 1\right)a_{u^*} + \left(n^2(b-1) + b - n\right)c + \left(-b(1+n^2)\right)F^w + (n+1)(n+1-bn)\bar{a} = 0.$$

Then the worst-case forward trading for homogeneous firms is obtained as stated in the proposition. ∎

It is worthwhile to mention that, contrary to the risk-neutral stochastic model, the position in the worst-case forward trade for homogenous firms depends on both $\bar{a}$ (the expected values of the demand parameters) and a_{u^*} (the value of the intercept of the demand function in the scenario in which at least one of the firms obtains its lowest profit).

We carry on the analytical results by comparing the equilibria obtained under the risk-neutral stochastic model and the robust analysis. We can derive the specific conditions under which the expected production in the robust analysis is higher or lower than the one in the risk-neutral stochastic model, as expressed in Proposition 4.

Proposition 4. *For any homogeneous firm in equilibrium, the inequality* $\dfrac{E[Q_i^w]}{E[Q_i^s]} > 1$ *holds if any of the*

following conditions is satisfied: i.) $\bar{a} > a_{u^*}$ *and* $b < \dfrac{n+1}{n-1}$, *or ii.)* $\bar{a} < a_{u^*}$ *and* $b > \dfrac{n+1}{n-1}$.

Proof: From the equilibrium conditions of the risk-neutral stochastic model for homogeneous firms,

we have $E[Q_i^s] = \dfrac{n\big(E[a_u]-c\big)}{(n^2+1)b} = \dfrac{n(\bar{a}-c)}{(n^2+1)b}$. Similarly, the worst-case production level under scenario u for

homogenous firms is $Q_{iu}^w = \dfrac{a_u - c_i + F_i^w}{n+1}$. When substituting F_i^w in Q_{iu}^w and taking its expected value, we

obtain $E[Q_i^w] = \dfrac{\big(b(1-n)+1+n\big)\big(\bar{a}-a_{u^*}\big)+n(n+1)(\bar{a}-c)}{(n^2+1)(n+1)b}$. The ratio of $E[Q_i^w]$ to $E[Q_i^s]$ is obtained as

$\dfrac{E[Q_i^w]}{E[Q_i^s]} = 1 + \left(\dfrac{b(1-n)+1+n}{n(n+1)}\right)\left(\dfrac{\bar{a}-a_{u^*}}{\bar{a}-c}\right)$. Recall that $\bar{a} > c$ and u^* denotes a scenario selected such a way

that the corresponding the Lagrangian multiplier in Equation (12) is positive. The demand scenario at u^* is

a_{u^*} in which at least one of the homogenous firms achieves the lowest profit. The inequality $\dfrac{E[Q_i^w]}{E[Q_i^s]} > 1$ is

valid only when $b(1-n)+1+n > 0$ and $\bar{a}-a_{u^*} > 0$ or $b(1-n)+1+n < 0$ and $\bar{a}-a_{u^*} < 0$. This

completes the proof of the proposition. ∎

Proposition 4 establishes specific conditions when the expected production of a homogeneous firm under

the robust analysis is higher than the expected production of the same firm at the risk-neutral stochastic

model. The most common case arises when the expected demand intercept is larger than the worst-scenario

demand intercept (where the firms achieves the lowest profit) and the slope of demand is greater than one and

less than three (when the number of firms is as small as two).

Proposition 5. *For any homogeneous firm in equilibrium,* $\dfrac{F^w}{F^s} \geq 1$, *if and only if*

$$i.)\ \ \frac{\bar{a}-a_{u^*}}{\bar{a}-c} \geq \frac{(b-1)(n^2+1)}{n(1-b)+1+b}\ and\ b < \frac{n+1}{n-1}\quad or\quad ii.)\ \ \frac{\bar{a}-a_{u^*}}{\bar{a}-c} \leq \frac{(b-1)(n^2+1)}{n(1-b)+1+b}\ and\ b > \frac{n+1}{n-1}\ .$$

Proof: Homogeneous firms, under the equilibrium conditions for the worst-case analysis and the risk-

neutral stochastic model, achieve the forward trading, respectively, as

$$F^w = \Big[\big(b(n-1)-n-1\big)a_{u^*} + \big(n^2(b-1)+b-n\big)c + (n+1)(n+1-bn)\bar{a}\Big]\frac{1}{b(n^2+1)}\ \text{and}\ F^s = \frac{(n-1)(\bar{a}-c)}{(n^2+1)b}.$$

The ratio of F^w to F^s is then obtained as

$$\frac{F^w}{F^s} = \frac{-\big(b(1-n)+1+n\big)a_{u^*} - \big(n^2(b-1)+b-n\big)(\bar{a}-c) + \big(b(1-n)+1+n\big)\bar{a}}{(n-1)(\bar{a}-c)}.$$ From this it follows that

$\dfrac{F^w}{F^s} \geq 1$ if and only if $\dfrac{\big(b(1-n)+1+n\big)(\bar{a}-a_{u^*}) - \big(n^2(b-1)+b-n\big)(\bar{a}-c)}{\bar{a}-c} \geq n-1$, which is equivalent

to $\Big[n(1-b)+1+b\Big]\dfrac{\bar{a}-a_{u^*}}{\bar{a}-c} \geq (b-1)(n^2+1)$. The conditions for parameter b can be easily established. If

$n(1-b)+1+b > 0$, that is $b < \dfrac{n+1}{n-1}$, then the inequality $\dfrac{\bar{a}-a_{u^*}}{\bar{a}-c} \geq \dfrac{(b-1)(n^2+1)}{n(1-b)+1+b}$ holds. However, if

$n(1-b)+1+b < 0$, that is $b > \dfrac{n+1}{n-1}$, we obtain $\dfrac{\bar{a}-a_{u^*}}{\bar{a}-c} \leq \dfrac{(b-1)(n^2+1)}{n(1-b)+1+b}$. ∎

Proposition 5 derives the conditions under which the forward trading in the worst-case analysis is higher than the forward trading in the risk-neutral stochastic model. These conditions allow the decision maker to choose appropriate parameters. In order to analyse these two conditions (described in Proposition 5) further,

let's define a parameter $R = \dfrac{\bar{a}-a_{u^*}}{\bar{a}-c}$, and suppose that $R = M > 0$. In this case $\bar{a} \geq a_{u^*}$, and there exists

two possible cases; either $M \geq \dfrac{(b-1)(n^2+1)}{n(1-b)+1+b} > 0$ or $0 > \dfrac{(b-1)(n^2+1)}{n(1-b)+1+b}$ that is also equivalent to a case

of $R < 0$ when $\bar{a} < a_{u^*}$. From $\dfrac{(b-1)(n^2+1)}{n(1-b)+1+b} > 0$ we obtain $b \geq 1$ and $b < \dfrac{n+1}{n-1}$. On the other hand, the

latter case $\dfrac{(b-1)(n^2+1)}{n(1-b)+1+b} < 0$ leads to conditions of either $b < 1$ or $b > \dfrac{n+1}{n-1}$. These cases are depicted

in Figure 2 where the shaded areas display the interactions between b and R with $\dfrac{F^w}{F^s} \geq 1$.
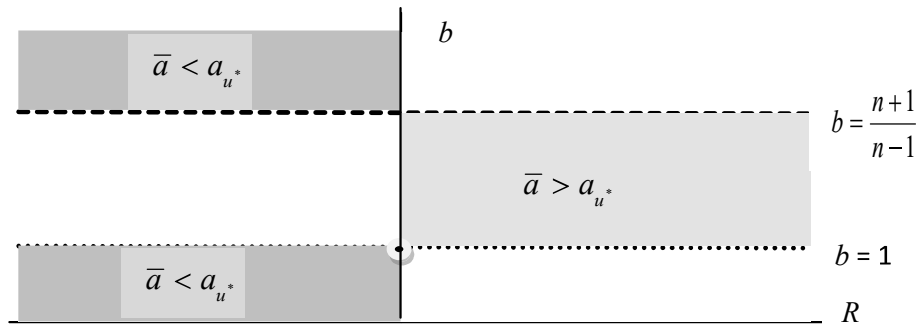
Figure 2: The cases described for forward trading obtained by the worst-case and stochastic models.

Our analysis proceeds by comparing the profits received by the firms under the worst-case and the risk-neutral analysis. The robustness in worst-case analysis is due to the inferiority of the min-max solution, e.g., Rustem & Howe (2002). As proved in Proposition 6, profit of any firm implementing the worst-case strategy is always lower than the one gained by the risk-neutral strategy.

Proposition 6. *The worst-case profit at player level is less than the expected profit.*

Proof: Let μ_i^* be the worst-case profit of player i. This is achieved by solving the nonlinear maximization problem (8), $\max_{F_i^w} \left\{ \mu_i \mid \pi_{iu}^w(F_i^w) \geq \mu_i, \quad u = 1, 2, \cdots, K \right\}$ For rival scenarios, $u = 1, 2, \cdots, K$, the inequality $\mu_i^* \leq \pi_{iu}^w$ holds. Therefore, $E[\pi_i^w] = \sum_{u=1}^{K} p_u \pi_{iu}^w \geq \sum_{u=1}^{K} p_u \mu_i^* = \mu_i^*$. ■

Finally, we determine the conditions, under which the proportion of forward trading in the worst-case analysis is higher (lower) than the one in the risk-neutral stochastic model.

Proposition 7. *For homogenous fi*rms, *one of the following conditions holds.*

i) $\dfrac{F_i^s}{E\left[Q_i^s\right]} \leq \dfrac{F_i^w}{E\left[Q_i^w\right]}$ *if and only if* $b \leq \dfrac{(R+n)(n+1)}{R(n-1)+(n+1)n}$ *or* $b > \dfrac{(R+n)(n+1)}{R(n-1)}$;

ii) $\dfrac{F_i^s}{E[Q_i^s]} > \dfrac{F_i^w}{E[Q_i^w]}$ *if and only if* $\dfrac{(R+n)(n+1)}{R(n-1)+(n+1)n} < b < \dfrac{(R+n)(n+1)}{R(n-1)}$.

Proof: Recall that for each demand scenario, $u = 1, 2, \cdots, K$, $a_u > c$. For homogenous firms, the stochastic model provides expected production and forward trading as $E[Q_i^s] = \dfrac{n(\bar{a} - c)}{(n^2 + 1)b}$, and

$F_i^s = \dfrac{(n-1)(\bar{a} - c)}{(n^2 + 1)b}$, respectively. Then we can obtain the ratio $\dfrac{F_i^s}{E[Q_i^s]} = \dfrac{n-1}{n}$, from which it follows that

$\dfrac{F_i^s}{E[Q_i^s]} \leq \dfrac{F_i^w}{E[Q_i^w]}$ if $\dfrac{F_i^w}{E[Q_i^w]} \geq \dfrac{n-1}{n}$. Furthermore, for homogenous firms that are implementing the worst-case strategy, the forward trading and expected production, respectively, are obtained as

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$$F^w = \frac{(b(n-1)-n-1)a_{u^*} + (n^2(b-1)+b-n)c + (n+1)(n+1-bn)\bar{a}}{b(n^2+1)} \text{ and}$$

$$E[Q_i^w] = \frac{(b(1-n)+1+n)(\bar{a}-a_{u^*}) + n(n+1)(\bar{a}-c)}{(n^2+1)(n+1)b}. \text{ Hence } \frac{F_i^s}{E[Q_i^s]} \le \frac{F_i^w}{E[Q_i^w]}, \text{ that is } \frac{n-1}{n} \le \frac{F_i^w}{E[Q_i^w]},$$

equals

$$\frac{n-1}{n(n+1)} \le \frac{(b(n-1)-n-1)a_{u^*} + (n^2(b-1)+b-n)c + (n+1)(n+1-bn)\bar{a}}{(b(1-n)+1+n)(\bar{a}-a_{u^*}) + n(n+1)(\bar{a}-c)} \tag{15}$$

Note that for the second part of the proposition, the sign of inequality in (15) needs to be changed. It is also

worthwhile to mention that, $\frac{n-1}{n(n+1)} \ge 0$ always holds. We now analyse (15) under two circumstances.

Suppose that $(b(1-n)+1+n)(\bar{a}-a_{u^*}) + n(n+1)(\bar{a}-c) > 0$. This leads to $b < \frac{(R+n)(n+1)}{R(n-1)}$. In this

case, by solving the inequality $\frac{n-1}{n} \le \frac{F_i^w}{E[Q_i^w]}$ we obtain $b \le \frac{(R+n)(n+1)}{R(n-1)+(n+1)n}$. Since

$b \le \frac{(R+n)(n+1)}{R(n-1)+(n+1)n} < \frac{(R+n)(n+1)}{R(n-1)}$ we can conclude that the inequality $\frac{n-1}{n} \le \frac{F_i^w}{E[Q_i^w]}$ is satisfied if

and only if $b \le \frac{(R+n)(n+1)}{R(n-1)+(n+1)n}$. Similarly, under this condition, we can show that the inequality

$\frac{F_i^s}{E[Q_i^s]} > \frac{F_i^w}{E[Q_i^w]}$ holds if and only if $\frac{(R+n)(n+1)}{R(n-1)+(n+1)n} < b < \frac{(R+n)(n+1)}{R(n-1)}$, as the two ratio inequalities

are complementary.

In the second case, $(b(1-n)+1+n)(\bar{a}-a_{u^*}) + n(n+1)(\bar{a}-c) < 0$ which is equivalent to

$b > \frac{(R+n)(n+1)}{R(n-1)}$. Similarly, by solving the inequality $\frac{F_i^s}{E[Q_i^s]} \le \frac{F_i^w}{E[Q_i^w]}$, it follows that

$b \ge \frac{(R+n)(n+1)}{R(n-1)+(n+1)n}$ which is smaller than $b > \frac{(R+n)(n+1)}{R(n-1)}$. Then we can then conclude that

$\frac{F_i^s}{E[Q_i^s]} \le \frac{F_i^w}{E[Q_i^w]}$ holds if and only if $b > \frac{(R+n)(n+1)}{R(n-1)}$. Under this circumstance, the inequality

$\dfrac{F_i^s}{E[Q_i^s]} > \dfrac{F_i^w}{E[Q_i^w]}$ (in the second part of the proposition) never holds as $\dfrac{(R+n)(n+1)}{R(n-1)} > \dfrac{(R+n)(n+1)}{R(n-1)+(n+1)n}$.

∎

Figure 3 illustrates how the relationship between the slope of demand, b, and the ratio R determines the circumstances in which the proportion of forward trading in the risk-neutral model is higher or lower than the one in the worst-case analysis. The frontier is convex for low number of firms (n). However, it becomes almost linear as n increases. As can be seen from the relationship stated in Proposition 7, the worst-case proportion traded forward depends on the slope of demand, the demand intercepts, and the production costs, see (15), as illustrated in Figure 3.
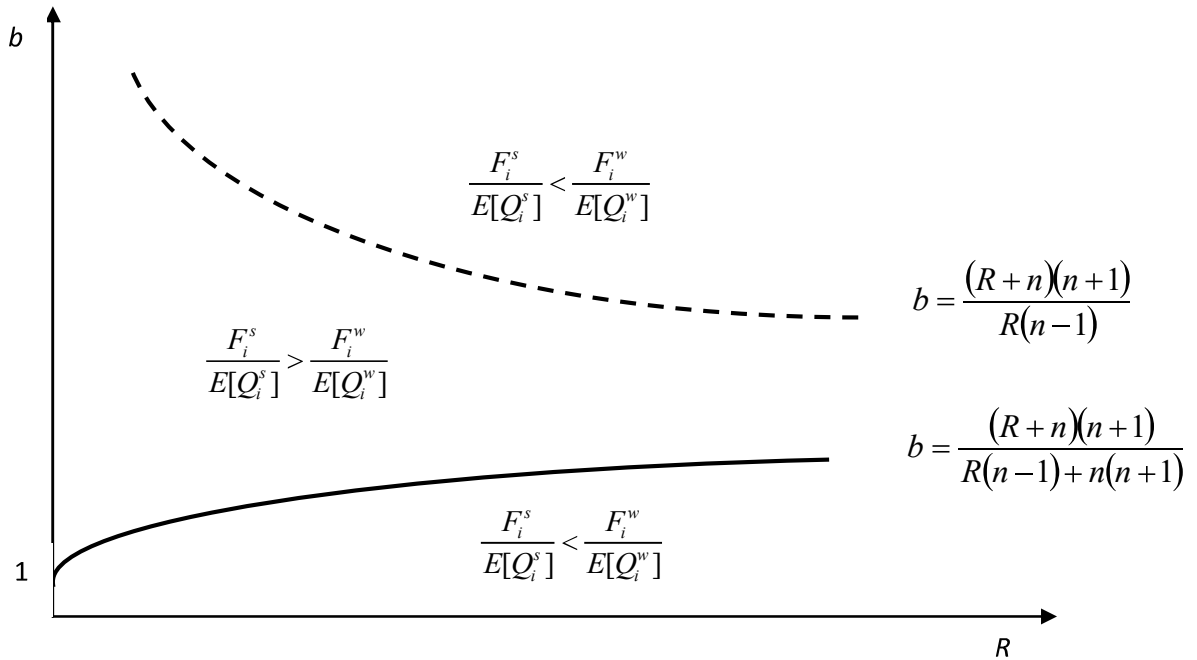
Figure 3: Relationship between the proportion of forward trading in the worst-case and risk-neutral analysis.

Next, we design a series of numerical experiments to illustrate the performance of optimization models as well as to validate the analytical results.

6. Computational Experiments

For illustration purposes, we consider a stylized industry with four firms that sell directly to the final consumers. The four players differ in the production technology. The marginal cost of production is assumed to be constant. We set up three different experiments. Table 1 summarizes the parameters used in three experiments for the portfolio of technologies. The first experiment assumes that all players are homogenous

with a marginal production cost equal to one. In the second and third experiments, the players are heterogeneous with different marginal costs of production.

Following Allaz (1992), we model consumer behaviour using a stochastic linear function, in which price is a function of production. Moreover, the level of demand is defined by the parameter a. For the deterministic model, we use mean demand (expected value of a) as $E[a] = 55$. For the stochastic and worst-case optimization models, uncertain demand parameter $\tilde{a}$ is represented by ten rival scenarios $u = 1, 2, \cdots, 10$ as $a_u \in \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$. Notice that demand varies between 10 and 100. Each scenario u is associated with the probability p_u whose values are 0.05, 0.05, 0.1, 0.1, 0.2, 0.2, 0.1, 0.1, 0.05, 0.05 for scenarios, respectively. In addition, we fix $b = 1$ through all experiments.

Table 1: Players' marginal costs per experiment

Experiment ID	Player 1	Player 2	Player 3	Player 4
Experiment 1	1	1	1	1
Experiment 2	1	2	3	4
Experiment 3	1	2	4	6

We analyze the performance of each player (denoted as P_1, P_2, P_3 and P_4) under three models; deterministic, stochastic and worst-case analysis. Their results are presented in Tables 2, 3, and 4 in terms of expected production (EPROD), expected forward trade (EFT), expected proportion traded forward (EPTF), expected profit (EP) and minimum profit (MP). We implement all optimization problems in the General Algebraic Modeling System (GAMS) (http://www.gams.com), and integrate it with nonlinear optimization solver CONOP, Drud (1992). It is worthwhile to mention that the CPU time, taking to solve the optimization problems, is in the range of one to three seconds.

Table 2 shows that, as proved in Proposition 1, the quantity traded in the spot and forward markets in the deterministic model are equal to the expected spot trading and forward trading in the stochastic risk-neutral model since the expected value of the intercept of demand equals its value in the deterministic model. Similarly, for heterogeneous players in Experiment 2, the levels of trading in the spot and forward markets are the same in the deterministic and risk-neutral stochastic models. However, Proposition 1 does not hold for Experiment 3 as the equilibrium quantity for player 4 is zero in the very low demand scenario and the capacity constraint is binding. Consequently, quantities traded in the stochastic and deterministic models are different: 3.4 in the deterministic model and 4.0 in the stochastic model for player 4.

Furthermore, in all experiments and for all the players, the expected profit in the risk-neutral stochastic model is higher than the expected profit in the deterministic model. This result arises from the fact that the forward prices in the deterministic model (4.18, 5.59 and 6.29, for Experiments 1, 2 and 3, respectively) are lower than the corresponding forward prices in the risk-neutral model (4.64, 5.83 and 6.34, for Experiments 1, 2 and 3, respectively), due to the existence of scenarios in this second model with prices equal to zero.

Table 2: Results obtained by deterministic model

	Experiment 1				Experiment 2				Experiment 3			
	P_1	P_2	P_3	P_4	P_1	P_2	P_3	P_4	P_1	P_2	P_3	P_4
EPROD	12.7	12.7	12.7	12.7	17.2	14	10.8	7.6	19.4	16.2	9.8	3.4
EFT	9.5	9.5	9.5	9.5	13.8	10.8	7.8	4.8	15.9	12.9	6.9	0.9
EPTF	75	75	75	75	80.2	77.1	72.2	63	81.9	79.6	70.3	26
EP	40.4	40.4	40.4	40.4	78.7	50.1	27.8	12.0	102.6	69.5	22.4	1.0
MP	40.4	40.4	40.4	40.4	78.7	50.1	27.8	12.0	102.6	69.5	22.4	1.0

Table 3: Results of scenario based risk-neutral stochastic model

|	Experiment 1				Experiment 2				Experiment 3			
	P_1	P_2	P_3	P_4	P_1	P_2	P_3	P_4	P_1	P_2	P_3	P_4
EPROD	12.7	12.7	12.7	12.7	17.2	14	10.8	7.6	19.4	16.2	9.8	4
EFT	9.5	9.5	9.5	9.5	13.8	10.8	7.8	4.8	15.9	12.9	6.9	0.9
EPTF	75	75	75	75	80.2	77.1	72.2	62.4	81.9	79.6	70.3	22
EP	62.9	62.9	62.9	62.9	101	71.6	48.6	31.7	122.8	89.5	42.2	16.8
MP	34.7	34.7	34.7	34.7	67.1	41.3	22	9	85.9	56.2	16.2	-0.8

Comparing performance of stochastic and worst-case models presented in Tables 2 and 3, we observe that demand uncertainty leads to a significant increase in the value of the different technologies, and this effect is particularly evident in the case of the most expensive ones. For example, in Experiment 3, the expected profit of player P_4 increased from 1 in the deterministic model to 16.8 in the risk-neutral stochastic model. Furthermore, the introduction of uncertainty increases risk, obviously, and the minimum profit is lower for all the players in all the experiments.

Table 4: Results of worst-case analysis

	Experiment 1				Experiment 2				Experiment 3			
	P_1	P_2	P_3	P_4	P_1	P_2	P_3	P_4	P_1	P_2	P_3	P_4
EPROD	13.1	13.1	13.1	13.1	21.1	15.5	9.1	5.4	24.4	18	6.7	2.4
EFT	11.2	11.2	11.2	11.2	19.1	13.7	7.5	3.7	22.2	16	4.9	-0.1
EPT	86.1	86.1	86.1	86.1	90.5	88.4	82.4	68.6	91.1	89	74	-2.5
EP	48.4	48.4	48.4	48.4	93.9	57.9	31.8	18.9	118.1	74	24.8	9.7
MP	29.6	29.6	29.6	29.6	69.7	37.4	12.6	2.1	91.7	51.6	5.4	-0.3

From the comparison of Tables 2 and 4, which presents the worst-case analysis results, we conclude that if the players are risk averse they tend to have higher expected profits than in the deterministic case. Moreover, in Experiment 3, the player with the very expensive technology P_4 sells rather than buying in the forward market as the expected price is 5.1, below his marginal cost.

We also compare the performance of the risk-neutral stochastic model (in Table 3) with worst-case analysis (in Table 4). Overall, the computational experiments confirm the analytical results proved in the previous section. The worst-case profit is lower than the expected profit, as proved in Proposition 5. If all the firms are homogeneous as in Experiment 1, then under worst-case analysis there is more trade in the forward market (Proposition 4) and higher production in the spot market (Proposition 4). Moreover, the homogeneous players trade a higher proportion of their production in the forward market under worst-case analysis than in risk-neutral analysis. However, as Experiments 2 and 3 clearly show, these results are not extendable to the heterogeneous case.
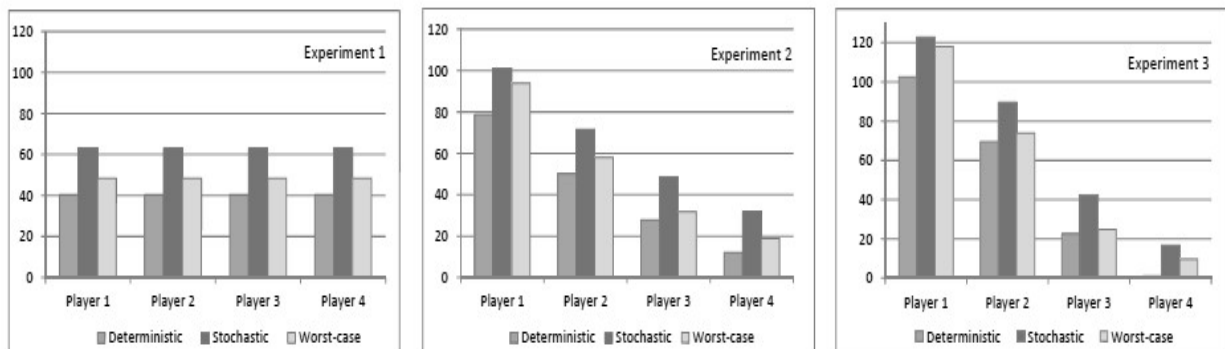

Figure 4: Expected profits of players obtained by different models at various market structures.

The performance of each player in terms of profit (that is obtained by deterministic, stochastic and worst-case strategies) at different market structure is presented in Figure 4. The left plot assume that all players in the market are homogenous and have unit marginal production cost whereas the middle and right plots consider heterogonous players facing various production costs. We observe that the risk-neutral model provides the highest expected profit for both homogeneous and heterogeneous players with any marginal production cost. As all players under the worst-case analysis consider a conservative approach, their expected profits are lower than the ones obtained by the stochastic strategy. However, the worst-case profits at any market structure are still higher than the ones obtained by the deterministic strategy. In particular, the worst-

case strategy outperforms at the catastrophic market conditions like the highest production cost as illustrated by player 4 in Experiment 3.
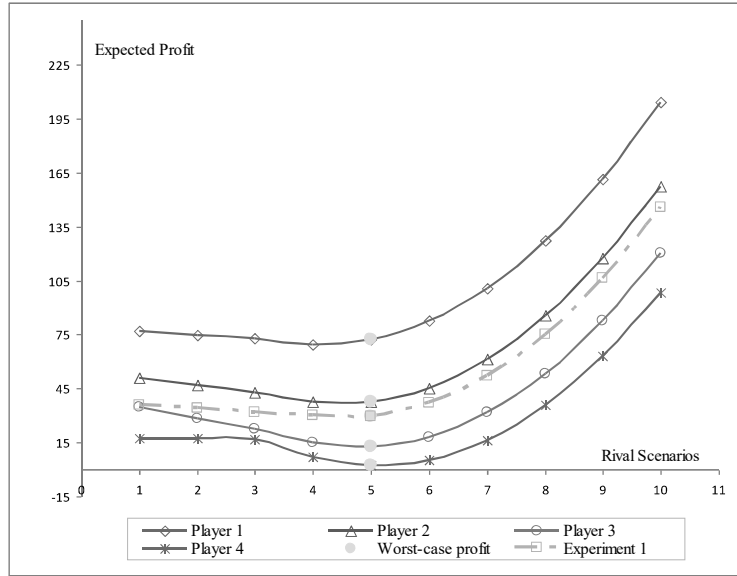
Figure 5: Robustness of minimax strategy.

In Figure 5, we present the robustness of the minimax strategy for all players over ten scenarios for Experiment 1 (represented by just one line as all the players are homogeneous) and Experiment 3 (in which each player is has his own line, Player 1, Player 2, Player 3 and Player 4). We have disregarded Experiment 2, in order to simplify the figure, as it is similar to Experiment 3. As it can be seen from Figure 5, the worst-case profit is achieved at fifth scenario ($a_5 = 50$) for all experiments. This shows that expected profit for all players will improve if the worst-case scenario is not realised.

7. Conclusions

In this paper, we consider robust Cournot dynamic games and model the interaction between forward contracts and spot prices in oligopolistic markets under uncertainty. We introduce discrete scenarios representing data uncertainty into the two-stage stochastic model that was previously developed by Allaz (1992), Allaz & Vila (1993), Dong & Liu (2007), and Mendelson & Tunca (2007). We then extend the risk-neutral stochastic model to the worst-case design of oligopolistic markets at the player level. Analytical results comparing the properties of deterministic, risk-neutral and worst-case models are derived.

We have proved that, ceteris paribus, the deterministic and risk neutral models lead to the same decisions regarding forward trading and expected productions, if the corresponding models have "equivalent" demand functions. We have further showed that the expected profit in the risk neutral model is equal to the profit in

the deterministic model for as long as the weighted prices in the risk-neutral scenarios are equal to the price in the deterministic spot market.

In addition, we have established the relationship between the risk-neutral and robust analysis under specific conditions depending on the specific levels of demand, demand slops and production costs. It is shown that the worst-case model provides higher expected production and forward trading than the risk neutral model. However, it is not clear in which model the proportion of forward trading on expected production is larger.

Finally, we have carried out numerical experiments, within a stylized representation of the electricity industry, in order to illustrate the performance of these models. Our findings can be summarised as follows;

- Demand uncertainty has a significant impact on the value of the different technologies.

- For all homogeneous firms in the market, the worst-case analysis provides more trade in the forward market (in absolute and as a proportion) and higher production in the spot market. However, these results do not hold for any heterogeneous player.

- The expected profits in the worst case analysis are lower than the in the risk-neutral model, due to the players' conservative policy, but are still higher than the ones obtained by the deterministic strategy.

Moreover, our results suggest that worst-case modelling of Cournot players in oligopolistic industries captures better behaviour of the generation companies in practice as in the real world prices tend to be lower (and firms tend to produce more) than suggested by the risk-neutral Cournot models (e.g., Bunn & Oliveira, 2008; and Green &March, 2004). As a result, we can conclude that the robust worst-case analysis at player level, as introduced in this article, is an important step forward in the modelling and understanding of how risk-averse oligopolistic firms behave in practice.

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