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Analysis of Relationship between Forward and Spot Markets in Oligopolies under Demand and Cost Uncertainty

N. Gulpinar . F. S. Oliveira

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Abstract: In this paper, we consider interaction between spot and forward trading under demand and cost uncertainty, deriving the equilibrium of the multi-player dynamic games. The stochastic programming and worst-case analysis models based on discrete scenarios are developed to analyze the impact of demand uncertainty and risk aversion on oligopoly (forward and spot) markets' structure in terms of the forwards and spot pricing, traded quantities and production. A real case of the Iberian electricity market is studied to illustrate performance of the models. The numerical experiments show that cost uncertainty impacts on the strategic decisions more than demand uncertainty.

Keywords: Forward contracts, oligopoly, uncertainty stochastic programming, worst-case analysis

1 Introduction

Market design has been the topic of analysis, for many years, of a large stream of literature researching how market rules, regulation, and behavioral issues, such as perfect vs. bounded rationality, interact in conditioning the companies' strategies and market performance, in terms of the consumer and firms' surpluses. This analysis has been particularly interesting in the case of oligopolistic markets in which a small number of firms interact in the production of a good or service delivered to the final consumers.

N. Gulpinar
Warwick Business School, The University of Warwick, Coventry, CV4 7AL, UK.
E-mail: Nalan.Gulpinar@wbs.ac.uk

F. S. Oliveira
ESSEC Business School, 100 Victoria Street, National Library Building.13-02, Singapore 188064
Email: Oliveira@essec.edu

An oligopolistic market (in which this analysis has been particularly active) and in the newly liberalized electricity markets in which a few large players (possibly holding a set of generation plans using different technologies) interact in proving electricity to final consumers. This industry is particularly difficult to analyze due to the very different technical features of the generation technologies and the nature of electricity which, at the same time, can be perceived as a commodity and as a service (as, currently, it is not economically viable to store in large scale required by the electricity distribution network).

Some of the major research issues addressed in the context of electricity market design have been: the nature of the electricity pricing as, for example, the relationship between day-ahead and real time energy markets (e.g., Bunn and Oliveira, 2001; Borenstein et al., 2008), of auction mechanism, such as supply function offers (e.g., Bunn and Oliveira, 2001; Anderson and Hu, 2008b); abuse of market power by the large incumbent firms (e.g., Bunn and Oliveira, 2003); investment incentives and real option games (e.g., Oliveira, 2008; Siddiqui and Fleten, 2010; Siddiqui and Takashima, 2012; Nagl et al., 2013); transmission constraints and their impact on the market clearing prices and on market power (e.g., Limpaitoon et al., 2011; Weijde and Hobbs, 2011; Rosellon et al., 2012; Schuler, 2012; Kunz, 2013); sustainable electricity generation and CO_2 emissions (e.g., Limpaitoon et al., 2011; Green and Vasilakos, 2012; Leuthold et al., 2012; Schuler, 2012; Kunz, 2013; Nagl et al., 2013; Henriot, 2014).

A comprehensive understanding of the relationship between futures and spot electricity markets is critical for market players. In particular, the relationship between oligopolistic structure and the trade of productive assets in the electricity market (e.g., Bunn and Oliveira, 2007, 2008; Downward et al., 2011) and the impact of the introduction of futures markets for electricity on the behavior of players, prices and production under uncertainty (e.g., Gulpinar and Oliveira, 2012; Oliveira et al., 2013) have been studied. This is also the main topic addressed in this article.

The relationship between spot markets and forward contracts, in oligopolistic industries, is a problematic one. On one hand it has been argued that the ability to engage in sequential forward contracts, or to trade in futures markets (organized exchanges for trading contracts on the future delivery of products and services in specific spot markets) ahead of the spot trading, can be profitable for the producers, in detriment of consumers: for example, Greenstone (1981) explained how some coffee producing countries, in the 1970s, used futures markets in their attempt to increase profits, buying their own production forward, and taking delivery.

On the other hand, in analyzing this relationship between forward contracts and spot markets, in the context of oligopolies, Allaz and Vila (1992, 1993) have shown that forward contracts increase competition between producers, reducing prices and profitability. On this topic, Thille (2003) has

included inventories in his analysis, showing that, in this case, prices tend to be higher than in the Allaz-Vila model, but lower than in the Cournot model without forward trading. Bushnell (2007) has provided a solution for the Allaz-Vila model with *N*-players that he uses to analyze electricity markets. The relationship between forward contracts and spot markets has been tested empirically by Herguera (2000) who found evidence supporting the hypothesis that, given a concentrated market structure, the introduction of forward contracts leads to lower spot prices; this thesis has also been corroborated by Le Coq and Orzen (2006) in laboratory experiments.

In the context of the electricity markets, this issue has been analyzed by Bushnell (2007) who developed a symmetric model of the relationship between futures and spot markets with multiple generators; the optimal trading strategies, in electricity markets, that account for the interaction between spot and futures markets have been developed by Carrión et al. (2007) for retailers, and Conejo et al. (2008) for generators. Anderson and Hu (2008), in a setting in which the generators trade with retailers, have shown that forward hedging mitigates market power; Murphy and Smeers (2005) and Kazempour et al. (2012) have analyzed how futures markets impact investment in electricity markets; Gulpinar and Oliveira (2012) have developed an algorithm for the worst-case analysis of the relationship between future and spot markets in the context of oligopolies, which they exemplify with the electricity industry; Oliveira et al. (2013) have analyzed the interaction between futures and spot markets in the electricity supply chain, taking into consideration two-part-tariffs and the contracts for differences; and Huisman and Kilic (2012) and Bunn and Chen (2013) have analyzed the forward risk premium (the difference between the average settlement price in the futures contract and the corresponding average spot price) in electricity markets, whereas Bunn and Chen (2013), using a MS-VAR model to account for endogenous and nonlinear price drivers, concluded that whereas daily premia tend to be driven by “operational aspects” the monthly premia tend to reflect “fundamental expectations”, Huisman and Kilic (2012) have emphasized that the pricing of forward contracts needs to take into consideration if the electricity is generated mainly from fossil fuels or from renewable (hydro, wind and solar generation) as way prices reflect expectations and risk premiums differs; Herraiz and Monroy (2013) have analyze the liquidity of the futures market in the MIBEL (Iberian Electricity Market), concluding that the market still has poor liquidity; Kalantzis and Milonas (2013) have analyzed the impact of futures trading on spot price volatility, focusing on the France and German electricity markets, concluding that in the integrated market (and in the French market) the introduction of a futures market reduced spot price volatility.

In this paper, we extend the stochastic programming and robust models (e.g., Gulpinar and Rustem, 2007a,b) introduced in Gulpinar and Oliveira (2012) by incorporating uncertainty not only on the demand intercept but also on the demand slope and in the production cost. We derive equilibrium conditions for the two-stage dynamic games of producers within oligopolistic markets, both for risk neutral and risk-averse players. Finally, we analyze the case of the Iberian electricity market to explain the strategic behavior of the main producers in terms of optimal and worst-case strategies.

The rest of the paper is organized as follows. In Section 2, we present a stochastic two-stage oligopoly market model. In Section 3 we model demand and production cost uncertainties using discrete scenarios and introduce scenario based stochastic programming model that maximizes expected total profit. Section 4 introduces a robust trading approach for two-stage oligopoly model using worst-case analysis over the scenarios. We present computational results in Section 5. Section 6 gives a short summary and our conclusions.

2 Problem Statement: Stochastic Two-stage Oligopoly Market Model

In the paper, we use tilde $(\tilde{*})$ to denote randomness; for instance "$\tilde{y}$" represents random variable y . A description of the notation is given in Table 1.

Table 1: Description of notation.

Notation	*Description*
N	number of producers that are represented by index of $i = 1, \cdots, N$
K	total number of discrete rival scenarios that are indexed by $u = 1, \cdots, K$
Q_i	total production of producer *i*
f_i	total forward trade in period 1 by producer *i*
P_1	forward price at period 1
P_0	equilibrium spot price at period 0
Q_0	total production of oligopolistic market
Π_i^s	profit of producer *i* in the spot market
Π_i^f	profit of producer *i* in the forward market
Π_i	total profit of producer *i* during the planning horizon
Random Variables	
$\tilde{C}_i$	marginal cost of producer *i*
$\tilde{a}$, $\tilde{b}$	intercept and factor loading of demand function

In this section we first formulate the two-stage model mathematically and then derive the equilibrium. For the two-stage model, we consider N producers, represented by indices $i = 1,\ldots, N$. Each producer trades in forward and spot markets within an oligopolistic industry during planning horizon. The game between producers starts under spot price uncertainty, at the first stage. The firms decide how much to contract forward for the production to be delivered at the second stage. Then, the spot trading takes place and production occurs.

Let Q_i be total production of producer i at time 1. The total production of oligopolistic market is determined as $Q_0 = \sum_{i=1}^{N} Q_i$. Let P_0 and P_1 denote equilibrium spot price (at time period zero) and forward price (at time period 1), respectively. Let f_i represent total forward trade of producer i for $i = 1,\cdots,N$. Unlike the model introduced by Allaz and Vila (1993), we relax the binding pre-commitment constraints in forward contracts. In other words, the producers are allowed to sell and buy forward. Therefore, forward trading variable f_i can be positive (or negative) that implies forward selling (or forward buying) for player i.

We consider two main sources of uncertainty on demand and marginal production cost. Demand uncertainty is described by parameters, $\tilde{a}$ and $\tilde{b}$, of the inverse demand function that is defined as

$$P_0 = \tilde{a} - \tilde{b} \cdot Q_0$$

The uncertain production cost $\tilde{C}_i$ for producer $i = 1,\ldots,N$ impacts on the variable cost, $\tilde{C}_i \cdot Q_i$. For the quantities traded in the spot market, $Q_i - f_i$, the profit of producer i gained in the spot market at time zero is computed as $\Pi_i^s = P_0 \cdot (Q_i - f_i) - \tilde{C}_i \cdot Q_i$. The forward profit at time 1 for producer i is $\Pi_i^f = P_1 \cdot f_i$. Then the total profit Π_i of producer i is computed as sum of profits earned in spot and forward markets during the planning horizon

$$\Pi_i = \Pi_i^s + \Pi_i^f = P_0 \cdot (Q_i - f_i) - \tilde{C}_i \cdot Q_i + P_1 \cdot f_i$$

Each producer aims to maximize the expected total profit. Then the stochastic two-stage problem is formulated as

$$\max_{f_i} \quad \mathrm{E}[\,\Pi_i\,] = E\left[\left(\tilde{a} - \tilde{b} Q_0\right) \cdot (Q_i - f_i) - \tilde{C}_i \cdot Q_i + P_1 \cdot f_i \right]$$

From a mathematical perspective, the process of finding a solution of such game starts at time zero, when we compute the equilibrium production and spot trading for each firm. Then, given these equilibrium outcomes, we compute the equilibrium forward trading at time 1. In the absence of arbitrage opportunities the transaction price, in each period, is equal to the spot price.

The Cournot-Nash equilibrium in the spot market is defined as a vector of outputs $(Q_1, Q_2, \cdots, Q_N)$ such that the first-order necessary and second-order sufficient conditions for all producers are satisfied. The optimality condition at time zero, $\frac{\partial \Pi_i^s}{\partial Q_i} = 0$, provides the reaction functions

$$Q_i = \frac{\widetilde{a} + \widetilde{b} f_i - \widetilde{C}_i - \widetilde{b} \sum_{\forall j \neq i} Q_j}{2\widetilde{b}}, \quad for \;\; i = 1, \cdots, N.$$

On the other hand, the Cournot-Nash equilibrium productions are obtained as

$$Q_i = \frac{\widetilde{a} - N\widetilde{C}_i + N\widetilde{b} f_i + \sum_{\forall j \neq i} \widetilde{C}_j - \widetilde{b} \sum_{\forall j \neq i} f_j}{(N+1)\widetilde{b}}$$

The spot price at equilibrium at time zero is computed by substituting equilibrium productions in $P_0 = \widetilde{a} - \widetilde{b} Q_0$ as $P_0 = \frac{\widetilde{a} + \sum_{i=1}^{N} \widetilde{C}_i - \widetilde{b} \sum_{i=1}^{N} f_i}{N+1}$.

In order to determine optimal trading strategies of the producers at various degree of risk aversion, we formulate the stochastic and robust profit maximization problems in view of rival scenarios for random demand and production cost.

3 Scenario-based Analysis

Stochastic programming requires a coherent representation of uncertainty. This is expressed in terms of a multivariate continuous distribution. Hence, a decision model is generated with internal sampling or a discrete approximation of the underlying continuous distribution. A method to obtain the discrete outcomes for the random variables is referred to as scenario generation. The discretization of the random values and the probability space leads to a framework in which a random variable takes finitely many values. Thus, the factors driving the risky events are approximated by a discrete set of scenarios, or sequence of events. In literature, there are several variants of the moment matching procedure (Hoyland et al. 2003) and simulation and clustering based approaches (Gulpinar et al. 2004) to generate scenarios.

In this section we describe scenario based formulation of the stochastic two-stage market model assuming that demand and cost random variables are discretised by a finite number of scenarios. Let's consider K number of scenarios that are represented as $u = 1, \cdots, K$ and the associated branching probabilities, p_u, so that $\sum_{u=1}^{K} p_u = 1$.

The two-stage stochastic market model takes into account a firm's attitude towards risk due to uncertainty on the spot price and production cost. Moreover, at time one, the optimal decisions (obtained by scenario based stochastic market model) on the quantity to trade forward and equilibrium price in the forward markets only depend on the expected value of random parameters. At time zero,the spot market takes place in which quantities produced and spot prices are decided already taking into account the specific realizations of the scenarios about demand and marginal production cost.

Total market production depends on scenario *u,* and is accumulated over all producers in spot market, $Q_{0,u} = \sum_{i=1}^{N} Q_{i,u}$. The inverse demand function in scenario u is defined as $P_{0,u} = a_u - b_u Q_{0,u}$. The spot profit of producer i at scenario u is formulated as $\Pi^s_{i,u} = P_{0,u} \cdot (Q_{i,u} - f_i) - C_u \cdot Q_{i,u}$. Under no-arbitrage condition, the forward price is $P_1 = \sum_{u=1}^{K} p_u P_{0,u}$. Allaz (1992) proved that for as long as there is a arbitrageur in the market, even with risk-averse decision makers, this condition holds. In fact, as arbitrage is the process of making a profit out of market inefficiencies, with no risk, the degree of risk aversion of the decision makers has no impact on this condition. Solving the profit maximization problem for each producer in the spot market, we find the reaction functions as

$$Q_{i,u} = \frac{a_u + b_u f_i - C_{i,u} - b_u \sum_{\forall j \neq i} Q_{j,u}}{2b_u}, \quad \text{for } i = 1,\cdots,N, \;\; u = 1,\cdots,K.$$

Similarly, the closed form of Cournot-Nash equilibrium productions for each producer i under scenario u can be computed as

$$Q_{i,u} = \frac{a_u - NC_{i,u} + Nb_u f_i + \sum_{\forall j \neq i} C_{j,u} - b_u \sum_{\forall j \neq i} f_j}{(N+1) b_u}, \;\; \text{for } i = 1,\cdots,N, \;\; u = 1,\cdots,K \qquad (1)$$

The scenario-based stochastic market model maximizes the expected profit of each producer in view of all discrete scenarios and is stated as follows;

$$
\begin{aligned}
&\max_{f_i} \quad \mathrm{E}[\,\Pi_i\,] = \sum_{u=1}^{K} p_u \Pi_{i,u} \\
&\text{subject to} \\
&\qquad \Pi_{i,u} = \left(P_1 - P_{0,u}\right)\cdot f_i + \left(P_{0,u} - C_{i,u}\right)\cdot Q_{i,u}, && u = 1,\cdots,K \\
&\qquad P_1 = \sum_{u=1}^{K} p_u P_{0,u}, && (2) \\
&\qquad P_{0,u} = a_u - b_u Q_{0,u}, && u = 1,\cdots,K \\
&\qquad Q_{i,u} = \frac{a_u - NC_{i,u} + Nb_u f_i + \sum_{\forall j\neq i} C_{j,u} - b_u \sum_{\forall j\neq i} f_j}{(N+1)b_u}, && u = 1,\cdots,K \\
&\qquad Q_{i,u} \geq 0, && u = 1,\cdots,K
\end{aligned}
$$

where the scenario based parameters $a_u, b_u, p_u, C_{i,u}$ as well as the pre-specified parameters K and N are inputs to the nonlinear program. Note that the size of this model increases as the number of producers and discrete scenarios increases.

Next we derive the trading strategy of producer i at time one using the optimality conditions. Substituting Cournot-Nash equilibrium productions in the inverse demand function provides the equilibrium spot price at time zero under scenario u as

$$P_{0,u} = \frac{a_u + \sum_{i=1}^{N} C_{i,u} - b_u \sum_{i=1}^{N} f_i}{N+1} \tag{3}$$

Using no arbitrage conditions, the expected profit objective function is simplified

$$
\begin{aligned}
\mathrm{E}[\,\Pi_i\,] &= \sum_{u=1}^{K} p_u \Pi_{i,u} = \sum_{u=1}^{K} p_u \left[\left(P_1 - P_{0,u}\right)\cdot f_i + \left(P_{0,u} - C_{i,u}\right)\cdot Q_{i,u}\right] \\
&= \left(P_1 - \sum_{u=1}^{K} p_u P_{0,u}\right)\cdot f_i + \sum_{u=1}^{K} p_u \left[\left(P_{0,u} - C_{i,u}\right)\cdot Q_{i,u}\right] = \sum_{u=1}^{K} p_u \left[\left(P_{0,u} - C_{i,u}\right)\cdot Q_{i,u}\right]
\end{aligned} \tag{4}
$$

Then the optimization problem (2) becomes

$$
\begin{aligned}
&\max_{f_i} \quad \sum_{u=1}^{K} p_u \left(P_{0,u} - C_{i,u}\right)\cdot Q_{i,u} \\
&\text{subject to} \\
&\qquad P_{0,u} = \frac{a_u + \sum_{i=1}^{N} C_{i,u} - b_u \sum_{i=1}^{N} f_i}{N+1}, && u = 1,\cdots,K \\
&\qquad Q_{i,u} = \frac{a_u - NC_{i,u} + Nb_u f_i + \sum_{\forall j\neq i} C_{j,u} - b_u \sum_{\forall j\neq i} f_j}{(N+1)b_u}, && u = 1,\cdots,K.
\end{aligned}
$$

The optimal forward trading can be obtained by using the optimality conditions of the unconstrained optimization problem that is derived by substituting (1) and (2) into (4). The solution of the first-order optimality condition

$$\frac{\partial}{\partial f_i}\left(\sum_{u=1}^{K} p_u \left(P_{0,u} - C_{i,u}\right)\cdot Q_{i,u}\right) = \sum_{u=1}^{K} p_u \left(\frac{\partial P_{0,u}}{\partial f_i}\cdot Q_{i,u} + \left(P_{0,u} - C_{i,u}\right)\cdot \frac{\partial Q_{i,u}}{\partial f_i}\right) = 0$$

provides the reaction functions for the forward contracts as follows

$$f_i = \frac{(N-1)\bar{a} - N(N-1)\overline{C}_i + (N-1)\sum_{\forall j\neq i}\overline{C}_j - (N-1)\bar{b}\sum_{\forall j \neq i} f_j}{2N\bar{b}}$$

where $\bar{a} = \sum_{u=1}^{K} p_u a_u$, $\bar{b} = \sum_{u=1}^{K} p_u b_u$, and $\overline{C}_i = \sum_{u=1}^{K} p_u C_{i,u}$.

The total forward trading of N producers in the oligopoly can be computed given the number of producers in the market and expected values of all random variables as follows;

$$\sum_{j=1}^{N} f_j = \frac{N(N-1)\bar{a} + (1-N)\sum_{j=1}^{N}\overline{C}_j}{(N^2+1)\bar{b}}$$

Using this property, we can compute the forward contracting in equilibrium in a closed form as

$$f_i = \frac{N-1}{(N^2+1)\bar{b}}\left[\bar{a} - (1+N^2)\overline{C}_i + N\sum_{j=1}^{N}\overline{C}_j\right]$$

that depends on only expected values of all random parameters and total number of producers. In the same manner, we can compute the expected spot price,

$$\begin{aligned} E[P_0] &= \sum_{u=1}^{K} p_u P_{0,u} = \frac{1}{N+1}\sum_{u=1}^{K} p_u \left(a_u + \sum_{i=1}^{N} C_{i,u} - b_u \sum_{i=1}^{N} f_i\right) \\ &= \frac{1}{N+1}\left(\bar{a} + \sum_{i=1}^{N}\overline{C}_i - \frac{N(N-1)\bar{a} + (1-N)\sum_{i=1}^{N}\overline{C}_i}{(N^2+1)}\right) = \frac{1}{(N^2+1)}\left(\bar{a} + N\sum_{i=1}^{N}\overline{C}_i\right) \end{aligned}$$

and the expected production of player i under scenario u as

$$E[Q_i] = \frac{1}{N+1}\left(E\left[\frac{\tilde{a}}{\tilde{b}}\right] - NE\left[\frac{\widetilde{C}_i}{\tilde{b}}\right] + \sum_{\forall j\neq i} E\left[\frac{\widetilde{C}_j}{\tilde{b}}\right] - \frac{(N-1)^2\,\overline{C}_i}{\bar{b}} + \frac{\left((1-N)\bar{a} + (N-1)(N(N-1)+1)\sum_{j=1}^{N}\overline{C}_j\right)}{(N^2+1)\bar{b}}\right)$$

It is worthwhile to mention that these calculations reduce impact of scenarios due to variability on the optimal strategy and also allow eliminating the computational burden of the scenario based market model to find the optimal trading strategy.

4 Robust Worst-case Analysis with Rival Scenarios

As described before, the scenario based stochastic model relies on the expected performance of the underlying system in view of rival scenarios. When making decisions under various uncertainties, it is reasonable to evaluate the best policy in view of the worst-case uncertain effect. Worst-case analysis is an important tool to address the inherent error for forecasting uncertainty. This entails minimax formulations of the stochastic system (see for instance, Rustem and Howe 2002).

The discrete minimax problem arises when the worst-case needs to be determined over a discrete set of rival scenarios. The optimal strategy corresponding to the worst-case is called minimax strategy and is determined by all scenarios simultaneously. The robust nature of minimax actually comes from the guaranteed best lower-bound performance in view of the worst-case. The minimax strategy ensures that the performance improves if the worst-case is not realized. Since minimax optimal strategy is determined in view of all the scenarios, it is robust to the realization of worst-case scenarios than considering a single scenario or an arbitrary pooling of scenarios. It is therefore suitable for situations which need protection against risk of adopting a strategy based on the wrong scenario, Gulpinar & Rustem (2007).

Let us now formulate worst-case two-stage market model. Each producer's aim is to optimize the worst-case total profit in spot and forward markets. Then max-min formulation of the two-stage problem is

$$\max_{f_i} \min_{u=1,\cdots,K} \Pi_{i,u}$$

where $\Pi_{i,u} = P_{0,u}\left(Q_{i,u} - f_i\right) - C_{,ui}Q_{i,u} + P_1 f_i$. This problem is equivalent to

$$\begin{aligned}
&\min_{f_i,\mu_i} \; -\mu_{\mathrm{i}} \\
&s.t \\
&\qquad \mu_{\mathrm{i}} - \Pi_{i,u} \le 0, \quad u = 1,\cdots,K \\
&\qquad \mu_{\mathrm{i}} \;\; \text{free}
\end{aligned}$$

Introducing Lagrangean multipliers $\lambda_{i,u} \ge 0$ associated with each constraint, we can construct the Lagrangean function as $L(f_i,\mu_i,\lambda_{i,u}) = -\mu_i + \sum_{u=1}^{K} \lambda_{i,u}(\mu_i - \Pi_{i,u})$. The KKT optimality conditions are derived as follows;

$$\frac{\partial L}{\partial f_i} = -\sum_{u=1}^{K} \lambda_{i,u} \frac{\partial \Pi_{i,u}}{\partial f_i} = 0$$
$$\frac{\partial L}{\partial \mu_i} = -1 + \sum_{u=1}^{K} \lambda_{i,u} = 0$$
$$\frac{\partial L}{\partial \lambda_{i,u}} = \mu_i - \Pi_{i,u} \le 0, \quad u = 1,\cdots,K$$
$$\lambda_{i,u}(\mu_i - \Pi_{i,u}) = 0, \qquad u = 1,\cdots,K$$
$$\lambda_{i,u} \ge 0, \qquad\qquad u = 1,\cdots,K$$

Note that the first condition also includes partial derivatives

.

$$\frac{\partial \Pi_{i,u}}{\partial f_i} = \frac{(b_u - \bar{b})}{N+1} f_i - \frac{\left(a_u + c_{i,u}N - \bar{a}(N+1)\right)}{N+1} + \frac{b_u}{N+1}\sum_{j \ne i}^{N} Q_{j,u} - \sum_{s=1}^{K} p_s b_s \left(\sum_{j=1}^{N} Q_{j,s}\right)$$

and equilibrium production under rival scenarios. It is not straightforward to find the robust forward trading in a closed form as in the case of the stochastic programming formulation. Instead, we construct a complementarity problem that consists of a system of nonlinear equations for the KKT conditions of all producers. For numerical experiments, we used the GAMS solver.

5 Computational Experiments in the Iberian Electricity Market

In order to test performance of the stochastic and robust optimization models, we design numerical experiments using real data of the Iberian electricity market. Specifically, the goal of conducting the experiments is to answer the following questions:

- What is the impact of demand and marginal cost uncertain parameters on trading strategies obtained at different degrees of risk aversion?
- How do the stochastic and robust profit maximization models perform at various seasonal days when the underlying uncertainty is represented by finite number scenarios?

5.1 Market Structure and Data used in the Case Study

The Iberian electricity market was created in the early 2000's and resulted from the merger of the Portuguese and Spanish electricity markets in the context of the larger plan to integrate the European electricity market in order to improve efficiency and reduce consumer prices. This market is characterized by a very marked seasonality in the electricity consumption and in the availability of the different technologies to generate electricity, due to the very high dependency on renewable electricity sources, hydro, wind, and solar power plants.

There are four major players in this market, Iberdrola (25 GW), Endesa (18 GW), Gas Natural Fenosa (13 GW), and EDP (17 GW). In Figure 1 (the sources are REE, 2012 and REN, 2012) we depict the relationship between the marginal cost and installed capacity for the four large firms in the market. All the other small generators are price takers. For this reason, we have analyzed the workings of our model in four different typical days, two summer days, one winter day and one spring day. We design simulation experiments to analyze the relationship between forward and spot trading in the electricity market (in which the demand is traded 1 month ahead of the actual delivery in the spot market).

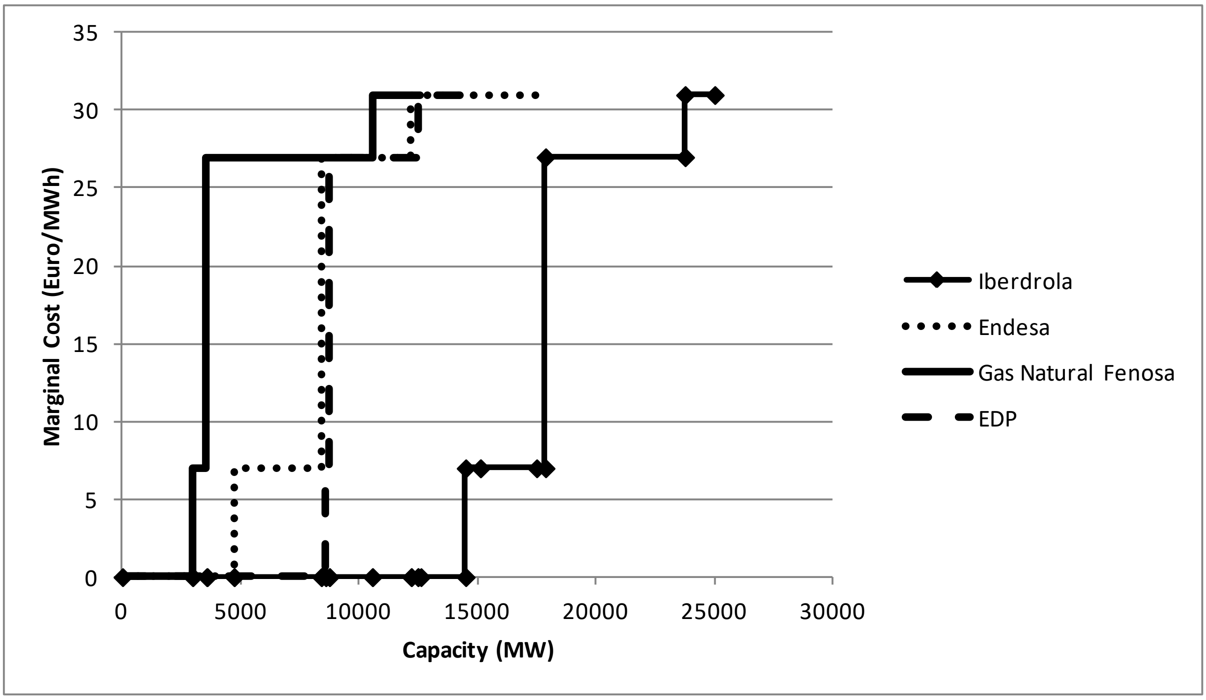

Figure 1: Relationship between Marginal Cost and Capacity for the four major companies.

The demand is uncertain and depends on the weather conditions. We assume that the demand slope is the same in all the days analyzed and a day is characterized by the change in the expected level of the

demand intercept and respective standard deviation. In this condition, for the same level of production, the elasticity of demand is larger in the days with lower demand, as the customers are more price-sensitive. In high demand periods most of the customers will need to consume, possibly for economic reasons, but also due to weather conditions (in this case for heating requirements) and the demand is less price-sensitive.

In all these experiments the demand intercept and slope are assumed to follow a normal distribution. Statistics of random parameters' distributions are presented in terms of expected value and standard deviation of demand (at various specific seasonal days) and marginal production cost (for different technologies) at each specific day in Table 2. The demand scenarios are based on the market data from the REN (2012) and the cost data was based on Brinckerhoff (2011) and the Royal Academy of Engineering (2004). The expected value of the demand intercept ranges from 230 GWh in the High Demand Summer day to 40 GWh in the Low Demand Winter day, with the respective standard deviations ranging from 4.8 GWh in the High Demand Summer day to 2.0 GWh in the Low Demand Summer day. The expected value of the demand slope is fixed at 3.0 for all the days, with a respective standard deviation set at 0.2.

Table 2: Statistics of Demand and Marginal Cost Random Parameters

Parameters	**Seasonal Days and Technologies**	**Expect Value**	**Standard Deviation**
Demand Intercept (a) (GWh)	*High Demand Summer*	230	4.8
	High Demand Spring	200	3.2
	Average Demand Summer	150	2.0
	Low Demand Winter	40	2.0
Demand Slope (b) (GWh)	*High Demand Summer*	3.0	0.2
	High Demand Spring	3.0	0.2
	Average Demand Summer	3.0	0.2
	Low Demand Winter	3.0	0.2
Marginal Production Cost (C) (Euros/MWh)	*Hydro-power/Wind/Solar*	0	0
	Nuclear	7	0.3
	Gas	27	4.2
	Coal	31	3.3

The marginal costs of the different technologies considered are also assumed to be normally distributed and are characterized by the expected value and standard deviation depicted in Table 2. The hydro-power, wind-turbines and solar have a zero marginal cost, nuclear has a generation cost of 7 euros/MWh (with a standard deviation of 0.3 euros/MWh), gas has a marginal cost of 27 euros/MWh (with a standard deviation of 4.2 euros/MWh) and coal has a marginal cost of 31 euros/MWh (with a standard deviation of 3.3 euros/MWh).

Table 3 (the source of which is REN, 2012) describes the percentage of the total capacity available for hydro and wind power plants in the four typical days considered. In the summer day only 3% of hydro capacity is available as water is scarce during this time of the year and used for human consumption and irrigation in agriculture. In the winter and spring water is very abundant but only 25% and 20% of the total capacity, respectively, can be used for electricity generation, due to the inter-temporal management of the water reserves.

Wind it is a very unstable resource, changing in intensity and regularity throughout the year. During the summer about 20% of the installed capacity can be used to generate electricity whereas during the winter and spring the winder is stronger and more regular allowing the use of, approximately, 30% and 25% of the installed capacity, respectively.

Table 3: Load factors (in %) for hydro and wind power plants for the four typical days.

Technologies	High Demand Summer	High Demand Spring	Average Demand Summer	Low Demand Winter
Hydro (%)	3	20	3	25
Wind (%)	20	25	20	30

Moreover, in order to generate the marginal costs for each one of the scenarios we need to consider the correlation between the different fuel prices. Whereas the prices of the renewable plants remain at zero, and the marginal cost of the nuclear power plants is also uncorrelated with the rest of the fuel prices, the correlation coefficient between gas and coal prices is about 0.93, and it needs to be considered when generating the marginal costs of these technologies in the different scenarios.

As not only the fuel costs but also the load factors for hydro and wind are stochastic (as well as possible maintenance and unscheduled downtime for the other plants are unpredictable), the actual technology setting the price (the marginal technology), at any given time, is also stochastic. There are six major technologies used to generate electricity in the Iberian electricity market (hydro, nuclear, wind, solar, gas and coal). In Table 4, for each typical day, we represent the probability that a given technology is the marginal plant, for a given generation firm. It should be noted that any plant with a marginal cost above (below) the one set by the marginal plant it will not produce (i.e. generate electricity at full capacity).

Table 4: Probability Distribution for the Marginal Plants, per Generator and Typical Day

Seasonal Days	Technology	EDP	Endesa	Gas Natural	Iberdrola
High Demand Summer	*Coal*	0.75	0.7	0.8	0.05
	Gas	0.25	0.3	0.2	0.95
High Demand Spring	*Coal*	0.3	0.3	0.6	0.0
	Gas	0.7	0.7	0.4	1.0
Average Demand Summer	*Nuclear*		1.0	0.7	0.4
	Wind	1.0		0.3	0.6
Low Demand Winter	*Hydro*	0.6	1.0	0.5	0.4
	Wind	0.4		0.5	0.6

For the High Demand Summer day the two possible marginal technologies are coal and gas, as the cheaper technologies are scarce (hydro) and demand is high: in this case, Iberdrola will have a gas marginal plant 95% of the time whereas the other three incumbents are more likely to have a coal marginal plant, EDP (75%), Endesa (70%), and Gas Natural (80%). In the spring day as hydro is relatively more abundant the gas plants have a higher probability of being marginal. In the Low Demand Winter day the price is set by the technologies with lower marginal costs: hydro and wind for EDP, Gas Natural and Iberdrola, and hydro for Endesa.

5.2 Simulation Results

We have the possible outcomes of the trading decisions in the Iberian electricity market using 20000 scenarios for the risk neutral and the worst case models, in four hypothetical periods High Demand Summer (HDS), High Demand Spring (HDP), Average Demand Summer (ADS), and Low Demand Winter (LDW). The price in the futures market (in euros/MWh) resulting from the simulations, per degree of risk aversion, and typical day, is described in Table 5. When the generators are risk averse the price are significantly higher across all the typical days, with a price increase of about 30% in the Average Demand Spring day to 122% in the Low Demand Winter day. This suggests that, across all the typical days, risk-averse generators produce less electricity, increasing prices.

Table 5: Future Price per Typical day and Degree of Risk Aversion

Seasonality Scenarios	**Risk Neutral Strategy**	**Risk Averse Strategy**
HDS	41.71	54.53
HDP	38.18	49.74
ADS	12.19	18.37
LDW	2.33	5.18

In Table 6, we present the amount of optimal trading in futures market (GWh) and expected trading in the spot market (GWh) obtained by solving the scenario based stochastic program and robust models. We observe that amount of trading for all companies by any strategy in both markets shows similar characteristics. More precisely, the risk neutral strategy for each company suggests trading in futures market (as well as expected trading in spot market) more than trading by the risk-averse strategy at any seasonal days apart from two cases. In these two cases, both EDP and Endesa’s trading in future markets obtained by the risk averse strategies (15.2 and 9, respectively) are more than their trading provided by the risk neutral strategies (12.3 and 5.31) at *Average Demand Summer,* as can be seen from Table 6. In addition, the risk averse strategy of both companies Gas Natural and Iberdrola does not trade at all in futures markets at the same seasonal typical day.

In totals, the trading in the futures market is larger in the risk neutral agents, for the same scenario, when compared with the risk-averse agents of any company. The same patterns are also observed for the expected spot trading.

Table 6: Optimal trading in the futures market and expected trading in spot markets (GWh)

	Risk Neutral Strategy				Risk Averse Strategy			
	HDS	**HDP**	**ADS**	**LDW**	**HDS**	**HDP**	**ADS**	**LDW**
Companies	*Trading in Future Market*							
EDP	11.71	10.17	12.34	2.37	5.99	5.02	15.29	1.18
Endesa	11.71	10.15	5.31	2.37	5.72	4.93	9	1.18
Gas Natural	11.33	8.96	7.43	2.37	6.4	4.91	0	1.18
Iberdrola	12.52	11.35	9.52	2.37	6.73	6.49	0	1.18
Total	47.27	40.63	34.6	9.48	24.84	21.35	24.29	4.72
Companies	*Expected Trading in Spot Market*							
EDP	15.62	13.56	16.46	3.15	14.24	12.26	21.46	2.92
Endesa	15.62	13.54	7.08	3.15	14.03	12.17	12.83	2.91
Gas Natural	15.1	11.94	9.91	3.15	14.58	11.75	4.54	2.92
Iberdrola	16.69	15.13	12.69	3.15	15.91	14.13	5.23	2.92
Total	63.03	54.17	46.14	12.6	58.76	50.31	44.06	11.67

In order to compare the performance of risk neutral and risk averse strategies of each company and to establish relationship between amounts of trading in futures market and expected trading in spot market, we compute ratios (%) that are presented in Table 7. The ratio between futures trading and expected spot trading is about 75% for the risk neutral agents, it ranges from 40.4% to 45.9% for the risk-averse agents, in the High and Low Demand days, and it is 0%, for Gas Natural and Iberdrola, and about 70% for EDP and Endesa, in the Average Demand day.

Table 7: Ratio of Futures and Expected Spot Trading (%) for companies at various seasonal days

	Risk Neutral Strategy				Risk Averse Strategy			
Company	**HDS**	**HDP**	**ADS**	**LDW**	**HDS**	**HDP**	**ADS**	**LDW**
EDP	75.0	75.0	75.0	75.2	42.1	40.9	71.2	40.4
Endesa	75.0	75.0	75.0	75.2	40.8	40.5	70.1	40.4
Gas Natural	75.0	75.0	75.0	75.2	43.9	41.8	0.0	40.4
Iberdrola	75.0	75.0	75.0	75.2	42.3	45.9	0.0	40.4
Total	75.0	75.0	75.0	75.2	42.3	42.4	55.1	40.4

Across all firms and typical days we find that the proportion of forward trading decreases with risk aversion. The exception to the general behavior of lower production by risk-averse agents happens in the spring period when EDP and Endesa increase dramatically their production and forward trading, affecting negatively Gas Natural and Iberdrola. These results are interesting as they complement Gulpinar and Oliveira (2012), who have analyzed the effect of demand uncertainty on futures trading, reporting that risk-aversion increases futures trading as the risk-averse generators, due to a prisoners' dilemma effect, would trend to increase trading in the futures market, increasing production and decreasing prices.

On the contrary, in this case study, which includes demand and cost uncertainty, the latter dominates the former, and the risk-averse generators tend to sell *lower proportion* in the futures market, increasing prices and decreasing production. The effect of this behavior on expected profit is reported in Table 8. In fact risk aversion, in the presence of cost uncertainty, leads to a similar effect to collusive behavior, increasing the expected profit, when comparing risk-neutral and risk-averse agents, across all scenarios. The relationship between demand, cost and profits is not linear, due to the different load factors in the different periods. EDP is a particularly interesting case as it makes the highest profit in the spring period (due to the abundance of water), especially in the simulation where the agents are risk-averse.

Table 8: Performance comparison of strategies in terms of expected profits at any typical day

	Expected Profit (in 000 Euros/hour)								**Ratio (%) of Expected Profits**			
	Risk Neutral Strategy				**Risk Averse Strategy**							
	HDS	**HDP**	**ADS**	**LDW**	**HDS**	**HDP**	**ADS**	**LDW**	**HDS**	**HDP**	**ADS**	**LDW**
EDP	220.9	199.9	202.0	7.5	388.9	328.2	395.4	15.2	56.8	60.9	51.1	49.1
Endesa	280.0	244.2	38.9	7.5	443.8	370.7	147.9	15.2	63.1	65.9	26.3	49.1
Gas Nat.	216.0	155.7	76.9	7.5	399.3	289.4	65.5	15.2	54.1	53.8	117.4	49.1
Iberdrola	335.0	323.8	126.5	7.5	540.7	475.5	88.7	15.2	62.0	68.1	142.6	49.1
Total	1052.0	923.6	444.4	29.8	1772.5	1463.7	697.6	60.8	59.3	63.1	63.7	49.1

As shown in ratios of expected profits (%) obtained by the risk-neutral and risk-averse strategies per company (see the last four columns) in Table 8, the industry as a whole makes larger profits when the agents are risk-averse (the risk-neutral profits range from about 49% to about 64% of the risk-averse profits), but in the spring day both Gas Natural and Iberdrola receive larger profits when players are risk-neutral (about 17% and 42% more, respectively) than being risk-averse. This is due to the effect of the increase of EDP and Endesa's production, in this period, which lead to a decreasing in production and futures trading both by Gas Natural and Iberdrola.

The results reported in Table 8 are particularly interesting as they illustrate how the risk-aversion in games increases the *expected profit*, whereas in single-decision maker problems the risk aversion tends to decrease the expected profit in order to decrease risk. This observation becomes even more relevant when we consider the worst-case profits obtained by risk-neutral and risk-averse strategies. As illustrated in Table 9, the worst-case profit has also increased with risk-averse decision making.

For the industry as a whole, the worst-case profit for the risk-neutral agents ranges from 33% (HDS) to 57% (ADS) of the worst-case profit received by the risk-averse agents.

Table 9: Performance comparison of risk-neutral and risk-averse strategies in terms of worst-case profits at any typical day

	Worst-case Profit (in 000 Euros/hour)								**Ratio (%) of Worst-case Profits**			
	Risk Neutral Strategy				**Risk Averse Strategy**							
	HDS	**HDP**	**ADS**	**LDW**	**HDS**	**HDP**	**ADS**	**LDW**	**HDS**	**HDP**	**ADS**	**LDW**
EDP	29.2	46.5	159.4	5.5	214.4	197.3	328.4	9.8	13.6	23.6	48.5	56.4
Endesa	134.8	128.5	23.7	5.5	300.0	271.9	107.6	9.8	44.9	47.3	22.0	56.4
Gas Nat.	26.8	15.9	34.2	5.5	217.9	159.2	17.1	9.8	12.3	10.0	200.6	56.4
Iberdrola	169.0	205.9	54.0	5.5	357.4	369.1	26.3	9.8	47.3	55.8	205.1	56.4
Total	359.8	396.8	271.3	22.1	1089.7	997.4	479.5	39.1	33.0	39.8	56.6	56.4

However, at the player level, in the ADS period both Gas Natural and Iberdrola managed to have a higher worst-case profit when the agents are risk-neutral (about double), due to the negative strong impact of their competitors on their expected profits, under risk-aversion.

6. Conclusions

In this article we analyze the strategic problem faced by producers of non-storable products and services (such as electricity), taking into account the interaction between forward contracts and spot markets, and the impact of demand and cost uncertainties on the firm's strategies. The major methodological contribution is to have derived a closed form solution to solve this problem.

We have applied our model to the analysis of the Iberian electricity market, in four different periods of the year, concluding that the effect of cost uncertainty dominates demand uncertainty. Contrary to a previous study on the issue of demand uncertainty (Gulpinar and Oliveira, 2012) who has reported that risk-averse agents trade more in the futures markets, decreasing spot prices and increasing production, we found that, when we consider cost uncertainty, the agents tend to *reduce* forward trading, increasing spot prices, and reducing production.

Moreover, in our case study, the industry when using risk-aversion, in general, was able to increase both expected profits and to increase the worst-case profit. This finding was also true at a player level, with only two exceptions in one of the scenarios. This result is very important from a policy perspective: when a regulator observes that the agents are having outputs below the expected from a more competitive market, it may not be enough evidence to prove collusion as this behavior may be the result of a risk-averse management.

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