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Recursive Expected Conditional Value at Risk in the Fleet Renewal Problem with Alternative Fuel Vehicles

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Abstract: We study the fleet portfolio management problem faced by a firm deciding which alternative fuel vehicles (AFVs) to choose for its fleet to minimise the weighted average of cost and risk, in a stochastic multi-period setting. We consider different types of technology and vehicles with heterogeneous capabilities. We propose a new time consistent recursive risk measure, the Recursive Expected Conditional Value at Risk (RECVaR), which we prove to be coherent. We then solve the problem for a large UK based company, reporting how the optimal policies are affected by risk aversion and by the clustering for each type of vehicle.

Keywords: Risk Management, Fleet Replacement, Stochastic Programming, Conditional Value at Risk (CVaR).

1. Introduction

The transportation sector is responsible for a large proportion of CO_2 emissions worldwide (Schiffer, 2008) due to its dependence on fossil fuels. For this reason, it is facing a big challenge regarding the adoption of alternative fuel vehicles (AFVs). As a result, car-makers are under pressure to develop fuel-efficient vehicles such as hybrid vehicles (HVs) and electric vehicles (EVs) as an alternative solution to reduce fossil fuel consumption and greenhouse gas emission levels. This is the main motivation for this work: to analyse the viability of the introduction of alternative technologies in a large fleet of vehicles used in service provision.

In the operations research literature, the problem of renewing an existing fleet is commonly referred to as the fleet renewal or replacement problem (Patricksson et al., 2015). A fleet vehicle replacement optimization framework requires three types of inputs: economic factors, vehicular characteristics, and initial fleet configuration (Feng and Figliozzi, 2013). Economic factors include the planning time horizon, the annual demand for vehicles, the annual mileage driven, and the energy price forecast. Vehicular factors include the vehicle type, the age, capital costs, fuel consumption, and annual utilization (miles travelled). The initial fleet configuration includes the number of vehicles per type, and their ages. Once all of the inputs are specified, the model can provide an optimal solution, together with cost breakdown and usage statistics.

In the traditional fleet replacement models, the main concern for a fleet manager is to focus on the minimisation of expected cost over a planning horizon. However, there are uncertainties in fuel and carbon emission prices, environmental regulations, and the technology's life cycle, all of which impact leasing and ownership costs. Therefore, it is essential to view the fleet replacement problem from an uncertainty perspective using risk management methodologies.

A novel contribution of this article is to provide an optimization framework to fully take into account risk management in fleet replacement. We view risk as a manifestation of uncontrollability rather than merely a downside possibility, as defined by Arrow (1970).

This article is the result of a study of the problem faced by a large company in the UK that is aiming to reduce CO_2 emissions by optimally choosing which vehicles to include in their fleet. To this effect we have developed a new risk measure for the dynamic stochastic problem, the Recursive Expected Conditional Value at Risk (RECVaR), and analyse how the optimal policies are affected by risk aversion and by clustering the different types of vehicles.

This article extends the work by Ansaripoor et al. (2014) which considered: a) a two stage model; b) one vehicle analysed individually; c) only one decision is made at time zero. In this article: a) we present a multi-stage model; b) we have a large number of different types of vehicles at each node of a decision tree; c) at each node the firm needs to decide how many vehicles to lease of each type; d) the demand for the different types of services (in terms of mileage and technical requirements) is stochastic. Finally, this paper also provides a methodological contribution by introducing a new time consistent version of CVaR, the RECVaR. We have found that RECVaR takes into account the risks that exist in the middle stages of the scenario tree and is a major contribution to the use of risk analysis in dynamic multi-stage problems.

This paper proceeds as follows. In section 2 we introduce the case study. In section 3 we review the existing literature. In section 4 we present a multi-stage stochastic programming model for minimising the weighted average of the expected cost and risk, considering the existing constraints and uncertainties in the market. In section 5 we derive the analytical results for the RECVaR. In sections 6 we present the computational results and, finally, we present the main conclusions in section 7.

2. A Case of the Fleet Renewal Problem with AVFs

This article is motivated by the problem faced by a large firm in UK that aims to reduce the carbon footprint of its vehicle fleet. The firm leases a large fleet of vehicles used by its engineers. Generally an engineer is assigned a vehicle for the whole lifetime of the lease. Vehicles are assigned depending on the engineer's specialization.

Currently the fleet has only diesel vehicles with different capacities (small, light, medium size vehicles). The size of the vehicle is an important characteristic because, depending on their specialization, the engineers have to carry different materials and, thus, have to drive a vehicle with sufficient capacity. For instance, power engineers need light vans with enough carrying capacity. We consider small vans as weighing 300 kg and that light vans weigh 500 kg and have a greater carrying capacity. Medium vans can be used on any type of function, but there are only a few because they are far more expensive to lease and maintain.

In this case study, we consider another four additional possible technologies: petrol, hybrid-petrol, hybrid-diesel, and EVs. Each vehicle type can be leased with different capacities, as they are used for different job types.

An important issue, when developing a model which aims to solve a real-world problem, is to determine whether it is an accurate representation of the system studied, i.e., if it is valid (Landry et al., 1983; Law and Kelton, 1991; Landry and Oral, 1993). The term "accurate representation" is used to mean the extent to which the model fits the real system either in terms of structure and mechanism or in terms of output, depending on the context of the problem. In order to have a representation of the real-world as accurate as possible we have used: 1) data from the fleet analyzed, including mileage and consumption per vehicle; 2) data on the leasing costs for different types (capacities) of vehicles; 3) forecasts for the fuel prices for the planning horizon considered, based on real data; and 4) a model for CO_2 prices estimated from real data. Moreover, validity is also tested by comparing the optimal decisions with the current fleet used by the company, as reported in section 6.3.

3. Review of the Fleet Replacement Problem and Time Consistency of Dynamic Risk Measures

Decision support systems for fleet operations, capacity decisions, routing problems, and humanitarian operations are well developed in the logistics literature (e.g., Couilard, 1993; Lau et al., 2003; Ghiani et al., 2004; Ruiz et al., 2004; Figliozzi 2009, 2010, 2011). As the literature on replacement strategies in fleet operations is closely related to our work, we focus our review on this topic. Specifically, our concern is related to parallel replacement models, which are for replacement plans of vehicles that are economically interdependent. In addition, we introduce two important risk measures and then we consider the time consistency issue in a dynamic setting, which is essential for the fleet renewal model presented in the article.

3.1. Parallel Replacement Models

These models can generally be categorised in two main groups based on different fleet (asset) characteristics: homogenous and heterogeneous. In the homogeneous replacement model, a group of similar vehicles, in terms of type and age that form a cluster (each cluster or group cannot be decomposed into smaller clusters) has to be replaced at the same time. In the heterogeneous model, multiple heterogeneous assets, such as fleets with different vehicle types, have to be optimised simultaneously. For instance, vehicles of the same type and age may be replaced in different periods (years) because of the restricted budget for the procurement of new vehicles. The heterogeneous model is closer to the real world commercial fleet replacement problem. This model is solved by integer programming and, generally, the input variables are assumed to be deterministic (e.g., Simms et al., 1984; Karabakal et al., 1994; Hartman, 1999, 2000, 2004). The methodology used to solve the homogenous model is dynamic programming. The advantage of the homogenous model is to assume probabilistic distributions for input variables in the optimisation model (e.g., Bellman, 1955; Bean et al., 1984; Oakford et al., 1984; Hartman, 2001; Hartman and Murphy, 2006).

As an example of a parallel replacement problem Feng and Figliozzi (2013) developed a model for considering the economic and environmental optimization of vehicle replacement decisions from a fleet manager's viewpoint. They introduced an integer programming vehicle replacement model that is used to evaluate the current environmental and governmental intervention, such as greenhouse gases, taxes, and fiscal incentives for EVs purchases. As another example of parallel replacement, Patricksson et al. (2015) developed a model to deal with stricter emission regulations for new vessels. They used a stochastic programing model which considers stochastic fuel prices. There is, currently, a lack of research on the dynamic fleet size and mix problem, see, e.g., the discussion by Pantuso et al. (2014).

Moreover, although the fleet (asset) replacement or renewal literature is rich in models dealing with budget constraints (Karabakal et al., 1994, 2000; Chand et al., 2000), variable utilisation (Bethuyne, 1998), stochastic demands (Hartman, 2001), heterogeneous vehicle types (Hartman, 2004), to our knowledge risk measure issues have not been considered. Specifically, from our analysis of the literature, it is evident that there is a gap, which we aim to address in this article: How to explain fleet replacement from an uncertainty perspective using conditional value at risk in a dynamic multi-stage setting model which differs from recent literature in adoption of

AFVs (e.g., Kleidorfer et al., 2012; Wang et al., 2013) and in maritime fleet renewal problems (e.g., Pantuso et al., 2015a, Pantuso et al., 2015b, Fagerholt et al., 2015).

3.2. Risk Measures and Time Consistency

The definition of Value at Risk (VaR) (e.g., Anderson, 2014) is the maximum potential loss that a financial sector can tolerate, with some likelihood, during a finite period at a certain confidence level over a finite time horizon. For instance, if VaR is -$150 for a portfolio with confidence level of 99% and a time horizon of 20 days, we can say that there is a 1% probability that a loss of at least $150 is incurred over the next twenty days.

The Conditional Value at Risk (CVaR) gives us information about the magnitude of the losses beyond VaR. Mathematically, CVaR is derived by taking the average of losses exceeding the VaR. For instance, in our previous example CVaR is the average of the losses that are beyond -$150. For example, if the calculated CVaR is -$180, we can say that the expected loss for the worst 1% scenarios is $180. Obviously, the loss associated with the CVaR is always higher or equal to the loss associated with the respective VaR.

When analyzing dynamic systems, time consistency is an important requirement to get appropriate optimal decisions. Articles on time consistency can be categorized in two different approaches: the first one concentrates on risk measures and the second one on optimal policies (Rudloff et al., 2014).

The first approach says that, in a dynamic setting, if some random payoff A is always riskier than a payoff B, at time t+1, then A should also be riskier than B, at time t. It has been shown that this property leads to the so called time consistent dynamic risk measure proposed by several authors, e.g., Detlefsen and Scandolo (2005). Other definitions, such as acceptance and rejection consistency, are also developed in the literature, e.g., Cheridito et al. (2006); Kovacevic and Pug (2009).

The second approach, defined by Shapiro (2009), is the time consistency of optimal policies in multistage stochastic programming models. A basic concept of the multistage stochastic programming model is the requirement of *nonanticipativity* (Shapiro, 2009). That is, our decisions should be a function of the history of the data process available at time t, when decisions are made. We also have an idea of which scenarios may and may not be observed in the future. Thus, it is natural to consider the conceptual requirement the optimality of our decision at state n should only involve future children nodes (m) of state n. This principle is called *time consistency* (Shapiro, 2009). This time consistency requirement is closely related to, although not the same as, the Bellman's principle used to derive dynamic programming equations. The standard risk neutral formulation of multi-stage programming problems satisfies this principle. On the other hand, some risk-averse stochastic programming problems do not satisfy this requirement (Shapiro, 2009). Other alternatives also have been proposed, e.g., Boda and Filar (2006), however none of them used the recursive set up of time consistent dynamic risk measures which is addressed in this article.

4. A General Model for a Fleet Management System

In this section, we introduce a multi-stage stochastic programming model to obtain the optimal number of vehicles to be leased, taking into account the constraints that exist to minimise a cost function, which considers both expected cost and risk during the planning horizon. The notation used is summarized in Tables 1(a) and 1(b). In Figure 1, at each node, we have a vector of stochastic processes, namely, fuel prices, CO_2 prices, mileage driven, and fuel consumption for fossil fuel technologies per 100 miles. We consider five technologies: fossil fuels (petrol, diesel), hybrids (petrol, diesel), and EVs. The leasing costs are obtained from current market prices, for each capacity and technology, and are assumed to be constant during the time horizon considered due to mass production, over stocking, and maturity of the production processes in a very competitive industry, as observed in the market.

$$r_{ni} = o_n (f_{ni} + c_n^p c_i^g) \qquad \forall n \in N, \ i = \text{fossil fuels, hybrids} \qquad (1)$$

$$r_{ni} = f_{ni} + 100 c^e c_n^p \qquad \forall n \in N, \ i = \text{electric} \qquad (2)$$

$$\lambda_{ni} = \frac{r_{ni}}{100} D_n \qquad \forall n \in N, i \qquad (3)$$

$$\mu_i = l_i \qquad i = \text{fossil fuels, hybrid} \qquad (4)$$

$$\mu_i = l_i + M_e \qquad i = \text{electric} \qquad (5)$$

In equations (1) and (2), we calculate the running cost for fossil fuel, hybrid vehicles and EVs, per 100 miles, at each node, r_{ni}. In these equations, o_n denotes the fuel consumption of fossil fuels and hybrids per 100 miles at each node, f_{ni} stands for the fuel prices for each technology and c_n^p are the CO_2 emissions at each node. At the moment, in the UK, there is no mandatory CO_2 trading scheme for fleet system of the company, however, the CO_2 emission costs, c_n^p, enter in the running cost equation due to the fact that firm wishes to compensate them by managing an efficient fuel consumption of its fleet. Finally, c^g denotes the CO_2 emissions (g/litre) for fossil fuels and hybrids, and c^e shows the CO_2 emissions for EVs (g/mile).

Table 1(a): Indices and variables

$i \in I$={fossil fuels, hybrids, and electric}
$a \in A$= {1, 2,.., A} index for age of the vehicles
$n \in N$= {1,2,…,N} index for nodes in scenario tree
$t \in T$= {1, 2, ..,T} index for time periods in year over planning horizon
$s_t \in S$= {1, 2,.., S_t} index for number of branches (states) at each stage
$k \in K$={1, 2,.., K} index for different clusters of vehicles
$\Psi_{n,m}$: Tree structure for parent nodes n and child nodes m
$\Omega_{t,n}$: Structure for the association of each node n and each stage of t
x_{nia}: Total number of vehicles with technology i and age a currently leased at node n
y_{ni}: Number of new replaced vehicles with technology i that company leases at node n
α_n^β: Value at risk at confidence level of β at node n
ϕ_n^β: Conditional value at risk at confidence level of β at node n
z_n: Positive stochastic variables for loss function at node n

As a result, based on equations (1) and (2), we calculate the total running cost at each node, λ_{ni}, using equation (3), in which D_n represents the annual mileage driven at node n. The total investment cost is represented by (4) for fossil fuels and hybrid technologies and by (5) for EVs. Because we take into account the leasing contracts for providing different vehicle types in the fleet system, we use the annual lease cost, represented by l_i, to obtain the fixed cost at each node. Moreover, for EVs we have an extra investment cost, which is the annual lease cost for batteries, represented in equation (5) by M_e.

Table 1(b): Parameters of the model.

ω: Parameter for trade-off of risk and cost in the objective function

β: Confidence level for calculating CVaR and VaR

G_k: Number of vehicles in cluster k

h_{na}: Number of vehicles at node n, age a

f_{ni}: Fuel price for technology i at node n

o_n: Fuel consumption, liters per 100 mile, at node n

D_n: Annual mileage driven at node n

r_{ni}: Running cost per 100 miles for technology i at node n

c_n^p : CO_2 prices, per g, at node n

c^e: CO_2 emissions, g per mile, for electrical technology

c_i^g : CO_2 emissions, g per liter, for fossil fuel/hybrid technology i

l_i: Annual lease cost for technology i

M_e: Annual lease cost for EV batteries

λ_{ni}: Total annual running cost for technology i at node n

μ_i: Total annual fixed cost for technology i

The firm aims to solve the mixed integer multi-stage stochastic programming model represented in equations (6)-(17). The objective function (6) is the weighted average of expected cost, Q_1, and RECVaR, ϕ_1^β, at the root node. If ω equals 1, only the expected cost is minimised and if ω is equal to zero, only RECVaR is minimized. We have three main decision variables. The first decision variable is y_{ni}, which denotes the number of new vehicles with technology i that are replaced, at each node, at the beginning of each year, due to the end of the lease contracts. The second one is VaR at each node, α_n^β. Finally, the third one is z_n which represents the losses that are beyond VaR at each node. The total number of the vehicles x_{nia}, at each node is a variable because the demand for vehicles changes every year, due to different requirements both in terms of tasks and mileage.

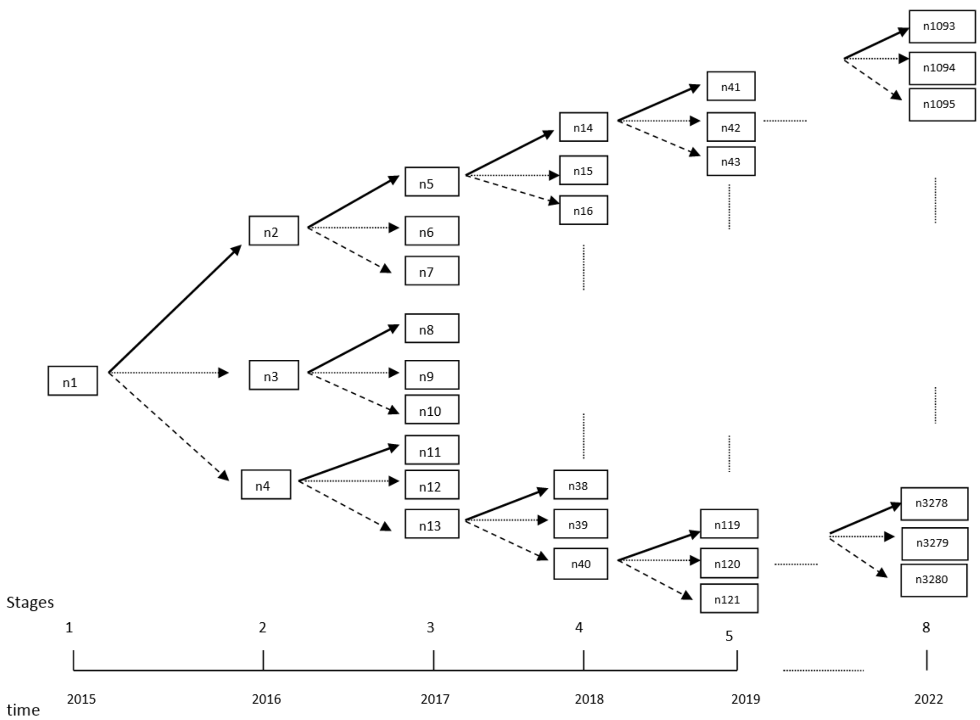

Figure 1: The node-based tree model and corresponding stages (3 branches at each node).

$$\underset{y_{ni},\alpha_n^{\beta},z_n}{Min}\ \omega Q_1 + (1-\omega)\phi_1^{\beta} \tag{6}$$

s.t.

$$x_{1ia} = h_{1a} \quad \forall \mathrm{a} \in A, i \tag{7}$$

$$y_{ni} = x_{ni1} \quad \forall n \in N, i \in I \tag{8}$$

$$y_{ni} = 0 \quad \forall n \in \Omega_{t,n} \text{ if } t \geq T - A - 1$$

(9)

$$x_{mia} = y_{mi} + x_{ni(a-1)} \quad \forall a \in A, \forall (n,m) \in \Psi_{n,m}$$

(10)

$$\sum_i \sum_a x_{nia} \geq \sum_a h_{na} \quad \forall n \in N \tag{11}$$

$$L_n = \sum_i \sum_a (\lambda_{ni} + \mu_i) x_{nia} / 10^6 \quad \forall n \in N \tag{12}$$

$$Q_n = L_n + \frac{1}{S_t} \sum_{\Psi(n,m)} (Q_m) \qquad \forall (n,m) \in \Psi_{n,m} \tag{13}$$

$$z_m \geq L_m - \alpha_n^\beta \qquad \forall (n,m) \in \Psi_{n,m} \tag{14}$$

$$\phi_n^\beta = \alpha_n^\beta + \frac{1}{S_t(1-\beta)} \sum_{\Psi(n,m)} (z_m) + \frac{1}{S_t} \sum_{\Psi(n,m)} (\phi_m^\beta) \qquad \forall (n,m) \in \Psi_{n,m} \tag{15}$$

$$\alpha_n^\beta = 0 \quad \forall n \in \Omega_{t,n} \tag{16}$$

$$x_{nia}, y_{ni} \in N \cup \{0\}, \text{ and } \alpha_n^\beta, z_n \in R^+ \tag{17}$$

Constraints (7) impose the initial condition of the fleet system, h_{1a}, which comprises vehicles with different ages, should be equal to total number of the vehicles, x_{1ia}, at the root node. In constraints (8) we determine the number of new leased vehicles at each node, y_{ni}, required to replace the segment of new vehicles of the total vehicles in the fleet system, x_{ni1}, due to retirement of the older vehicles at the corresponding node. In addition, constraints (9) limit the planning horizon for decision variable y_{ni} to A years, after which there will be no new leased vehicles in the fleet system. Constraints (10) limit the total number of vehicles at each child node, x_{mia}, to equal to the number of new leased vehicles, y_{mi}, plus the number of vehicles in the predecessor node, x_{nia}. Moreover, constraints (11) require that the total number of vehicles for all technologies and ages, $\sum_i \sum_a x_{nia}$, at each node should be greater than, or equal to, the number of vehicles needed, $\sum_a h_{na}$, at the corresponding node. Constraints (12) represent the loss function (total cost), L_n, at each node, which is the sum of the running cost, λ_{ni}, and fixed cost, μ_i, at the corresponding node. In addition, constraints (13) represent the recursive formula for calculating the expected cost function at each node, Q_n, which is equal to the loss function, L_n, at the corresponding node plus the average of cost functions, $\frac{1}{S_t}\sum_{\Psi(n,m)}(Q_m)$, in the successor nodes. Constraints (14) represent the calculation of decision variable z_m at each child node. It stands for the losses that are beyond VaR at each child node which come from its corresponding parent node, α_n^β. To take into account the time consistency issue of CVaR (Shapiro, 2011), we use constraints (15) which account for the effect of risk at the child nodes. In addition, constraints (16) impose that the VaR, α_n^β, at the final stage to be zero, because there is no uncertainty at this stage, and all the values of the stochastic processes are realised. Finally, (17) are the constraints for the integer, x_{nia} and y_{ni}, and non-negative, α_n^β and z_n, decision variables.

5. Analysing the Main Properties of the Model

Consider a T-stage scenario tree representing the evolution of the corresponding data process (Figure 1). This scenario tree represents a finite number of possibilities of what can happen in the future. At stage (time) $t = 1$, we have one root node denoted by n_1. At stage $t \geq 2$, we have S_t branches, at each node n (in Figure 1, we have assumed S_t to be equal to 3). Each branch is connected to the predecessor node by an arc. By $\Omega_{t,n}$ we denote the set of all nodes at stage t=1, 2,..., T. Moreover, $\xi_{ni} = (c_n^p, f_{ni}, D_n, o_n)$ denotes the random process for the realization of the stochastic parameters of CO_2 prices (c_n^p), fuel prices for each technology (f_{ni}), mileage driven

(D_n), and fuel consumption (o_n), at each node n, and stage t. In addition, we denote $\Psi_{n,m}$ as the set of nodes and respective children. The children nodes (m) of node n, at stage t, are nodes that may be realized at next stage t+1. A scenario representing a particular realization of the data process is a sequence of nodes and children nodes, such that $(n,m) \in \Psi_{n,m}$.

Now we are ready to consider the time consistency of the risk measure in (15). The static value of ϕ^{β} is obtained by the following system for the confidence level β (Rockafellar and Uryasev, 2000). In addition, γ_s represents the discrete scenarios sampled from the distribution of the stochastic processes in the model, S is the number of scenarios, $v = \left[(1-\beta)S\right]^{-1}$, x is the vector of decision variables, z_s are positive variables, and L denotes the loss function. Solving the following LP model gives the optimal value of $\phi^{\beta*}$, the decision variable x^*, and $\alpha^{\beta*}$.

$$\min_{x,\alpha^{\beta}} \phi^{\beta} = \min_{x,\alpha^{\beta}} \ \alpha^{\beta} + v\sum_{s=1}^{S} z_s$$

$s.t.$

$$z_s \geq L(x,\gamma_s) - \alpha^{\beta} \quad z_s \geq 0, \ s = 1,2,...,S$$

Boda and Filar (2006) and Shapiro (2009) have shown that, in general, in a dynamic setting the above model for calculating ϕ^{β} is not time consistent. For this reason, we present a dynamic risk measure of ϕ^{β} denoted by ϕ_n^{β} (equation (15)) which is the value of ϕ^{β} at each node $n \in \Omega_{t,n}$, and is time consistent by construction and we name it RECVaR which we prove is coherent. In the next part we consider the coherency property of equation (15) and the effect of clustering on the RECVaR. In the context of the overall optimization problem we can define the risk measures in (15) as:

$$\phi_n^\beta = \underset{\alpha_n^\beta}{Min}(\alpha_n^\beta + \frac{1}{S_t(1-\beta)}\sum_{\psi(n,m)} z_m + \frac{1}{S_t}\sum_{\psi(n,m)} \phi_m) \tag{18}$$

We prove that equation (18) is a coherent risk measure. Indeed, we want show that the dynamic risk measure ϕ_n^β in (18) satisfies all properties of a coherent risk measure.

Proposition 1: *The dynamic recursive risk measure in (18) is coherent.* [Proved in the appendix.]

Our risk measure is different from the one in Shapiro (2011), where the concept of cost-to-go function was used to satisfy the time consistency principle. We provide a recursive risk measure (RECVaR) for a scenario tree, explicitly computing the risk of the parent node as a function of the RECVaRs and expected conditional expectations of the extreme cost of the respective children nodes. The RECVaR provides more intuitive and robust results, because it takes into account the risks that exist in the middle stages of the scenario tree. However, the formulation by Shapiro (2011) is not sensitive to this kind of risk because it is only obtained based on recursive cost-to-go functions at different stages. It also differs from Boda and Filar (2006), in which the target-percentile approach was applied to follow the time consistency principle. Furthermore, we consider the effect of clustering on the RECVaR, which we analyze in Proposition 2.

Proposition 2: *Let $E(\phi_k^\beta)$ represent the expected RECVaR for each one of k clusters, and ϕ^β denote the RECVaR for the combined clusters. Then,*

$$E(\phi_k^\beta) \le \phi^\beta \le \sum_k \phi_k^\beta \quad .$$

Proposition 2 states that the amount of RECVaR per vehicle for the combined cluster is less than the sum of the RECVaR of other clusters (subadditivity property) and it is more than the average RECVaR of each sub-cluster, for each value of ω. In other words, by using clustering, the total risk decreases due to diversification of fleet system in different clusters, with different technologies and capacities.

6. Implementation and Computational Experiments

In Table 2 we present the average correlation coefficient, in each year, between fuel prices, from Jan. 2000 to Dec. 2014: the diesel and petrol prices have a very high correlation coefficient of 0.876 but a negative correlation of (-0.126) and (-0.162) with electricity, respectively. (In this article, we use electricity and electricity charge prices interchangeably. However, the price of electricity is the price of each charge for a 22 kWh battery for 100 miles autonomy). Based on this observation, we can generate the simulated scenarios for the fuel prices with the above correlation matrix using the expected forecasted prices from 2015 to the end of 2022 (Table 3).

The distribution per age group for vehicles, from year one to four of the leasing contract is [22%, 40%, 30%, 8%]. The CO_2 emissions for petrol, diesel, hybrid-petrol and hybrid-diesel are 2310, 2680, 1719, and 2177 (g/litre), respectively. Moreover, the CO_2 emissions for EVs are 130 g/mile. The leasing costs for each capacity and technology are summarised in Table 4.

Table 2: Correlation matrix for fuel prices from Jan. 2000 to Dec. 2014 in each year

	Petrol	**Diesel**	**Electricity**
Petrol	1	0.876	-0.126
Diesel	0.876	1	-0.162
Electricity	-0.126	0.162	1

Table 3: Average forecasted fuel prices (£) from 2015 to 2022

	2015	2016	2017	2018	2019	2020	2021	2022
Petrol	1.338	1.386	1.435	1.483	1.532	1.58	1.628	1.677
Diesel	1.387	1.439	1.49	1.542	1.593	1.645	1.696	1.748
Electricity	2.476	2.581	2.685	2.789	2.894	2.998	3.102	3.206

Table 4: The annual leasing costs (£) for vehicles with different capacities and technologies

Technology	Small (£)	Light(£)	Medium(£)
Petrol	2508	2736	3192
Diesel	2640	2880	3360
Hybrid-Petrol	3588	3912	4572
Hybrid-Diesel	3828	4176	4872
EVs	4572	4980	5808

We now proceed with the clustering analysis, using a combination of the hierarchical and of *K*-means clustering algorithms (e.g., MacQueen, 1967; Hand et al., 2001; Bacher et al., 2004; Schonlau, 2004) to find patterns in the consumption rates for the different vehicles. We perform a separate clustering analyisis for each of the three vehicle types (small, light, and medium vans).

The total number of vans at the start of the planning horizon is 2369, comprising 1137 (48%) small, 1077 (45%) light, and 155 (7%) medium vans. The result of clustering analysis for the small vans, light vans and medium vans are presented in Tables 5, 6 and 7.

Table 5: Cluster analysis for mileage and fuel consumption data for small vans

Types of vehicles	Cluster	Annual Mileage (miles)		Fuel Consumption (litres/100 miles)		Vehicles	
		Mean	S.D.	Mean	S.D.	Number	%
Small Vans	1	6976.8	3213.6	10.1	0.6	135	12%
	2	9470.4	3058.8	8.5	0.3	381	34%
	3	10147.2	3068.4	7.2	0.6	297	26%
	4	19260.0	3763.2	8.0	0.6	324	28%
	Combined	12140.4	5646.0	8.2	1.1	1137	100%

Table 6: Cluster analysis for mileage and fuel consumption data for light vans

Types of vehicles	Cluster	Annual Mileage (miles)		Fuel Consumption (litres/100 miles)		Vehicles	
		Mean	S.D.	Mean	S.D.	Number	%
Light Vans	1	7628.4	2900.4	11.4	0.8	130	12%
	2	8652.0	2473.2	9.6	0.5	292	27%
	3	13324.8	3045.6	8.6	0.8	324	30%
	4	16521.6	2803.9	10.6	0.8	189	18%
	5	23090.4	3756.0	9.3	0.6	142	13%
	Combined	13212.0	5734.8	9.7	1.1	1077	100%

Table 7: Cluster analysis for mileage and fuel consumption data for medium vans

Types of vehicles	Cluster	Annual Mileage (miles)		Fuel Consumption (litres/100 miles)		Vehicles	
		Mean	S.D.	Mean	S.D.	Number	%
Medium Vans	1	8824.8	2943.6	16.6	1.8	42	27%
	2	11576.4	2316.0	12.8	1.9	68	44%
	3	19416.0	3566.4	15.0	2.1	45	29%
	Combined	13106.4	5093.0	14.5	2.5	155	100%

6.1. Solving the Fleet Replacement Problem

The model presented in Section 4 has been implemented and solved using CPLEX running on a laptop with a 4.2 GHz processor and 30 GB RAM. The average computational times are presented in Table 8.

We aim to minimise the weighted average of expected costs and RECVaR during the planning horizon (2016 to 2019). Note that in order to calculate this policy we need to forecast the stochastic variables from 2019 to the end of 2022 (until the end of life of the vehicles leased in the period of analysis). Moreover, at each node we have generated a random vector of normal distribution for fuel prices, monthly mileage driven, and fuel consumption. For the means and standard deviations of corresponding distributions, we have used the information in Tables 5, 6, and 7. For simulating the CO_2 prices, we have assumed a uniform distribution between £5×10^{-6}/g and £20×10^{-6}/g. The number of vehicles at each node for each cluster is normal distributed with the mean equal to the number of vehicles in each cluster (Tables 5, 6, and 7).

We used the method of Høyland and Wallace (2001) to generate scenarios; the general method of scenario generation by matching statistical properties to specific targets. The method generates a set of discrete scenarios so that statistical properties of the random variables match specified target values. We have used Tables 2, 3 for fuel prices and Tables 5, 6, and 7 for mileage and fuel consumption.

Table 8: Effect of branching (S_t) on the convergence of RECVaR.

Case/year	1	2	3	4	5
S_1	10	15	20	30	40
S_2	5	10	10	10	10
S_3	3	5	5	5	5
S_4	2	2	2	2	2
S_5	2	2	2	2	2
S_6	2	2	2	2	2
S_7	2	2	2	2	2
S_8	2	2	2	2	2
# nodes	4711	23416	31221	46831	62441
# scenarios	2400	12000	16000	24000	32000
Ave. computational time (sec.)	15	195	375	873	1530
Variation of RECVaR	-	1.47%	0.43%	0.37%	0.10%

We also investigated the behavior of the model presented in Section 4 with respect to the number of scenarios. In order to validate the results we must show that the scenario generation method used is "neutral"; that is, it does not influence the results by causing instability (Kaut et al., 2007).

In the last row of Table 8 it can be seen that by increasing the number of scenarios the RECVaR converges (the percentage shows the change of the RECVaR when compared to the branching in the previous column). We have tested the sensitivity of the results to the number of scenarios (sampling size), by trial and error, concluding that if we consider a branching factor larger than 4, with exponentially more scenarios, the results remain the same. Note that this percentage of variation of RECVaR (proxy of instability measure for the model) monotonically decreases with increasing number of scenarios. This is a helpful consistency check. We see that as we increase the number of scenarios we can indeed achieve both in-sample and out-of-sample stability (Kaut et al., 2007). Thus, the scenario generation method is effective and "neutral", in the sense that it does not cause any instability in the solutions of the RECVaR model and we attain both in-sample and out-of-sample stability. As a compromise between accuracy in stability and computational speed, we decided to use trees with 24000 scenarios.

6.2. Result of Optimal Policies for AFVs

The results for the optimal number of vehicles, for each cluster, are summarized in Table 9. We consider three values for the risk preference parameter ω (0, 0.5, and 1) and denote the different technologies by D, P, H-P, H-D, and E for Diesel, Petrol, Hybrid-Petrol, Hybrid-Diesel, and Electric, respectively. For example, for small vans, Table 9(a), in cluster 1, for a ω equal to 0.5, we lease 11, 38, 4, and 0 diesel vehicles and 0, 2, 50, and 30 petrol vehicles at the beginning of each year, between 2016 and 2019, respectively. For ω equal to 0.5, 61% of the vehicles are petrol based. For clusters 2 and 3, at the beginning of 2019, for ω equal to 0, we lease a small number of hybrid-petrol and hybrid-diesel vehicles. For cluster 4, for ω equal to 0 and 0.5, we lease a small number of hybrid-petrol, hybrid-diesel and EVs.

Table 9(a). Optimal number of vehicles per cluster for small vans. For each cluster, and ω equal to 0, 0.5 and 1, we report the technologies leased and the number of vehicles per technology.

(a) Small Vans							
		Cluster 1			Cluster 2		
ω		0	0.5	1	0	0.5	1
Year	2016	34	11, 0	11	80, 0, 0, 0	30, 0	30
	2017	44	38, 2	41	125, 0, 0, 0	115, 0	115
	2018	44	4, 50	54	101, 27, 0, 0	14, 138	152
	2019	13	0, 30	29	26, 16, 3, 3	3, 81	84
Tech.		D	D, P	D	D, P, HP, HD	D, P	D
		Cluster 3			Cluster 4		
ω		0	0.5	1	0	0.5	1
Year	2016	60, 0, 0, 0	23, 1	24	73, 0, 1, 0, 0	25, 1, 0, 0, 0	26
	2017	94, 0, 0, 0	83, 6	89	97, 0, 0, 0, 0	99, 0, 0, 0, 0	97
	2018	78, 26, 0, 0	8, 111	119	98, 8, 4, 3, 2	84, 42, 1, 1, 1	130
	2019	20, 14, 3, 2	2, 63	65	20, 6, 4, 4, 4	22, 44, 1, 2, 1	71
Tech.		D, P, HP, HD	D, P	D	D, P, HP, HD, E	D, P, HP, HD, E	D

We now analyse the light van results. Given different values of ω (0, 0.5, and 1) we have computed the optimal choice of technology and number of new leased vehicles, Table 9(b). We can see that in higher mileage clusters the penetration ratio of EVs is higher. For example, in cluster 5 of light vans during 2016 to 2019, for ω equal to 0, the penetration ratio of diesel, petrol, hybrid-petrol, hybrid-diesel, and EVs are 81%, 1%, 2%, 3%, and 13%, respectively.

Table 9(b). Optimal number of vehicles per cluster for light vans. For each cluster, and ω equal to 0, 0.5 and 1, we report the technologies leased and the number of vehicles per technology.

(b) Light Vans							
		Cluster 1			Cluster 2		
ω		0	0.5	1	0	0.5	1
Year	2016	30	10, 0	10	55, 0, 0, 0	23, 0	23
	2017	43	39, 0	39	96, 0, 0, 0	88, 0	88
	2018	44	6, 46	52	82, 21, 0, 0	9, 108	117
	2019	13	0, 29	29	21, 13, 2, 2	0, 64	64
Tech.		D	D, P	D	D, P, HP, HD	D, P	D
		Cluster 3			Cluster 4		
ω		0	0.5	1	0	0.5	1
Year	2016	69, 0, 0, 0	25, 0	26	40, 0, 0, 0, 0	15, 0, 0, 0, 0	15
	2017	99, 0, 0, 0	99, 0	97	58, 0, 0, 0, 0	57, 0, 0, 0, 0	57
	2018	100, 15, 1, 1	24, 105	130	61, 3, 1, 2, 2	57, 17, 1, 1, 0	76
	2019	23, 11, 3, 2	2, 69	71	13, 3, 2, 2, 3	14, 24, 1, 1, 1	41
Tech.		D, P, HP, HD	D, P	D	D, P, HP, HD, E	D, P, HP, HD, E	D
		Cluster 5					
ω		0	0.5	1			
Year	2016	30, 0, 0, 0, 0	10, 0, 0	11, 0			
	2017	41, 0, 0, 0, 3	44, 0, 0	43, 0			
	2018	36, 1, 1, 2, 10	37, 16, 4	56, 1			
	2019	8, 1, 2, 2, 5	3, 27, 1	29, 2			
Tech.		D, P, HP, HD, E	D, P, E	D, E			

Next, we consider the medium van results presented in Table 9(c). For clusters 1 and 2 the optimal choice of vehicle does not include EVs. However, for the third cluster, due to higher investment costs when compared with small and light vans, the optimal policy is to include EVs are part of the of the medium vans.

Table 9(c). Optimal number of vehicles per cluster for medium vans. For each cluster, and ω equal to 0, 0.5 and 1, we report the technologies leased and the number of vehicles per technology.

(c) Medium Vans							
		Cluster 1			Cluster 2		
ω		0	0.5	1	0	0.5	1
Year	2016	10	3, 0	3	16,0, 0, 0	5	5
	2017	14	14, 0	13	18, 1, 0, 0	20	20
	2018	14	6, 10	17	20, 4, 0, 0	18, 9	27
	2019	4	1, 8	9	5, 2, 1, 1	6, 10	16
Tech.		D	D, P	D	D, P, HP, HD	D, P	D
		Cluster 3					
ω		0	0.5	1			
Year	2016	2, 0, 0, 10	3, 0, 0, 0, 1	4, 0, 0, 0			
	2017	5, 0, 0, 6	10, 0, 0, 0, 3	13, 0, 0, 0			
	2018	7, 1, 1, 6	11, 1, 1, 1, 4	14, 1, 1, 3			
	2019	2, 1, 1, 3	3, 2, 1, 1, 3	6, 0, 1, 2			
Tech.		D, HP, HD, E	D, P, HP, HD, E	D, HP, HD, E			

6.3. Portfolio Effect of Clustering

In this section we test whether clustering decreases risk and/or expected cost. Table 10 depicts the results for different capacities of vehicles for combined clusters. It can be seen that, for all vehicle types, the portion of diesel technology is the highest, for all values of ω. For example, for ω equal to 0, the percentage of new leased diesel vehicles from 2016 to 2019, with respect to other technologies, is 94%, 96%, and 83% for small, light, and medium vans, respectively.

We now consider the effect of clustering, for each vehicle capacity, on risk and on the expected cost per vehicle. As the number of vehicles in each cluster is different, and as we have a different expected mileage, and fuel consumption, for each cluster, in order to compare the risk values of the different clusters, we use the RECVaR per vehicle, in each cluster, for three values of ω. We can do this by subtracting the RECVaR from the expected cost of each cluster, and dividing the result by the number of vehicles in the cluster. In so doing, the amount of loss (risk) for each cluster is normalised, and this value is comparable among clusters. In addition, the expected cost analysis is also on a per vehicle basis.

Table 10: Optimal number of vehicles and technologies for combined clusters. For each cluster, and ω equal to 0, 0.5 and 1, we report the technologies leased and the number of vehicles per technology.

		Small Vans			Light Vans		
ω		0	0.5	1	0	0.5	1
	2016	283, 4, 3, 0	82, 7, 3	92, 0	273, 3, 3, 0	82, 3, 2	87, 0
Year	2017	376, 17, 2, 0	362,12, 2	321, 20	365, 8, 2, 0	356, 6, 2	318, 5
	2018	321, 29, 2, 2	71, 345, 3	428, 26	306, 19, 4, 3	102, 287, 1	417, 14
	2019	91, 7 , 0, 0	4, 276, 0	233, 17	87, 4, 0, 0	5, 231, 0	226, 10
Tech.		D, P, HP, HD	D, P, HP	D, P	D, P, HP, HD	D, P, HP	D, P
Total		1137	1137	1137	1077	1077	1077
		Medium Vans					
ω		0	0.5	1			
	2016	41, 1, 1, 0, 0	11, 1, 0, 0	12, 0, 0, 0			
Year	2017	42, 4, 4, 2, 1	46, 2, 1, 1	47, 0, 0, 0			
	2018	35, 5, 3, 3, 1	40, 16, 3, 2	56, 4, 1, 1			
	2019	10, 1, 1, 0, 0	10, 18, 2, 2	29, 3, 1, 1			
Tech.		D, P, HP, HD, E	D, P, HP, HD	D, P, HP, HD			
Total		155	155	155			

Next, we analyse the patterns observed in Figures 2, 3, and 4. For all capacities, and for all values of ω, the RECVaR decreases when we have clustering. For example, for the case of ω equal to 0.5, there is a reduction in RECVaR per vehicle of £1.14K (29.73%), £1.70K (44.44%), £1.94K (50.64%), and £1.16K (30.40%) for clusters 1, 2, 3, and 4, when compared to combined cluster in small vans, respectively. Moreover, as proved in Proposition 2, the amount of RECVaR per vehicle for the combined cluster is less than the sum of the RECVaRs of other clusters (subadditivity property) and is more than the average RECVaR of each sub-cluster, for each value of ω.

In addition, the expected cost per vehicle decreases when using clustering, for the different vehicle capacities (except for high mileage clusters) when compared to combined cluster. For instance, for the case of ω equal to 0.5, there is a reduction in expected cost per vehicle of £3.72K (13.7%), £4.07K (15.2%), and £1.18K (4.33%) for clusters 1, 2, and 3 of light vans. However, for the high mileage clusters, the expected cost per vehicle increases by using clustering, for the different vehicle capacities. One reason for this increase is that the adoption of EVs in high mileage clusters increases the expected cost per vehicle. Another reason is that, in these clusters, we have a high percentage of diesel and petrol vehicles and these vehicles are not cost efficient at high mileages. For instance, in cluster 4 for small vans, the expected cost per vehicle increases for all values of ω. This is the same in clusters 4 and 5 for light vans, and cluster 3 for medium vans. For example, for the case of ω equal to 0.5, there are increases of £4.53K (19.45%) in cluster 4 for small vans, £4.53K (16.70%) and £7.25K (26.71%) in clusters 4 and 5 for light vans, £9.66K (27.52%) in cluster 3 for medium vans, respectively.

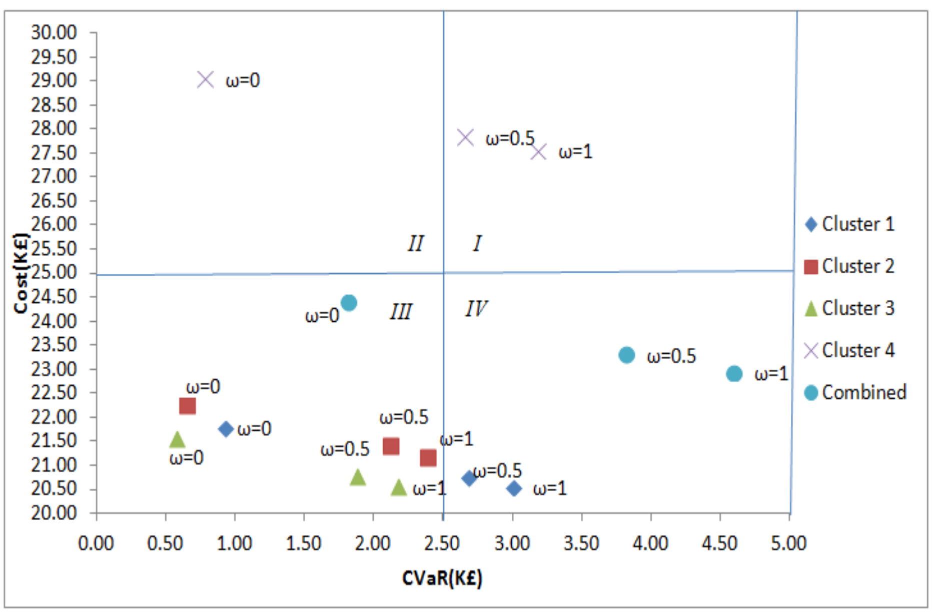

Figure 2: Portfolio Effect on RECVaR and Expected Cost for Small Vans

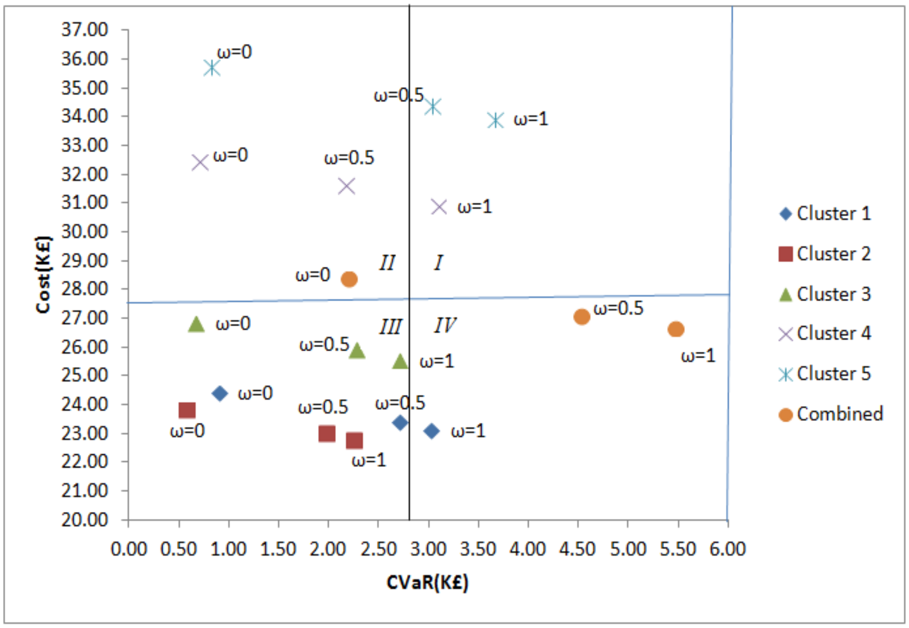

Figure 3: Portfolio Effect on RECVaR and Expected Cost for Light Vans

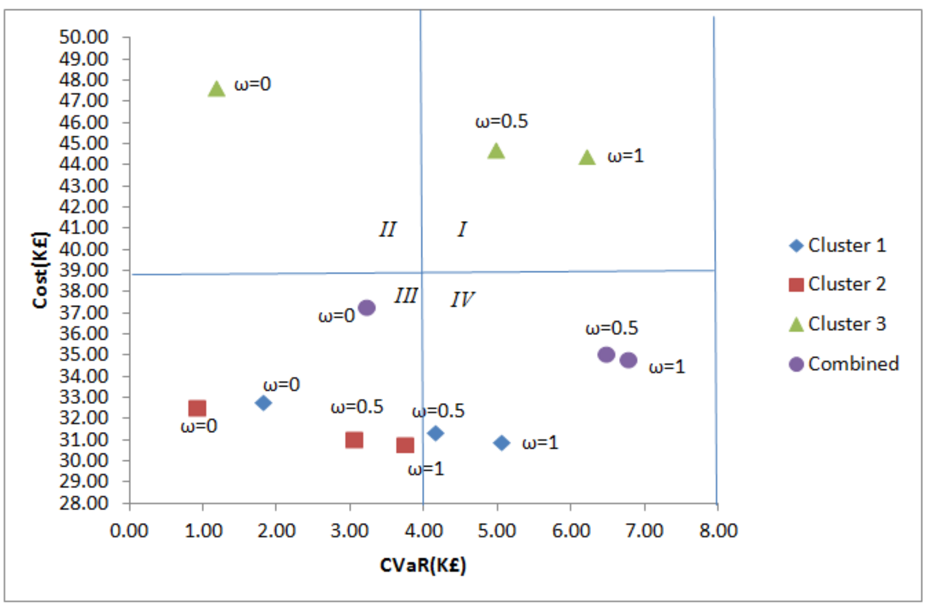

Figure 4: Portfolio Effect on the RECVaR and the Expected Cost for Medium Vans

7. Conclusions

In this article we presented a general model for the fleet replacement problem, considering alternative fuels and, simultaneously, taking into account the minimisation of both expected cost and risk. As methodological contribution, we have developed a time consistent, dynamic risk measure, the Recursive Expected CVaR and we have analysed how clustering affects the risk, proving that the average risk of the clusters is always less or equal to the risk without clustering.

We have tested our framework in the context of a case study, concluding that:

1. While the diesel vehicles are still the dominant choice in average and high mileage clusters, the petrol vehicles are the optimal choice in the low mileage clusters.
2. The hybrid-petrol and hybrid-diesel technologies are still not competitive, and their optimal utilization ratio ranged from 1% to 3%, in the different clusters.
3. EVs are, in general, an optimal choice for the high mileage clusters.
4. For all risk preferences, and all capacities, clustering reduces risk.
5. The risk per vehicle, without clustering, is above average for the separate clusters, and is less than the sum of risks of the clusters.

Appendix

Proposition 1: *The dynamic recursive risk measure in* (*18*) *is coherent.*

Proof: First, we consider the Positive Homogeneity property. By using (18), $\phi_n^\beta(L) = \underset{\alpha_n^\beta}{Min}\left[\alpha_n^\beta(L) + \frac{1}{S_t(1-\beta)}\sum_{\psi(n,m)} z_m(L) + \frac{1}{S_t}\sum_{\psi(n,m)}\phi_m(L)\right]$, we have:

$$\phi_n^\beta(hL) = \underset{\alpha_n^\beta}{Min}\left[\begin{array}{l}(\alpha_n^\beta(hL) + \frac{1}{S_t(1-\beta)}\sum_{\psi(n,m)}(hL_m - \alpha_n^\beta(hL))^+ \\ +\frac{1}{S_t}\sum_{\psi(n,m)}\left[\alpha_m^\beta(hL) + \frac{1}{S_t(1-\beta)}\sum_{\psi(n,m)}(hL_{mm} - \alpha_m^\beta(hL))^+\right]\end{array}\right] \quad \text{(A.1)}$$

In (A.1), L_{mm} is the loss function at child of child m. Because VaR has Positive Homogeneity property we can write (A.1) as:

$$\phi_n^\beta(hL) = h\underset{\alpha_n^\beta}{Min}\left[\begin{array}{l} \alpha_n^\beta(L) + \dfrac{1}{S_t(1-\beta)}\displaystyle\sum_{\psi(n,m)}(L_m - \alpha_n^\beta(L))^+ \\ +\dfrac{1}{S_t}\displaystyle\sum_{\psi(n,m)}\left[\alpha_m^\beta(L) + \dfrac{1}{S_t(1-\beta)}\displaystyle\sum_{\psi(n,m)}(L_{mm} - \alpha_m^\beta(L))^+\right] \end{array}\right] \tag{A.2}$$

and it follows :

$$\phi_n^\beta(hL) = h\phi_n^\beta(L)$$

Next, for proving Translation Invariance property we have:

$$\phi_n^\beta(L+h) = \underset{\alpha_n^\beta}{Min}\left[\begin{array}{l} \alpha_n^\beta(L+h) + \dfrac{1}{S_t(1-\beta)}\displaystyle\sum_{\psi(n,m)}(L_m + h - \alpha_n^\beta(L+h))^+ \\ +\dfrac{1}{S_t}\displaystyle\sum_{\psi(n,m)}\left[\alpha_m^\beta(L+h) + \dfrac{1}{S_t(1-\beta)}\displaystyle\sum_{\psi(n,m)}(L_{mm} + h - \alpha_m^\beta(L+h))^+\right] \end{array}\right] \tag{A.3}$$

Because VaR has Translation Invariance property we can write (A.3) as:

$$\phi_n^\beta(L+h) = \underset{\alpha_n^\beta}{Min}\left[\begin{array}{l} \alpha_n^\beta(L) + h + \dfrac{1}{S_t(1-\beta)}\displaystyle\sum_{\psi(n,m)}(L_m + h - \alpha_n^\beta(L) - h)^+ \\ +\dfrac{1}{S_t}\displaystyle\sum_{\psi(n,m)}\left[\alpha_m^\beta(L) + h + \dfrac{1}{S_t(1-\beta)}\displaystyle\sum_{\psi(n,m)}(L_{mm} + h - \alpha_m^\beta(L) - h)^+\right] \end{array}\right] \tag{A.4}$$

And it follows:

$$\phi_n^\beta(L+h) = \phi_n^\beta(L) + 2h$$

And if we consider until the last stage n, we have:

$$\phi_n^\beta(L+h)=\phi_n^\beta(L)+nh \tag{A.5}$$

In (A.5), we can say that by adding h through the stages (time periods), the risk will be higher. Indeed, it increases uncertainty about the future. We can call it time-adjusted Translation Invariance property. So, the equation in (18) satisfies Translation Invariance property.

Next, for proving convexity property, we use discrete static formulation of CVaR, (Rockafellar and Uryasev, 2000), in (A.6).

$$\phi^\beta(\mathrm{X}_i)=\underset{\alpha_i^\beta}{Min}\left[\alpha_i^\beta+\frac{1}{1-\beta}E[\mathrm{X}_i-\alpha_i^\beta]^+\right] \tag{A.6}$$

Let $f(\mathrm{x})=[x-\alpha]^+$. Because, f(x) is a convex function we use it for proving the convexity of CVaR in (24) and we can write its definition by:

$$\forall x_1,x_2\in[a,b],\ \exists\lambda\in[0,1],\ f(\lambda x_1+(1-\lambda)x_2)\le\lambda f(x_1)+(1-\lambda)f(x_2)$$

By replacing $[x-\alpha]^+$ for f(x) we have:

$$[\lambda x_1+(1-\lambda)x_2-\alpha]^+\le\lambda[x_1-\alpha]^++(1-\lambda)[x_2-\alpha]^+ \tag{A.7}$$

We can use the inequality in (A.7) for proving the convexity of the static formulation of CVaR, (Rockafellar and Uryasev, 2000), in (A.7).

$$\begin{aligned}&\phi^\beta(\lambda X_1+(1-\lambda)X_2)=\\&\underset{\lambda\alpha_1^\beta+(1-\lambda)\alpha_2^\beta}{Min}\left[\lambda\alpha_1^\beta+(1-\lambda)\alpha_2^\beta+\frac{1}{1-\beta}E[\lambda X_1+(1-\lambda)X_2-\lambda\alpha_1^\beta-(1-\lambda)\alpha_2^\beta]^+\right]\end{aligned} \tag{A.8}$$

By using property of convexity of $f(x)$ in (A.7) we have:

$$\lambda\alpha_1^\beta + (1-\lambda)\alpha_2^\beta + \frac{1}{1-\beta}E[\lambda X_1 + (1-\lambda)X_2 - \lambda\alpha_1^\beta - (1-\lambda)\alpha_2^\beta]^+$$

$$\leq \lambda\alpha_1^\beta + (1-\lambda)\alpha_2^\beta + \frac{\lambda}{1-\beta}E[X_1 - \alpha_1^\beta]^+ + \frac{1-\lambda}{1-\beta}E[X_2 - \alpha_2^\beta]^+ \tag{A.9}$$

And by sorting the terms in (A.9) we have:

$$\lambda\alpha_1^\beta + (1-\lambda)\alpha_2^\beta + \frac{\lambda}{1-\beta}E[X_1 - \alpha_1^\beta]^+ + \frac{1-\lambda}{1-\beta}E[X_2 - \alpha_2^\beta]^+$$

$$\leq \lambda(\alpha_1^\beta + \frac{1}{1-\beta}E[X_1 - \alpha_1^\beta]^+) + (1-\lambda)(\alpha_2^\beta + \frac{1}{1-\beta}E[X_2 - \alpha_2^\beta]^+)$$

And finally, by using the definition of static CVaR in equation (A.6) we have:

$$\lambda(\alpha_1^\beta + \frac{1}{1-\beta}E[X_1 - \alpha_1^\beta]^+) + (1-\lambda)(\alpha_2^\beta + \frac{1}{1-\beta}E[X_2 - \alpha_2^\beta]^+)$$

$$\leq \lambda\phi^\beta(X_1) + (1-\lambda)\phi^\beta(X_2) \tag{A.10}$$

Equation (A.10) clearly shows that RECVaR satisfies convexity property. As a result, the static part in equation (18) is coherent. Now, for the dynamic part, $E(\sum_{\psi(n,m)} \phi_m) = \frac{1}{S_t}\sum_{\psi(n,m)} \phi_m$, we can say that it is the definition of static RECVaR at child node *m*. Because, we have proved already the static CVaR in parent node *n* is convex, we can conclude that static CVaR at child *m*, inherits the convexity from its parent node. As a result, the equation in the dynamic part is also convex and the summation of two convex functions is also convex. ■

Next, we consider the effect of clustering on CVaR. In Proposition 2, we use the static definition of CVaR for considering the portfolio effect of clustering (Rockafellar and Uryasev, 2000).

Proposition 2:. *Let* $E(\phi_k^\beta)$ *represent the expected RECVaR for each one of k clusters, and* ϕ^β *denote the RECVaR for the combined clusters. Then,*

$$E(\phi_k^\beta) \le \phi^\beta \le \sum_k \phi_k^\beta \quad . \tag{A.11}$$

Proof: Let S_k represent the number of observations in cluster k and $S_1 + S_2 + ... + S_k = S$ in which S is the total number of observations in the combined set. By using static definition of CVaR, (Rockafellar and Uryasev, 2000), we have

$$\phi^\beta = \underset{\alpha^\beta}{Min}\left[\alpha^\beta + \frac{1}{(1-\beta)S}\sum_{j\in S}(L_j - \alpha^\beta)^+\right] \tag{A.12}$$

in which L_j represents the loss function associated with observation j which belongs to set S.

The upper limit of ϕ^β in inequality (A.11) can be obtained directly from subadditivity property of CVaR (Artzner et al., 1999). Then, for the lower limit

$$E(\phi_k^\beta) = \frac{S_1\phi_1^\beta + S_2\phi_2^\beta + ... + S_k\phi_k^\beta}{S_1 + S_2 + ... + S_k}\ .$$

Hence, by using equation (A.12):

$$E(\phi_k^\beta) = \frac{S_1\alpha_1^\beta + \frac{1}{1-\beta}\sum_{j\in s_1}(L_j - \alpha_1^\beta)^+ + ... + S_k\alpha_k^\beta + \frac{1}{1-\beta}\sum_{j\in s_k}(L_j - \alpha_k^\beta)^+}{S_1 + S_2 + ... + S_k}$$

$$= \frac{S_1\alpha_1^\beta + S_2\alpha_2^\beta + ... + S_2\alpha_2^\beta}{S} + \frac{1}{(1-\beta)S}\left[\sum_{j\in s_1}(L_j - \alpha_1^\beta)^+ + ... + \sum_{j\in s_k}(L_j - \alpha_k^\beta)^+\right]$$

from which we derive

$$E(\phi_k^\beta) = E_k(\alpha_k^\beta) + \frac{1}{(1-\beta)S}\left[\sum_{j\in s_1}(L_j - \alpha_1^\beta)^+ + \ldots + \sum_{j\in s_k}(L_j - \alpha_k^\beta)^+\right]. \qquad \text{(A.13)}$$

Now, we consider two separate cases: (a) $\forall k,\ \alpha_k^\beta = \alpha^\beta$, which means that, with clustering the decisions are the same in all the clusters and that the VaR doses not change. As a result, the α_k^β for all the clusters is equal to the α^β of the combined set. Then equation (A.13) can be represented as

$$E(\phi_k^\beta) = \alpha^\beta + \frac{1}{(1-\beta)S}\sum_{j\in S}(L_j - \alpha^\beta)^+ \rightarrow E_k(\phi_k^\beta) = \phi^\beta.$$

(b) $\forall k,\ \alpha_k^\beta \neq \alpha^\beta$, in this case, by clustering we gain more degrees of freedom; therefore, by optimality the loss (cost) at every observation, in each cluster, is less or equal to the cost in the combined cluster. Therefore, α_k^β, in each cluster, is less than or equal to the α^β in the combined cluster and, consequently, the expected value of the CVaR of the clusters is less than CVaR for the combined set. Indeed, if we compare equation (A.12) with (A.13), each term in (A.13) is less than the corresponding term in (A.12). As a result $E(\phi_k^\beta) < \phi^\beta$. By taking into account both cases of (a) and (b) the inequality (A.11) is proved. ■

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