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Wake oscillator equations in modelling vortex-induced vibrations at low mass ratios

Victoria Kurushina and Ekaterina Pavlovskaia
Centre for Applied Dynamics Research
School of Engineering, University of Aberdeen, Aberdeen, Scotland, AB24 3FX
Email: v.kurushina@abdn.ac.uk, e.pavlovskaia@abdn.ac.uk

***Abstract*—The current research explores the capabilities of Rayleigh and Van der Pol equations from the standpoint of accuracy of vortex-induced vibrations (VIV) modelling for low mass ratio cases. The two degree-of-freedom rigid structure model suggested by Postnikov et al. [1] is used as the base case, and the fluid equations are modified to create 7 options as alternatives to this model. The considered options constitute variation in damping terms, including introduction of additional damping coefficients (as different Van der Pol or Rayleigh parameters). Then the calibration is performed to identify the best set of coefficients to provide accurate match with the experimental data. The main aim is to predict correctly the development of the super-upper branch [2]. Experimental results by Stappenbelt and Lalji [3] for mass ratio 2.36 are utilised for the model calibration. Then the obtained models are validated using data from the series of experiments by Stappenbelt and Lalji [3] and published experimental data from other sources [2, 4]. The obtained results demonstrate the advantages of changes in damping terms. Overall, it is concluded that Rayleigh oscillator can be recommended to approximate the lift coefficient for low mass ratios.**

***Keywords*— elastically-mounted cylinder, low mass ratio, wake oscillator model, vortex-induced vibration, calibration of the model**

I. Introduction

Vortex-induced vibrations (VIV) develop as a result of interaction between slender structure and fluid flow. Experienced by risers, free spans of pipelines in water, and by antennas, suspension bridges in the air, they can lead to increased fatigue and shorter service life of the structures. Accurate estimation of the oscillations of such structures due to pressure variations caused by the vortex formations in the flow is the main concern in the design process. To ensure their safe and reliable operations, displacements of the structure, applied fluid forces and probability of failure need to be modelled appropriately.

The highest safety risks appear when the resonance condition (or so-called "lock-in", or synchronization state) develops between the fluid and the structure. In the case of increasing flow velocity, the amplitude of displacement significantly increases at lock-in, reaches its peak value and then reduces back to very small values. This phenomenon is observed when the frequency of vortex formation matches the natural frequency of the structure. It also manifests in peak values of fluid force fluctuations.

Govardhan and Williamson in 2001 [5], Jauvtis and Williamson in 2004 [2] collected the evidence to distinguish development of the resonance in cases with low and high mass ratio. Figure 1a illustrates the displacement amplitudes observed for mass ratio below 6 (low): the start of amplitude increase constitute initial branch, the peak values form super-upper branch, and the resonance decays as a lower branch. Figure 1b represents development of the resonance state for mass ratio higher than 6 (high), characterised by much lower maximum amplitudes of displacement, and smaller jump from the upper to the lower branch. Overall, their series of experiments demonstrate an apparent difference between the shapes of peak tops: low mass ratio provides an "angular" top, with sudden drop from the highest values to the lower branch; and high mass ratio implies a more "circular" top.

This well known classification of cases has a significant meaning not only for experimental branch of VIV studies, but also for modelling. The existing experimental evidence of different development of resonance for low and high mass ratio suggests that it might be essential to apply either different models for these cases, or different modifications of the same model, or different sets of empirical coefficients, instead of one model with the same coefficients. The attempt to develop the most accurate modifications of the same model for low mass ratio is performed by authors of the current study using existing model for 2DOF rigid structure developed by Postnikov et al. [1] as a starting point.

The paper is organised as follows. Section II gives a brief overview of wake oscillator method. Section III illustrates the influence of fluid damping terms on the accuracy of prediction utilising the original model for 2 degrees-of-freedom rigid structure [1], and provides the details of the considered fluid equations. Section IV discusses the conducted calibration for all the suggested versions of the model. Results of the following validation with the data from different experimental set-ups are given in Section V. The main research outcomes are summarised in Section VI.

II. Wake oscillator method

The current study is performed using wake oscillator approach. This semi-empirical method allows to evaluate fluid forces acting on the structure directly without calculating characteristics of the fluid flow around the structure. Phenomenon of VIV is three-dimensional, but most of the early models of this type, like Hartlen and Currie [6], Iwan and Blevins [7] and others, were able to describe 1 degree-of-freedom systems where the only oscillations of the structure in the direction

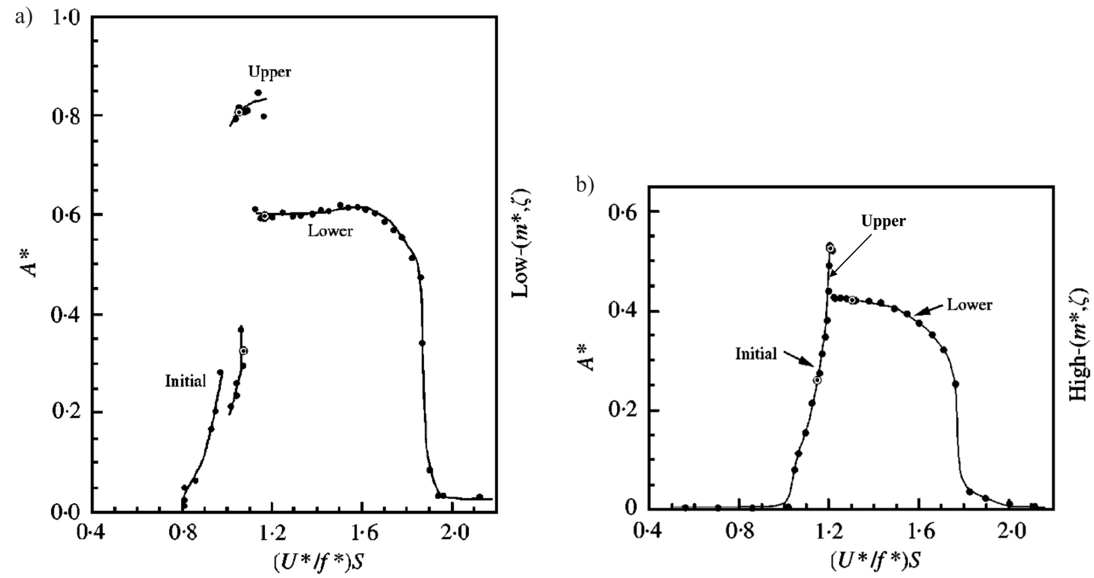

Fig. 1. Development of the resonance state with low (a) and high (b) mass ratio according to Govardhan and Williamson [5].

perpendicular to the flow were considered. Modern models are able to predict behaviour of the structure with 2 or 3 degrees-of-freedom, as it is given in Ge et al. [8], Bai and Qin [9], Zanganeh and Srinil [10], to name a few.

Typical wake oscillator equations are constructed to estimate lift and drag forces F_L and F_D which are functions of their dimensionless parts, or lift coefficient C_L and drag coefficient C_D, and according to Green [11]:

$$F_L = F(\ddot{C_L}, \dot{C_L}, C_L); \quad F_D = F(\ddot{C_D}, \dot{C_D}, C_D). \tag{1}$$

And the fluid forces are commonly defined as:

$$F_L = \frac{1}{2}\rho_f D C_L U^2; \quad F_D = \frac{1}{2}\rho_f D C_D U^2, \tag{2}$$

where ρ_f is the fluid density, D is the structure diameter, and the velocity of the flow is U.

The basic idea of wake oscillator method is that fluctuations of fluid coefficients C_D and C_L can be evaluated using oscillators which can generate self-excited limit cycle, for example, using Rayleigh or Van der Pol equations. These fluctuations are presented in many studies as wake coefficients w and q :

$$w = \frac{2C_D^{fl}}{C_{D0}^{fl}} = \frac{2(C_D - C_{D0})}{C_{D0}^{fl}}; \quad q = \frac{2C_L}{C_{L0}}, \tag{3}$$

where C_{D0}, C_{D0}^{fl} and C_{L0} are drag, fluctuating drag and lift coefficients of stationary structure. Thereby, wake oscillator of Van der Pol type for the transversal direction reads as [12]:

$$\ddot{q} + \varepsilon\Omega_F(q^2 - 1)\dot{q} + \Omega_F^2 q = A_y \ddot{y}. \tag{4}$$

This equation contains the coupling coefficient A_y; transversal acceleration $\ddot{y}$; dimensional vortex shedding frequency Ω_F; and fluid damping (Van der Pol) parameter ε. In wake oscillator method, fluid equation is applied for each direction and they are coupled with the structural equations. Motion of the structure, for example, in transversal direction, contains a transversal projection of the fluid force vector $(\mathbf{F}_{fl})_y$ in the RHS:

$$m\ddot{y} + r_s\dot{y} + hy = (\mathbf{F}_{fl})_y. \tag{5}$$

In this simplified equation of motion of the rigid structure, m_* represents the sum of mass of the structure and mass of the displaced fluid; y is transversal displacement; r_s is structural damping, and h is structural stiffness. Equations for in-line motion of the structure and drag fluctuations are designed in the similar way.

The current research is focused on modification of damping terms in two fluid equations in already developed model for 2 degree-of-freedom rigid structure [1], detailed in the next section.

III. Van der Pol vs. Rayleigh damping

The model by Postnikov et al. [1] was derived for the case of the rigid bare cylinder fixed on elastic supports. The structure is capable to oscillate in the direction of the flow (in-line) and perpendicular to the flow (cross-flow), and, therefore, it has 2 degrees-of-freedom. Thereby, the initial model employs two structural equations for non-dimensional displacements X and Y with combination of the fluid forces in the RHS, and two wake equations. Van der Pol damping is applied in the original model in both in-line and cross-flow fluid equations as terms

TABLE I

WAKE OSCILLATOR EQUATIONS EMPLOYED IN THIS RESEARCH

Option	**Equations**
Van der Pol - Van der Pol (Original model)	$\ddot{w} - 2\varepsilon_x\Omega_R\dot{w} + 2\varepsilon_x\Omega_R\dot{w}w^2 + 4\Omega_R^2 w = A_x\ddot{X};$ $\ddot{q} - \varepsilon_y\Omega_R\dot{q} + \varepsilon_y\Omega_R\dot{q}q^2 + \Omega_R^2 q = A_y\ddot{Y};$
Rayleigh - Rayleigh	$\ddot{w} - 2\varepsilon_x\Omega_R\dot{w} + 2\frac{\varepsilon_x}{\Omega_R}\dot{w}^3 + 4\Omega_R^2 w = A_x\ddot{X};$ $\ddot{q} - \varepsilon_y\Omega_R\dot{q} + \frac{\varepsilon_y}{\Omega_R}\dot{q}^3 + \Omega_R^2 q = A_y\ddot{Y};$
Rayleigh - Van der Pol	$\ddot{w} - 2\varepsilon_x\Omega_R\dot{w} + 2\frac{\varepsilon_x}{\Omega_R}\dot{w}^3 + 4\Omega_R^2 w = A_x\ddot{X};$ $\ddot{q} - \varepsilon_y\Omega_R\dot{q} + \varepsilon_y\Omega_R\dot{q}q^2 + \Omega_R^2 q = A_y\ddot{Y};$
Van der Pol - Rayleigh	$\ddot{w} - 2\varepsilon_x\Omega_R\dot{w} + 2\varepsilon_x\Omega_R\dot{w}w^2 + 4\Omega_R^2 w = A_x\ddot{X};$ $\ddot{q} - \varepsilon_y\Omega_R\dot{q} + \frac{\varepsilon_y}{\Omega_R}\dot{q}^3 + \Omega_R^2 q = A_y\ddot{Y};$
Modified Van der Pol - Modified Van der Pol	$\ddot{w} - 2\varepsilon_{x2}\Omega_R\dot{w} + 2\varepsilon_{x1}\Omega_R\dot{w}w^2 + 4\Omega_R^2 w = A_x\ddot{X};$ $\ddot{q} - \varepsilon_{y2}\Omega_R\dot{q} + \varepsilon_{y1}\Omega_R\dot{q}q^2 + \Omega_R^2 q = A_y\ddot{Y};$
Modified Rayleigh - Modified Rayleigh	$\ddot{w} - 2\varepsilon_{x1}\Omega_R\dot{w} + 2\frac{\varepsilon_{x2}}{\Omega_R}\dot{w}^3 + 4\Omega_R^2 w = A_x\ddot{X};$ $\ddot{q} - \varepsilon_{y1}\Omega_R\dot{q} + \frac{\varepsilon_{y2}}{\Omega_R}\dot{q}^3 + \Omega_R^2 q = A_y\ddot{Y};$
Modified Rayleigh - Modified Van der Pol	$\ddot{w} - 2\varepsilon_{x1}\Omega_R\dot{w} + 2\frac{\varepsilon_{x2}}{\Omega_R}\dot{w}^3 + 4\Omega_R^2 w = A_x\ddot{X};$ $\ddot{q} - \varepsilon_{y2}\Omega_R\dot{q} + \varepsilon_{y1}\Omega_R\dot{q}q^2 + \Omega_R^2 q = A_y\ddot{Y};$
Modified Van der Pol - Modified Rayleigh	$\ddot{w} - 2\varepsilon_{x2}\Omega_R\dot{w} + 2\varepsilon_{x1}\Omega_R\dot{w}w^2 + 4\Omega_R^2 w = A_x\ddot{X};$ $\ddot{q} - \varepsilon_{y1}\Omega_R\dot{q} + \frac{\varepsilon_{y2}}{\Omega_R}\dot{q}^3 + \Omega_R^2 q = A_y\ddot{Y};$

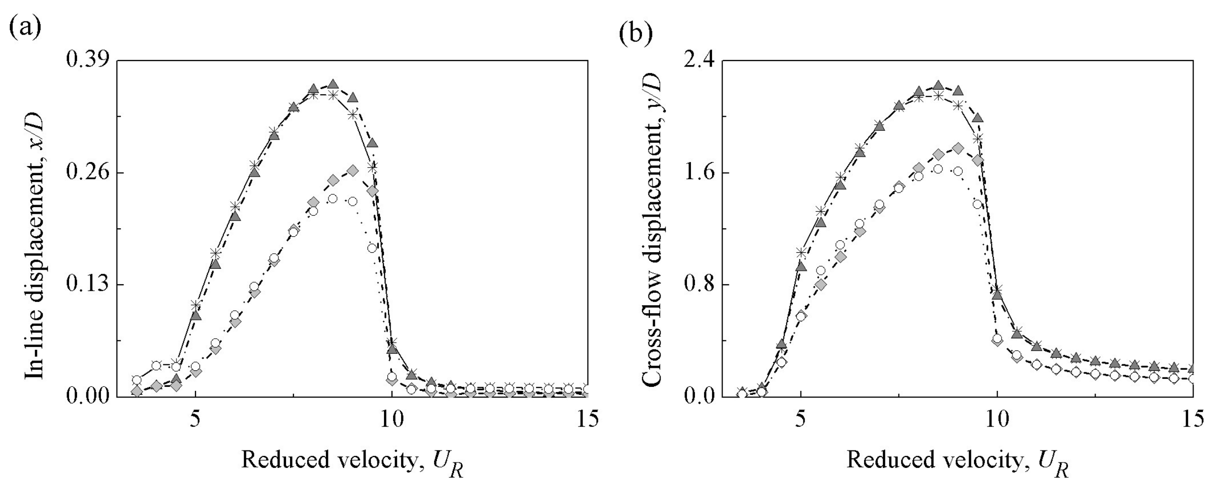

Fig. 2. Influence of the damping terms with the same set of coefficients onto in-line (a) and cross-flow (b) displacements: ∗ - system with Van der Pol in-line and Van der Pol cross-flow equations; ▲ - system with Rayleigh in-line and Van der Pol cross-flow equations; ◆ - system with Rayleigh in-line and Rayleigh cross-flow equations; ○ - system with Van der Pol in-line and Rayleigh cross-flow equations.

TABLE II
SELECTED CALIBRATED VERSIONS OF THE MODEL APPLICABLE AT LOW MASS RATIO (FROM 2 TO 5) ON THE SET-UP BY STAPPENBELT AND LALJI [3] TO PREDICT BOTH IN-LINE AND CROSS-FLOW RESPONSE

Option	C_{L0}	C_{D0}	C_{D0}^{fl}	ε_x	ε_y	A_x	A_y	C_A	K
Van der Pol - Rayleigh	1.87	2.30	0.47	2.1276	0.0644	12.24	3.78	4.06	0.94
Modified Van der Pol - Modified Rayleigh	1.01	2.03	0.22	0.6584, 0.6651	0.1130, 0.0326	11.97	4.51	2.42	1.19
Modified Rayleigh - Modified Rayleigh	0.77	2.12	0.20	0.7270, 0.7046	0.0042, 0.0125	11.95	5.27	1.84	1.04

$2\varepsilon_x\Omega_R(w^2-1)\dot{w}$ and $\varepsilon_y\Omega_R(q^2-1)\dot{q}$. Full basic model in non-dimensional form reads as:

$$\begin{aligned}\ddot{X}+2\zeta\dot{X}+\omega_{st}^2X &= \frac{a}{2\pi St}\Omega_R^2+\frac{b}{4\pi St}\Omega_R^2 w- \\ &\quad -2a\Omega_R\dot{X}+\frac{c}{2}\Omega_R q\dot{Y}+a\pi St\dot{Y}\dot{Y}+ \\ &\quad +2a\pi St\dot{X}\dot{X}-b\Omega_R w\dot{X}; \end{aligned} \tag{6}$$

$$\begin{aligned}\ddot{Y}+2\zeta\dot{Y}+\omega_{st}^2Y &= \frac{c\Omega_R^2}{4\pi St}q-a\Omega_R\dot{Y}+ \\ &\quad +2a\pi St\dot{X}\dot{Y}-\frac{b}{2}w\dot{Y}\Omega_R-cq\dot{X}\Omega_R; \end{aligned} \tag{7}$$

$$\ddot{w}+2\varepsilon_x\Omega_R(w^2-1)\dot{w}+4\Omega_R^2 w = A_x\ddot{X}; \tag{8}$$

$$\ddot{q}+\varepsilon_y\Omega_R(q^2-1)\dot{q}+\Omega_R^2 q = A_y\ddot{Y}. \tag{9}$$

Symbol Ω_R denotes dimensionless vortex shedding frequency; and ζ is damping ratio. The dimensionless coefficients a, b and c depend on the Strouhal number St, which is assumed 0.2 in this research:

$$a=\frac{C_{D0}\rho_f D^2}{4\pi m_* St};\quad b=\frac{C_{D0}^{fl}\rho_f D^2}{4\pi m_* St};\quad c=\frac{C_{L0}\rho_f D^2}{4\pi m_* St}. \tag{10}$$

Original damping terms of Van der Pol type in fluid equations can be modified in the way given in the Table I, forming alternative damping couples. For example, option Rayleigh - Van der Pol from the Table I means that Rayleigh damping is applied in in-line direction, and Van der Pol damping is applied in cross-flow direction. Modified Rayleigh and Modified Van der Pol dampings are suggested by the authors of this research to investigate if there are benefits in separate calibration of fluid damping coefficients $\varepsilon_{x1}, \varepsilon_{x2}, \varepsilon_{y1}, \varepsilon_{y2}$ versus classic $\varepsilon_x, \varepsilon_y$.

The influence of damping terms in the fluid equations on the predicted development of the resonance state in terms of displacement amplitudes in in-line and cross-flow direction is illustrated in Figure 2. This figure shows that when applying the same set of coefficients, Van der Pol damping allows to obtain a more "circular" shape of the top of the resonance peak, while Rayleigh damping provides the peak with a more abrupt drop. This influence can be observed in displacement statistics for both directions.

8 alternative versions of the existing model listed in the Table I are calibrated using the optimization algorithm described in the next section.

IV. CALIBRATION RESULTS

Each modification of the model developed in the previous section is calibrated using constrained nonlinear minimization from Matlab Optimization Toolbox. Identification of the working sets of coefficients is performed utilising the empirical data from [3] for low mass ratio 2.36 in cross-flow direction only. The result for in-line direction is just printed out and evaluated for the error. The displacement amplitudes provided by the model are calculated using the standard deviation of the generated displacement signal. The minimized error between the model prediction and experimental data for cross-flow direction is estimated as a sum of absolute squared differences. The weighting coefficients are applied in calculation of the error to emphasize the importance of correct prediction of the super-upper branch with the abrupt drop from the highest peak value to lower branch.

The resulting best sets of coefficients are listed in the Table II, identified by the least delivered errors in both in-line and cross-flow directions. Parameter K in the Table II constitutes the calibrated shift in terms of reduced velocity of the flow of the point of resonance start. Figure 3 demonstrates the results of calibration of one of the identified options - system of Modified Van der Pol - Modified Rayleigh equations. All options listed in the Table II provide approximately the same high quality of the fit for mass ratio 2.36.

V. VALIDATION RESULTS

After calibration, all the best fit options for mass ratio 2.36 have been tested on different sources of experimental data. It was confirmed, that modifications of the original model provide a decent fit with experimental data from the same set-up [3] as the calibration was performed, and with the data from other experiments conducted by Jauvtis and Williamson [2] and by Blevins and Coughran [4].

Demonstration of the quality of prediction is given on example of the system Modified Van der Pol - Modified Rayleigh from the Table II. Figure 4 shows the results of comparison between the model prediction and actual measurements for mass ratio 3.68 [3] on the same experimental set-up for both in-line and cross-flow direction. Figure 5 provides illustration how the selected modification of the model is performing on the experimental set-up utilised by Jauvtis and Williamson [2] for mass ratio 2.6 for both in-line and cross-flow direction. And Fig. 6 demonstrates the successful fit for cross-flow

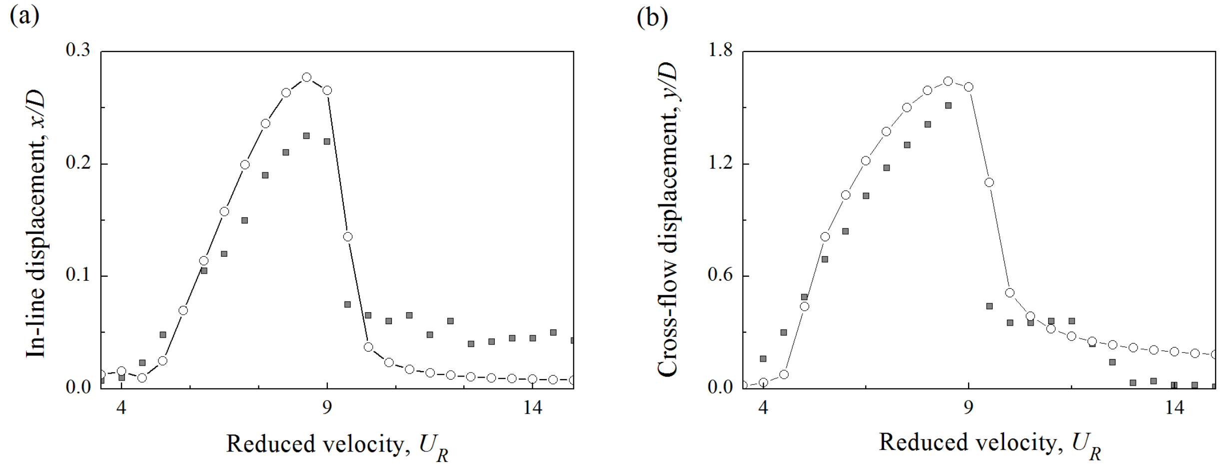

Fig. 3. Modified Van der Pol - Modified Rayleigh system calibrated using the data for mass ratio 2.36 by Stappenbelt and Lalji [3]: a) obtained in-line response; b) calibrated cross-flow response. -○- is the model prediction, and ▪ denotes experimental measurements.

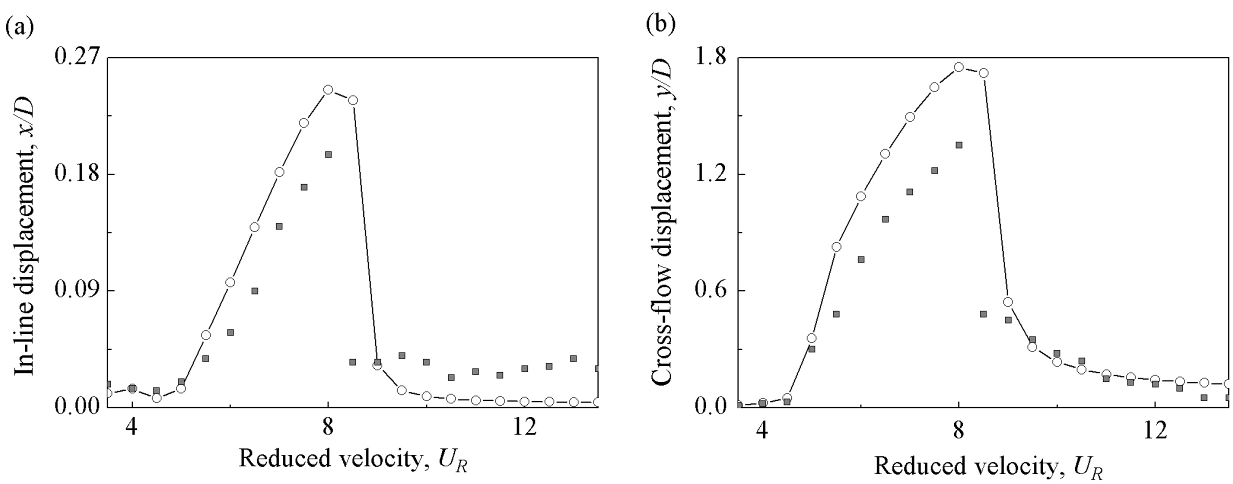

Fig. 4. Obtained in-line (a) and cross-flow (b) response for mass ratio 3.68 from [3] utilising the system of Modified Van der Pol - Modified Rayleigh equations, calibrated with respect to mass ratio 2.36 from [3]. -○- is the model prediction, and ▪ denotes experimental measurements.

direction only with the data collected in experiments by Blevins and Coughran [4] for mass ratio 2.8.

Undertaken verification proves that the borders of application of the selected calibrated modifications of the original model are from mass ratio 2 to approximately mass ratio 5.

VI. Conclusions and future work

The main conclusion from the conducted research is that it is possible to develop a modification of the model for 2 degree-of-freedom rigid structure which will provide accurate prediction throughout different experimental set-ups for cases with mass ratio from 2 to 5 by improving wake equations.

The second outcome is that, for low mass ratio cases, all successful combinations of oscillators, presented in the Table II, contain cross-flow Rayleigh equation in original or modified version. Hence, Rayleigh damping can be recommended as the best option of damping term in cross-flow wake equation to correctly model raise and fall of the super-upper branch.

The next stage of this research is to extended this analysis for a wider range of available oscillators and mass ratios, and explore different calibration algorithms.

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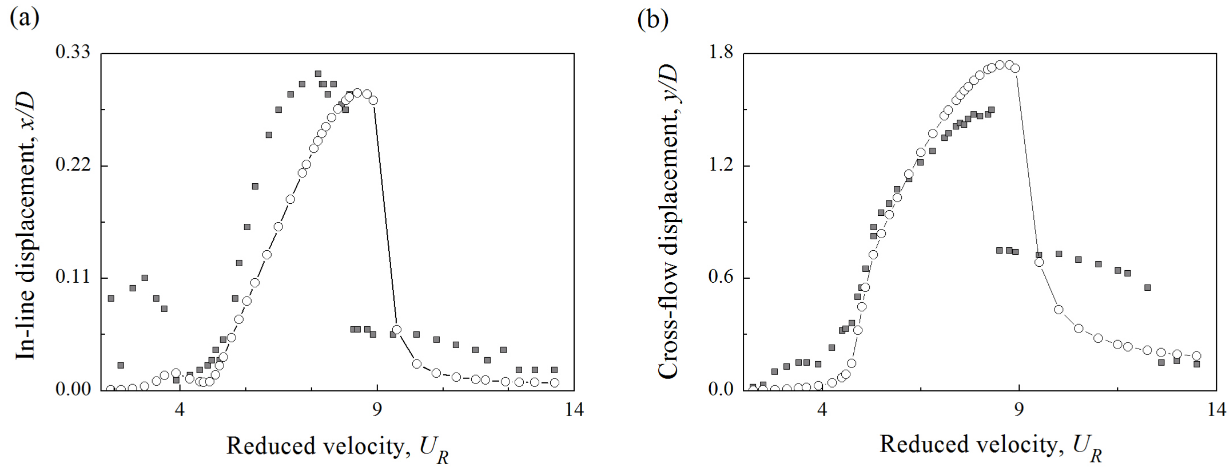

Fig. 5. Comparison of the model prediction and experimental data by Jauvtis and Williamson for mass ratio 2.6 [2] for: a) in-line, and b) cross-flow direction. -○- is the model prediction, and ▪ denotes experimental measurements.

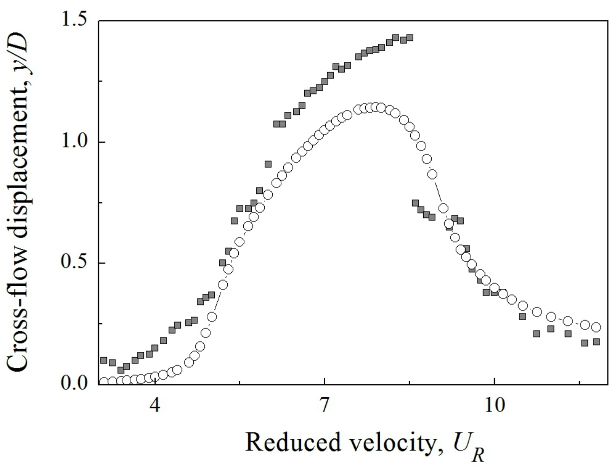

Fig. 6. Comparison of the model prediction and experimental data by Blevins and Coughran for mass ratio 2.8 [4] for cross-flow direction only. -○- is the model prediction, and ▪ denotes experimental measurements.

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