A generalised seasonality test and applications for stock market seasonality.

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A Generalised Seasonality Test and Applications for Stock Market Seasonality

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Abstract

This study develops a novel generalised seasonality test that utilises sequential dummy variable regressions for seasonality periodicity equal to prime numbers. It allows both to test for existence of any seasonal patterns against the broad null hypothesis of no seasonality and to isolate most prominent seasonal cycles while using harmonic mean p-values to control for multiple testing. The proposed test has numerous applications in time series analysis. As an example, it is applied to identify seasonal patterns in 76 national stock markets to detect trading cycles, determine their length, and test the weak-form efficient market hypothesis.

Keywords: seasonality, seasonality test, market efficiency, return seasonality

JEL codes: C22, C58, G14

Introduction

The study of seasonal patterns in time series for economic and financial datasets spans at least a century. Kondratieff (1925) is perhaps the most famous early piece of research on identifying short-, medium-, and long-term cyclical patterns in macroeconomic aggregates. In finance, seasonality has been thoroughly investigated at least since the 1970s, with calendar anomalies such as weekend effect (Cross, 1973), turn-of-the-month effect (Ariel, 1987), holiday effect (Ariel, 1990), and January effect (Keim, 1983) being discovered in stock price movements[1]. International evidence on calendar anomalies has been accumulating steadily, with mixed results from various national stock markets, suggesting seasonality patterns might differ substantially across similar datasets (Gutelkin and Gutelkin, 1983; Cadsby and Ratner, 1992; Kunkel et al., 2003). Most recently, weekly and monthly seasonality has been shown to contribute to abnormal returns of factor portfolios (Zaremba, 2017; Long et al., 2020). Nevertheless, the statistical and econometric tools used by researchers to mine for seasonal effects are still fragmented and largely depend on the pre-assumed cyclical patterns. There has not yet been developed a general procedure that would allow to test for existence of *any* seasonality in data against a broad null hypothesis of no seasonality. For example, to ensure that a stock return time series is not affected by calendar anomalies, a researcher has to undertake a separate test for each of the suspected seasonality types. This is problematic for two reasons: first, it raises multiple testing concerns, and second, the nature of seasonality might differ from the commonly established weekly, monthly, or annual patterns. As such, Alves and Reis (2020) identify half-year and quarterly seasonality in ETF returns that does not correspond to the "classical" calendar anomalies, while Tse (2018) and Alves and Reis (2020) also detect an April effect in foreign exchange and ETF returns, respectively.

[1] For a more detailed survey of calendar anomalies on financial markets, see Shanaev and Ghimire (in press)

This study proposes a generalised seasonality test that does not require any strong presumptions regarding potential seasonal patterns. It utilises sequential regressions with dummy variables representing all possible prime cycle periodicity and uses Wilson (2019) harmonic mean p-values to control for family-wise error rate. Such a design allows for degrees of freedom preservation and near-zero multicollinearity, even when testing for long-term seasonality in reasonably small datasets. The results of the test can be easily visualised and related to the common cyclical patterns (weekly, monthly, or annual) using prime factorisations.

The rest of the paper is organised as follows. In the next section, the testing procedure is outlined and elaborated on. Next, the applicability of the test is showcased on an example of seasonality detection in daily data for 76 national stock market indices over a 5-year period. The final section concludes, suggesting further potential applications of the developed test.

Methodology

To test for seasonality in a time series of n observations one should start with choosing the maximum period $m \leq n$. The number of explanatory dummy variables is calculated as $k = \pi(m)$, where π is the prime-counting function. As $\pi(m) \rightarrow \frac{m}{\ln m}$ for large m, it allows for degrees of freedom preservation while also enabling the test to detect seasonality of cycle periodicity equal to any compound number representable as a product of primes $\pi \leq m$. For example, if $m = 7$, the test will be able to detect any seasonal cycles of lengths 2, 3, 5, 7 and their products. Therefore, cycles of length $3 \times 7 = 21$ and $2 \times 2 \times 3 \times 3 \times 7 = 252$ (typical number of trading days in a month and in a year, respectively) will also be identifiable. Alternatively, one can select $m \rightarrow n$ and test for all possible prime cycle lengths. This study shows both approaches.

Next, the set of explanatory dummy variables for $i \leq n$ and $j \leq k$ is constructed according to the procedure $X_{ij} = I(i \bmod \pi_j \ = 0)$. Therefore, the elements of the jth column are equal to 1 if the observation index i is wholly divided by the prime π_j and 0 otherwise, or, equivalently, for column X_j corresponding to prime π_j, every π_jth entry is a 1 and the rest are zeroes. An example of explanatory dummy variable construction for $n = 15,\ m = 7$, and $k = \pi(7) = 4$ can be seen in Table 1 below. A useful side property of this approach is absence of multicollinearity by design, as any two prime numbers are coprime.

Table 1. Explanatory variable construction (example).

prime number	2	3	5	7
prime index (π_j)	π_1	π_2	π_3	π_4
observation index (i)	X_1	X_2	X_3	X_4
1	0	0	0	0
2	1	0	0	0
3	0	1	0	0
4	1	0	0	0
5	0	0	1	0
6	1	1	0	0
7	0	0	0	1
8	1	0	0	0
9	0	1	0	0
10	1	0	1	0
11	0	0	0	0
12	1	1	0	0
13	0	0	0	0
14	1	0	0	1
15	0	1	1	0

When k is specified, multiple linear regressions are fitted sequentially for j ranging from 1 to $k = \pi(m)$, with the jth regression including explanatory variables X from 1 to j. Therefore, in this example, four regressions will be fitted, each testing for seasonality of cycle periodicity up to π_j. For every regression j, a regular F-test for joint significance is performed and a p-value p_j is calculated from the F-statistic F_j and degrees of freedom $(j, n - j - 1)$. Hence, every test returns an array of k p-values. As the F-tests cannot be assumed independent, the harmonic mean p-value is computed as in Wilson (2019) to control for multiple testing as per the formula:

$$p^* = \frac{k}{\sum_{j=1}^{k} 1/p_j}$$

If the harmonic mean p-value (Wilson p-value) p^* is below the selected threshold, the null hypothesis of no seasonality in the time series must be rejected in favour of the alternative hypothesis that seasonality is present[2]. Then, one can examine individual p_js and isolate the cycle periodicity with lowest p-values to determine which prime number cycles contributed to seasonal patterns the most. In the next section, the test is applied to detect seasonal effects in 76 national stock market indices.

Findings and Discussion

To showcase the applicability of the test, this study seeks to apply it to daily data on 76 country-specific stock market index returns provided by Morgan Stanley Capital International (MSCI) for the five-year observation period 24/12/2014 – 25/12/2019.

The test is applied for $m = 7$ ($k = \pi(7) = 4$) and $m = 499$ ($k = \pi(499) = 95$) to illustrate its ability to successfully capture both short-term and long-term trading cycles. The former arrangement ($m = 7$) allows to determine whether the seasonality is weekly, monthly, annual, or of other form by using prime factorisations of the number of trading days in a month ($21 = 3 \times 7$) and a year ($252 = 2 \times 2 \times 3 \times 3 \times 7$) as well as the fact that a typical week consists of a prime number of trading days (5) itself. The latter setup ($m = 499$) seeks to detect seasonality periodicity directly by observing the dynamics of p_j as 499 is a prime that is close to the number of trading days in two years.

For $m = 7$, the generalised seasonality test output is reported in Table 2 below. The null hypothesis had to be rejected for six out of 76 countries (Bangladesh, Belgium, Denmark,

[2] Monte Carlo simulation with random data has confirmed the harmonic mean p-value procedure successfully controls for family-wise error rate, particularly for $\alpha < 0.2$, consistent with Wilson (2019). Data and code for the Monte Carlo simulation as well as the testing procedure itself is available on request.

Ireland, Israel, and Lithuania). Some individual F-tests for cycle length 2 (notably for Japan, Switzerland, and the United States) returned significant p-values, however the results were not significant after the adjustment for multiple testing. Among the significant results, seasonal patterns varied notably, with Israel being the case for the most short-term trading cycles, while Denmark and Lithuania demonstrated more long-term cyclical behaviour.

Table 2. Generalised seasonality test output for $m = 7$.

Market	Wilson p-value	most significant p-value	most significant cycle	Market	Wilson p-value	most significant p-value	most significant cycle
Argentina	0.9647	0.8766	7	Malaysia	0.8159	0.6552	2
Australia	0.9853	0.9715	2	Mauritius	0.3403	0.2049	3
Austria	0.5254	0.2999	2	Mexico	0.6738	0.4926	3
Bahrain	0.1797	0.1089	3	Morocco	0.3395	0.2088	2
Bangladesh	**0.0053**	**0.0021**	5	Netherlands	0.4375	0.2307	2
Belgium	**0.0720**	**0.0435**	3	New Zealand	0.2454	0.1751	3
Bosnia and Herzegovina	0.4068	0.2925	2	Nigeria	0.5352	0.3676	7
Botswana	0.1536	0.1038	3	Norway	0.6875	0.5996	3
Brazil	0.5841	0.3580	2	Oman	0.6684	0.4622	2
Bulgaria	0.6084	0.5266	3	Pakistan	0.5945	0.4461	2
Canada	0.9611	0.9061	2	Peru	0.6426	0.5349	3
Chile	0.7064	0.5037	5	Philippines	0.5600	0.3587	5
China	0.9567	0.9080	2	Poland	0.7343	0.5377	2
Colombia	0.7840	0.6064	3	Portugal	0.5465	0.4175	5
Croatia	0.6992	0.4566	2	Qatar	0.1677	0.1361	7
Czechia	0.1499	**0.0673**	5	Romania	0.6711	0.5145	2
Denmark	**0.0324**	**0.0192**	7	Russia	0.6829	0.5854	3
Egypt	0.3006	0.1600	2	Saudi Arabia	0.5868	0.4926	7
Estonia	0.5473	0.4536	2	Serbia	0.7634	0.5924	2
Finland	0.2214	0.1430	2	Singapore	0.8756	0.7407	3
France	0.3982	0.2609	2	Slovenia	0.2250	0.1114	5
Germany	0.1733	0.1090	2	South Africa	0.8090	0.5958	2
Greece	0.4127	0.2436	7	South Korea	0.4983	0.2782	5
Hong Kong	0.7827	0.6295	2	Spain	0.4700	0.3582	2
Hungary	0.1147	**0.0524**	5	Sri Lanka	0.3546	0.2021	2
India	0.6118	0.4518	2	Sweden	0.3310	0.1923	2
Indonesia	0.9637	0.9023	7	Switzerland	0.1355	**0.0837**	2
Ireland	**0.0050**	**0.0025**	5	Taiwan	0.3668	0.2069	3
Israel	**0.0206**	**0.0092**	2	Thailand	0.3952	0.2212	2
Italy	0.9242	0.8617	3	Trinidad and Tobago	0.2416	0.1204	5
Jamaica	0.3779	0.1908	7	Tunisia	0.3987	0.2257	2
Japan	0.1323	**0.0619**	2	Turkey	0.5577	0.4091	3
Jordan	0.2809	0.1707	5	Ukraine	0.5484	0.4285	7
Kazakhstan	0.5220	0.2741	7	United Arab Emirates	0.7897	0.7359	2
Kenya	0.7102	0.6465	2	United Kingdom	0.3476	0.3086	3
Kuwait	0.8425	0.8185	3	United States	0.2119	**0.0973**	2
Lebanon	0.7196	0.5049	2	Vietnam	0.9077	0.8542	7
Lithuania	**0.0611**	**0.0184**	7	Zimbabwe	0.3377	0.1684	2

Notes: significant (at 10%) p-values are reported **in bold.**

Next, for results significant as per the Wilson harmonic mean p-value, the individual p-values for cycle lengths can be examined to generate inferences regarding the nature of established seasonality. The significance of a 5-cycle would imply weekly seasonality, similar to Monday and Friday effects (Cross, 1973) or average same weekday dependence (Long et al., 2020). The joint significance of 3-cycle and 7-cycle would signal for monthly seasonal patterns as in turn-of-the-month effect (Ariel, 1987; Lakonishok and Smidt, 1988), or average same day of the month dependence (Zaremba, 2017). Annual effects, including January effect (Keim, 1983), varying average monthly returns throughout the year (Gutelkin and Gutelkin, 1983; Tse, 2018), or holiday effect (Ariel, 1990) would be manifested in significance for 2-cycle, 3-cycle, and 7-cycle simultaneously. Table 3 below relates the significance of individual prime length cycles to prominent seasonal patterns (weekly, monthly, or annual).

Table 3. Seasonal effects detected for $m = 7$.

Market	Seasonality				p-value				
	Overall	Weekly	Monthly	Annual	Wilson	2	3	5	7
Israel	Yes	Yes	Yes	Yes	**0.0206**	**0.0092**	**0.0236**	**0.0344**	**0.0682**
Denmark	Yes	Yes	Yes	No	**0.0324**	0.2105	**0.0409**	**0.0238**	**0.0192**
Ireland	Yes	Yes	Yes	Yes	**0.0050**	**0.0057**	**0.0164**	**0.0025**	**0.0063**
Lithuania	Yes	No	No	No	**0.0611**	0.3379	0.4155	0.1710	**0.0184**
Bangladesh	Yes	Yes	No	No	**0.0053**	0.5489	0.5747	**0.0021**	**0.0038**
Belgium	Yes	Yes	No	No	**0.0720**	0.1089	**0.0435**	**0.0663**	0.1202
Hungary	No	Yes	No	No	0.1147	0.7229	0.3188	**0.0524**	**0.0886**
Czechia	No	Yes	No	No	0.1499	0.9133	0.3420	**0.0673**	0.1281

Notes: significant (at 10%) p-values are reported **in bold.**

Among the six markets with seasonal effects identified to be significant as per the Wilson harmonic mean p-value, Bangladesh and Belgium demonstrate weekly seasonality only, while monthly cycles are also present for Denmark, and annual patterns are manifested in Israel and Ireland. For Lithuania, the nature of seasonality does not fall under either of the three prominent periodicities. Hungary and Czechia have individually significant p-values for some of the estimations, however the result ceases to be significant when controlled for multiple testing.

Table 4. Generalised seasonality test output for $m = 499$.

Market	Wilson p-value	most significant p-value	most significant cycle	Market	Wilson p-value	most significant p-value	most significant cycle
Argentina	0.2103	**0.0479**	59	Malaysia	0.8060	0.4802	43
Australia	0.7922	0.4542	73	Mauritius	0.5008	0.2002	479
Austria	0.5363	0.2794	173	Mexico	0.4831	**0.0841**	491
Bahrain	0.3970	**0.0906**	13	Morocco	0.6651	0.2034	29
Bangladesh	**0.0468**	**0.0021**	5	Netherlands	0.4562	0.1278	401
Belgium	0.2420	**0.0435**	3	New Zealand	0.3500	**0.0520**	491
Bosnia and Herzegovina	**0.0552**	**0.0077**	53	Nigeria	**0.0001**	**0.0000**	389
Botswana	0.2684	**0.0470**	47	Norway	0.6549	0.3464	41
Brazil	0.7862	0.3580	2	Oman	0.7738	0.4622	2
Bulgaria	0.8997	0.5266	3	Pakistan	0.5956	0.2674	89
Canada	0.6755	0.3796	283	Peru	**0.0965**	**0.0225**	359
Chile	0.1710	**0.0250**	89	Philippines	0.4843	0.2706	59
China	0.5583	0.2371	401	Poland	0.6724	0.3677	439
Colombia	0.7531	0.5002	311	Portugal	0.7789	0.4161	11
Croatia	0.2983	0.1140	47	Qatar	0.6365	**0.0974**	13
Czechia	0.4023	**0.0673**	5	Romania	**0.0108**	**0.0011**	353
Denmark	**0.0345**	**0.0017**	19	Russia	0.8624	0.5806	43
Egypt	**0.0000**	**0.0000**	487	Saudi Arabia	0.4439	0.1308	71
Estonia	0.5851	0.3597	167	Serbia	0.8389	0.5472	59
Finland	0.3058	0.1320	281	Singapore	0.2875	**0.0922**	439
France	0.4619	0.1560	401	Slovenia	0.6554	0.1114	5
Germany	0.5033	0.1090	2	South Africa	0.4895	0.1775	331
Greece	0.6927	0.2331	17	South Korea	0.6953	0.2782	5
Hong Kong	0.5456	0.2528	463	Spain	0.3440	0.1935	401
Hungary	0.5506	**0.0524**	5	Sri Lanka	0.4576	0.2021	2
India	0.5519	0.2075	11	Sweden	0.1860	**0.0322**	401
Indonesia	0.7734	0.5473	53	Switzerland	0.3743	0.0837	2
Ireland	**0.0053**	**0.0012**	281	Taiwan	0.7782	0.2069	3
Israel	**0.0535**	**0.0092**	2	Thailand	0.6243	0.2212	2
Italy	0.4187	0.2082	151	Trinidad and Tobago	0.4460	0.1204	5
Jamaica	0.7031	0.1908	7	Tunisia	0.7372	0.2257	2
Japan	0.4870	**0.0619**	2	Turkey	0.8911	0.4091	3
Jordan	**0.0906**	**0.0067**	13	Ukraine	0.7845	0.3629	47
Kazakhstan	0.4857	0.2741	7	United Arab Emirates	0.8012	0.4517	47
Kenya	0.3693	0.1230	131	United Kingdom	0.4670	0.1669	401
Kuwait	0.7690	0.4263	59	United States	0.2525	**0.0584**	283
Lebanon	**0.0000**	**0.0000**	269	Vietnam	0.8055	0.4873	197
Lithuania	**0.0799**	**0.0181**	61	Zimbabwe	0.3516	**0.0288**	79

Notes: significant (at 10%) p-values are reported **in bold.**

Table 4 above reports the generalised seasonality test results for $m = 499$. In this setting, the null hypothesis is rejected for 12 out of 76 markets (Bangladesh, Bosnia and Herzegovina, Denmark, Egypt, Ireland, Israel, Jordan, Lebanon, Lithuania, Nigeria, Peru, and Romania). Bangladesh, Denmark, and Israel persist as markets with strong short-term seasonality, while significant results that were not detected by the test with $m = 7$ stem from more complicated longer-term cyclical patterns on these markets, such as Bosnia and Herzegovina (periodicity of 53), Lebanon, Nigeria, Peru, and Romania (periodicities of longer than one year). For Ireland, long-term seasonality is prominent in addition to short-term effects identified previously.

The results of the generalised seasonality test for large values of m can be visualised with a dynamic p-value graph, which allows to illustrate where p-values for individual cycle periodicities decrease below the significance threshold. As such, Figures 1-6 below show examples of no seasonality (Australia, Figure 1), short-term seasonality (Denmark, Figure 2), medium-term seasonality (Bosnia and Herzegovina, Figure 3), long-term seasonality (Nigeria and Romania, Figures 4 and 5), and both short- and long-term seasonality (Ireland, Figure 6).

The findings of the test have multi-faceted implications. First, testing for general seasonal patterns in stock returns is a powerful test for market efficiency. The generalised seasonality test can thus augment the toolbox of efficiency tests that focus on time series dependence (such as runs test, variance ratio test, BDS test, or Hurst exponent). Second, the proposed test generalises and formalises the testing for the existence of calendar anomalies, while allowing to detect patterns without pre-assuming their structure nearly as strongly as in existing research on seasonality. It simultaneously addresses the multiple testing concerns in a conceptually and computationally simple way and enables to locate cyclicality that deviates from weekly, monthly, or annual structure commonly imposed on the data in the empirical literature. Finally, the test can inform trading strategies and help investors determine trading cycle lengths and exploit calendar anomalies with greater flexibility than usual tests allow. When applied to high-frequency data (e.g., 15-minute candles for most liquid instruments), the test can even assist intraday trading.

Figure 1. An example of no seasonality (Australia).

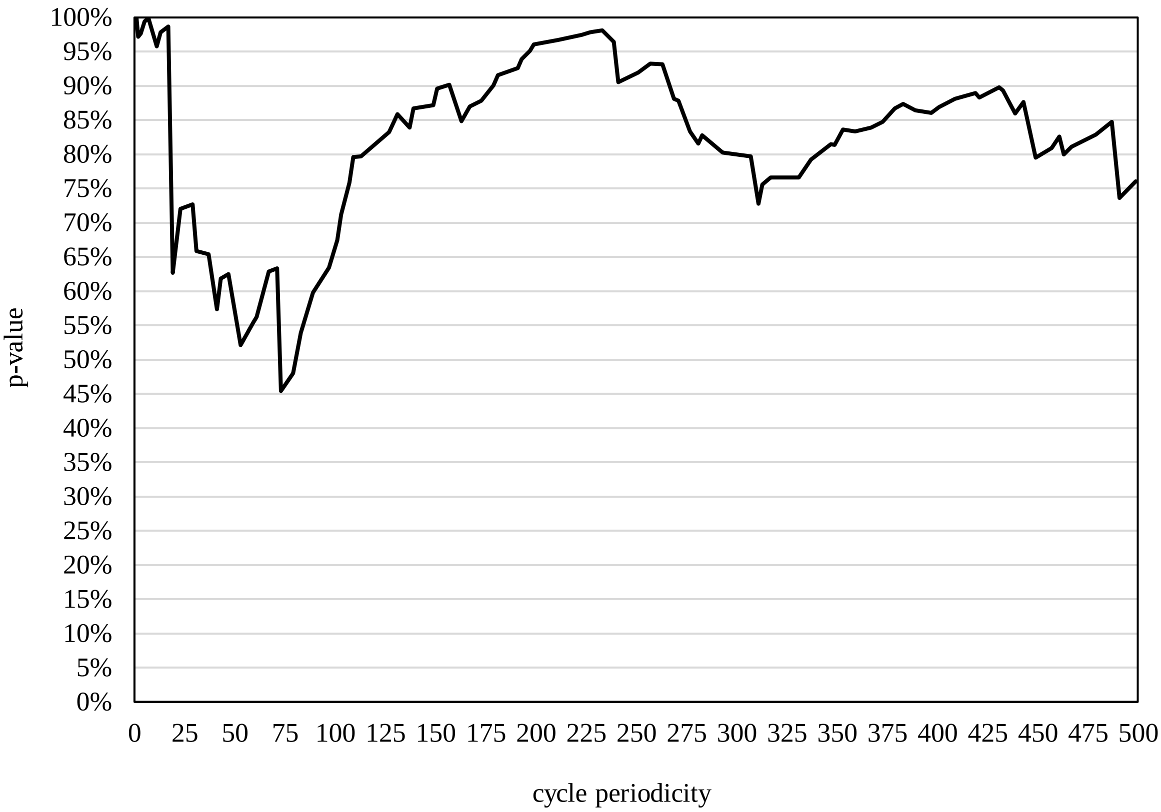


Figure 2. An example of short-term seasonality (Denmark).

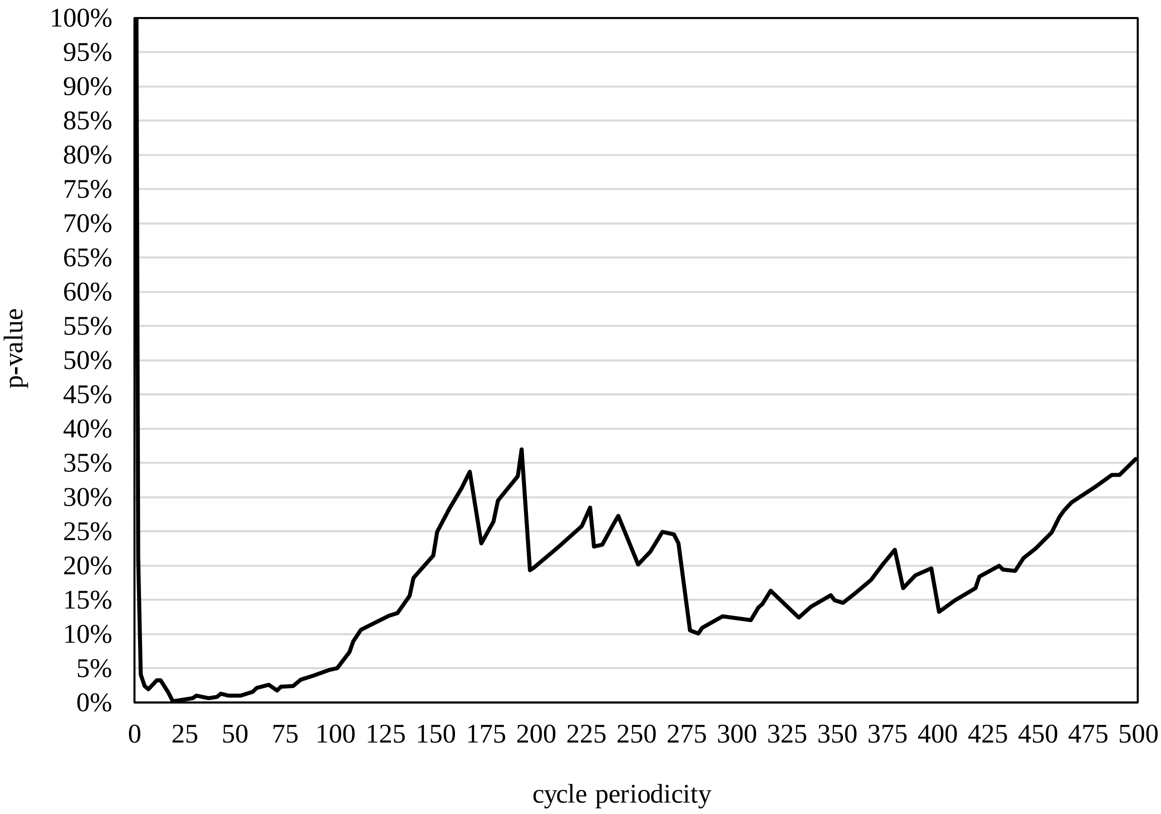


Figure 3. An example of medium-term seasonality (Bosnia and Herzegovina).

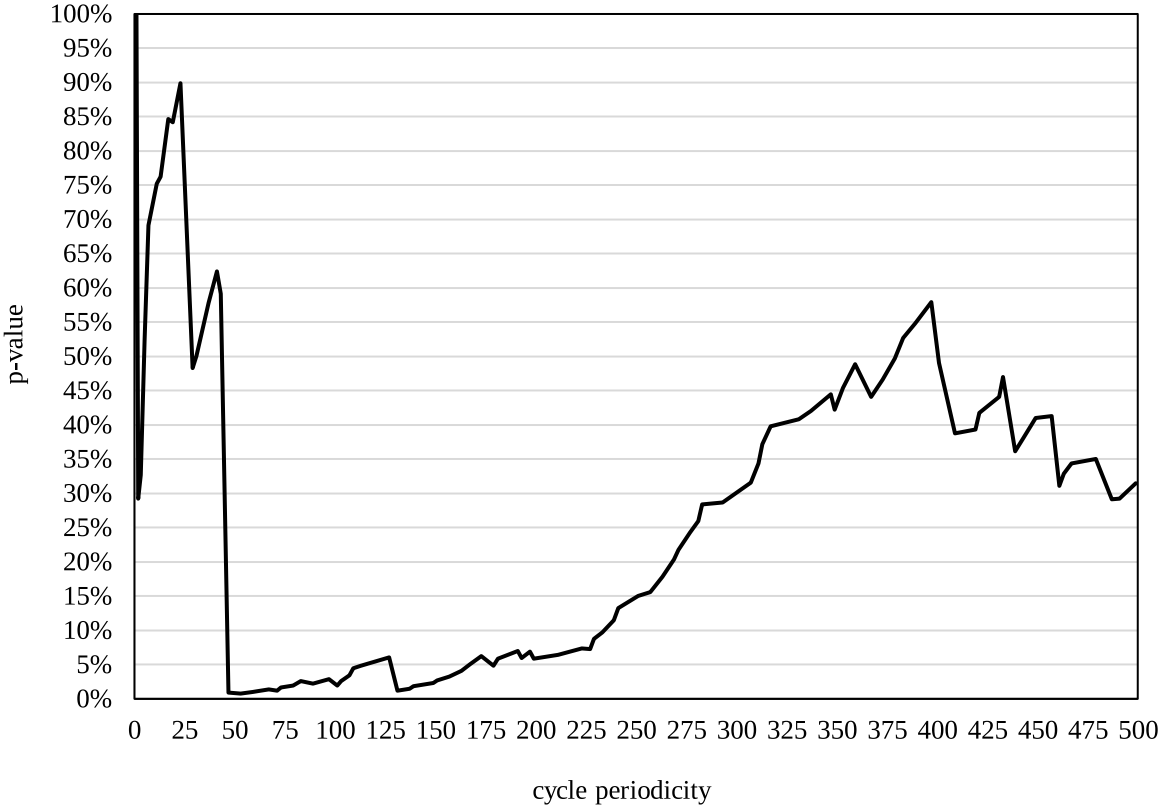


Figure 4. An example of long-term seasonality (Nigeria).

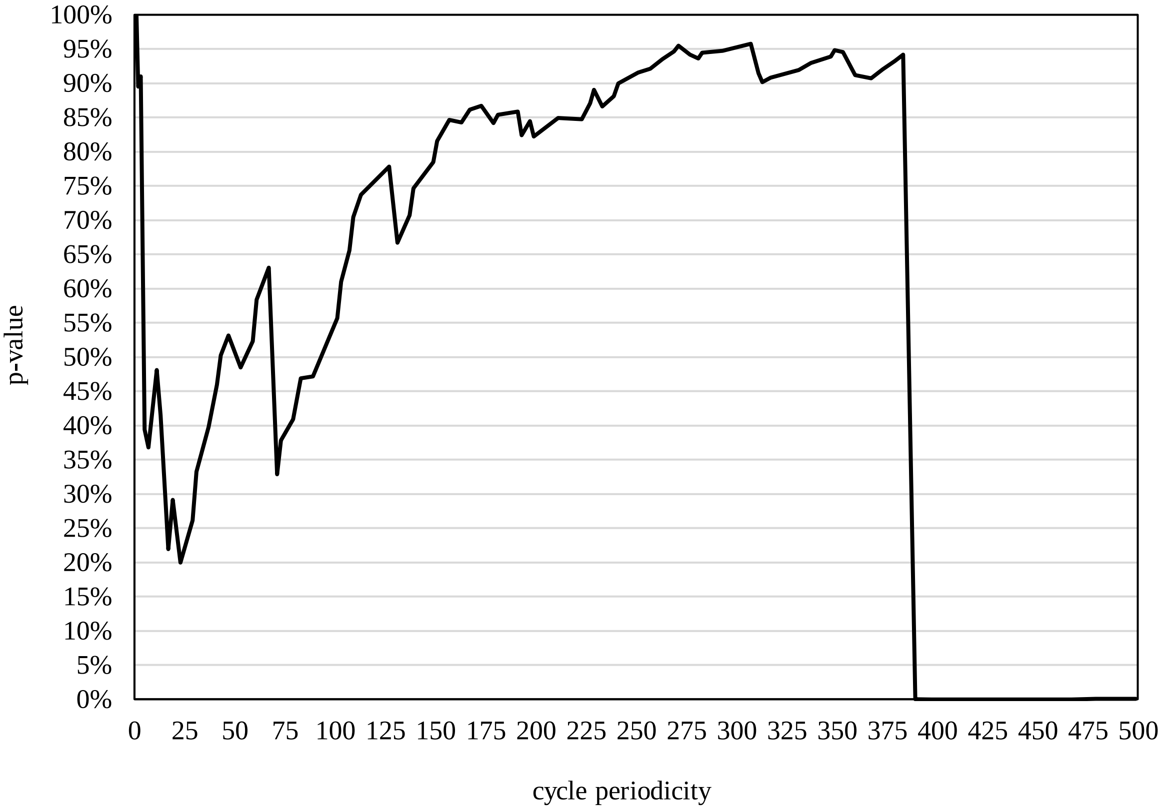


Figure 5. An example of long-term seasonality (Romania).

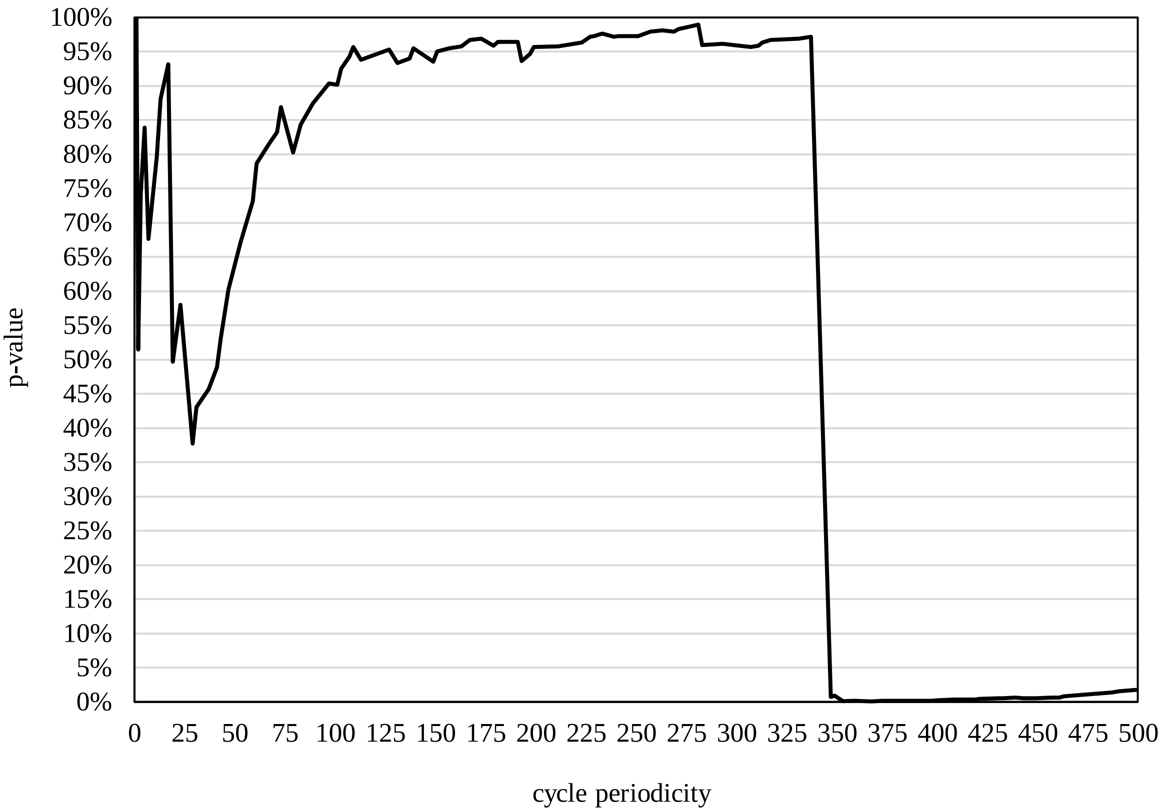
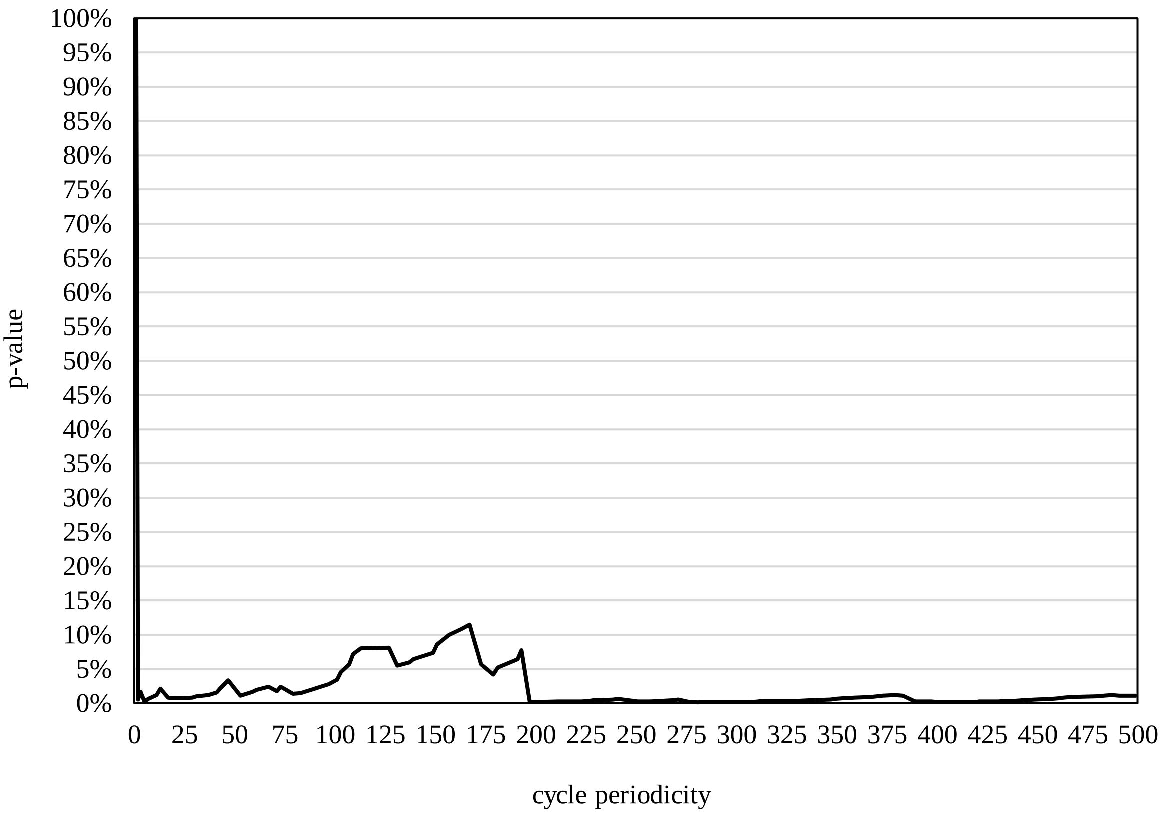


Figure 6. An example of short- and long-term seasonality (Ireland).



Conclusion

The generalised seasonality test developed by this study is a conceptually and computationally simple yet powerful econometric tool that allows to test for existence of any seasonal patterns in time series against the general null hypothesis of no seasonality. It utilises sequential regressions with dummy variables for cycle periodicity equal to prime numbers and applies Wilson (2019) harmonic mean p-value to control for family-wise error rate. Such test design allows to identify long-term cycles in data without substantial degrees of freedom loss or multicollinearity concerns. The study has evidenced the applicability of the test to seasonal effect detection in daily stock returns for 76 national stock market indices over a 5-year period. Potential further applications of the test are numerous. In finance, it can serve as an additional market efficiency test or as a tool for intraday traders and investors exploiting calendar anomalies. In economics, it can be used to detect business cycles or to generate seasonally adjusted data. For machine learning applications, the test can function as a pre-processing tool to identify anomalies or smoothen the data.

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