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Application of Aboutness to Functional Benchmarking in Information Retrieval

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Abstract

Experimental approaches are widely employed to benchmark the performance of an information retrieval (IR) system. Measurements in terms of recall and precision are computed as performance indicators. Although they are good at assessing the retrieval effectiveness of an IR system, they fail to explore deeper aspects such as its underlying functionality and explain why the system shows such performance. Recently, inductive (i.e., theoretical) evaluation of IR systems has been proposed to circumvent the controversies of the experimental methods. Several studies have adopted the inductive approach, but they mostly focus on theoretical modeling of IR properties by using some meta-logic. In this paper, we propose to use inductive evaluation for *functional benchmarking* of IR models as a complement of the traditional experimental based *performance benchmarking*. We define a functional benchmark suite in two stages: (a) the evaluation criteria based on the notion of “aboutness”; and (b) the formal evaluation methodology using the criteria. The proposed benchmark has been successfully applied to evaluate various well-known classical and logic-based IR models. The functional benchmarking results allow us to compare and analyze the functionality of the different IR models.

Categories and Subject Descriptions: [H.1.1] [Models and principles] Systems and Information Theory; [H3.3] [Information Storage and Retrieval] Information Search and Retrieval—*retrieval models; search process; selection process*; [H3.4] [Information Storage and Retrieval] Systems and Software—*performance evaluation (efficiency and effectiveness)*; [H3.m] [Information Storage and Retrieval] Theoretical Study of Information Retrieval

General Terms: Measurement, Performance, Theory

Additional Keywords and Phrases: Functional benchmarking, aboutness, logic-based information retrieval, inductive evaluation

1. Introduction

The information retrieval (IR) problem can be described as a quest to find the set of relevant information objects (i.e., documents, D) corresponding to a given information need, represented by a query, Q . The assumption is that the query Q is a good description of the information need N . An often used premise in IR is the following: if a given document D is *about* the request Q , then there is a high likelihood that D will be relevant with respect to the associated information need. Thus, the information retrieval problem is reduced to deciding the aboutness relation between documents and queries.

Articles on aboutness have appeared sporadically in the literature for more than two decades. Hutchins provides a thoughtful early study of the topic [Hutchins 1977]. This account attempts to define a notion of aboutness in terms of a combination of linguistic and discourse analyses of a text. At a high level of information granularity, e.g. a sentence, Hutchins introduces *themes* and *rhemes* as the carriers of the thematic progression of a text. Roughly speaking, the theme states what the writer intends to express in the sentence (i.e., what it is about), and the rheme is the “new” information. Thematic elements of a sentence are typically bound textually to the preceding text, or assumed as given within the current context. Hutchins also considers how sequences of sentences combine to form textual elements of lower information granularity such as an episode. In other words, sentences are considered to be a part of the micro structure of the text, whereas an episode is considered to be an element of its macro-structure. Themes and rhemes can be generalized to the macro level. Hutchins asserts “The thematic part of the text expresses what the text is ‘about’, while the rheme expresses what the author has to say about it” [Hutchins, 1977, p31].

Maron tackled aboutness by relating it to a probability of satisfaction [Maron 1977]. Three types of aboutness were characterized: S-about, O-about and R-about. S-about (i.e., subjective about) is a relationship between a document and the resulting inner experience of the user. O-about (i.e., objective about) is a relationship between a document and a set of index terms. More specifically, a document D is about a term set T if user X employs T to search for D . R-about purports to be a generalization of O-about to a specific user community (i.e., a class of users). Let t_i be an index term and D be a document, then D is R-about t_i is the ratio between the number of users satisfied with D when using t_i and the number of users satisfied by D . Using this as a point of departure, Maron further constructs a probabilistic model of R-aboutness. The advantage of this is that it leads to an operational definition of aboutness which can then be tested experimentally. However, once the step has been made into the probabilistic framework, it becomes difficult to study properties of aboutness, e.g., how does R-about behave under conjunction? By way of illustration, assume document D is characterized by the index terms K_1, \dots, K_n . From a logical point of view, D can be viewed as being represented by the conjunction $K_1 \wedge \dots \wedge K_n$. Assume that D is R-about index term t_i . One can translate this relationship between a document and term into a relation between the document representation $K_1 \wedge \dots \wedge K_n$ and term t_i (now viewed as an atomic logical formula). What happens to the aboutness relationship if information, represented by the term K_{n+1} is added to document D : Is $K_1 \wedge \dots \wedge K_n \wedge K_{n+1}$ about t_i ? In other words, is aboutness monotonic with respect to the composition of information? Such questions cannot be answered within a probabilistic framework. The underlying problem relates to the fact that

probabilistic independence lacks properties with respect to conjunction and disjunction. In other words, one's hands are largely tied when trying to express qualitative properties of aboutness within a probabilistic setting. (For this reason Dubois et al. [1997] developed a qualitative framework for relevance using possibility theory).

During the eighties and early nineties, the issue of aboutness remained hidden in the operational definitions of various retrieval models and their variations. For example, the vector space model represents both documents and queries as vectors in a high dimensional space whereby the dimensions correspond to information bearing terms. If the angle between the respective document and query vectors is above a certain threshold, the document is deemed to be about the query. This period also featured the emergence of sophisticated probabilistic retrieval models. Major effort was expended in producing ever more sophisticated matching functions between document and query representations. Such matching functions were evaluated by an experimental paradigm. The paradigm often has the following form: Given a set of test queries and a collection of documents, a set of relevant documents are *a priori* associated with each test query. In the actual experiment, a matching function produces a ranked list of documents descending on match score between a test query and a particular document representation. The performance of a matching function can be measured by studying the degree to which relevant documents are moved towards the top of the ranking produced by the matching function under observation. Statistical tests of significance can be applied to compare average performances of two ranking functions across the set of test queries, thus gaining some confidence that matching function A produces, on average, better rankings than matching function B. The experimental paradigm has long been one of the cornerstones of research into information retrieval, but it has long been debated as well. It is outside of the scope of this article to descend into the controversies surrounding experimental information retrieval, but we illustrate one of its manifestations. Many of the more sophisticated matching functions rely on constants. The values of these constants can greatly influence the performance of the matching function. The specific values of the constants are not derived from theory, but are "tuned" according to a particular document collection and test query set.

The emergence of logic-based information retrieval in the mid-eighties allowed the matching function between document and query to be seen in a new light. In one of the founding papers Van Rijsbergen states, "The single primitive operation to aid retrieval is one of uncertain implication" [Van Rijsbergen 1986]. In other words, retrieval could be viewed as a process of plausibly inferring the query from the document. This view spawned a number of attempts at implementing logic-based retrieval systems (see [Lalmas and Bruza 1998] for a survey and [Crestani, Lalmas and Van Rijsbergen 1998] for a compendium). Logic-based information retrieval also provided the framework to allow theoretical, rather than, experimental investigations in IR [Sebastiani 1998]. It planted the seed for fundamental investigations of the nature of aboutness [Bruza and Huibers 1994; Bruza and Huibers 1996; Hunter 1996; Nie *et al.* 1995] culminating in an axiomatic theory of information retrieval [Huibers 1996] and a characterization of aboutness in terms of commonsense rules [Bruza, Song and Wong, 2000]. Aboutness theory has also recently appeared in context of information discovery [Proper and Bruza 1999]. Broadly speaking, these works view information retrieval as a reasoning process, determining aboutness between two information carriers (e.g., document about a query, or document about a document). Work in this area attempted to symbolically characterize qualitative aspects of the

matching function, which, up to that point, were normally hidden in the numeric expressions of these functions. In a broad sense an attempt was made to flesh out the assumptions underpinning matching functions, and more generally to provide a symbolic, IR-centric account of “the most important relationship in IR – the one in which one object contains information *about* another” (italics ours)[Van Rijsbergen 1993]. An important consequence of logic-based information retrieval was that it allowed IR to be studied symbolically within a neutral framework, for example, researchers were free to posit question such as: Is aboutness transitive, or is the aboutness relationship preserved under the composition of information? Once properties of aboutness are described by a set of postulates, they can be used to compare IR models depending on which aboutness postulates they support [Bruza and Huibers 1994; Huibers 1996; Bruza, Song and Wong 2000]. This opens the door to an inductive, rather than, experimental theory of comparing matching functions. The development of an inductive theory of information retrieval evaluation parallels a similar development in the area of nonmonotonic reasoning. Through the nineteen eighties, a number of logics were proposed to model commonsense reasoning, for example, default logic, autoepistemic logic, circumscription etc. At that time, there was no way to compare these different logics until the meta-theory of non-monotonic reasoning appeared [Kraus, Lehman and Magidor 1990]. This theory embodied a suite of desired properties of nonmonotonic logic in terms of rules interpreted in a neutral framework (in this case, preferential models). By using this framework, the previously mentioned logics could be compared according to which properties they supported.

The theoretical analysis and comparison of information retrieval models need not take place within a logic-based framework. Losee provides an analytic theory [Losee 1997; Losee 1998]. He states that a theory of the operation of text filtering and retrieval systems should describe current performance, predict future performance and explain performance. The difference between Losee’s analytical theory and the logic-based inductive theory is more in approach and scope rather than philosophical point of departure. Both aim to gain understanding why particular IR systems perform the way they do. Losee’s analytic theory is statistically based. Measures such as the average search length (ASL - expected position of a relevant document) are used to analyze the quality of a ranking of documents in the context of a hypothesized database. For example, ASL can be plotted against the probability that a given term is in a relevant document yielding a surface. It has been shown that when this probability increases, the ASL steadily and more strongly decreases due to the increase in discrimination power of the terms. This is reflected in the plots by pivoting of the surface away from the median (random) performance of ASL. In this way, the Boolean and probabilistic retrieval models have been scrutinized from a theoretical point of view [Losee 1997]. In contrast to Losee’s analytical theory, the logic-based inductive theory focuses primarily on describing the aboutness properties embodied by a given matching function, and analyzing and comparing matching functions according to which aboutness postulates they support. “*Functional benchmarking*” is the general term coined for such analysis [Song et al. 1999].

The primary objective of this paper is to propose a formal methodology for functional benchmarking and apply it to inductive evaluate and compare various typical IR models. Our evaluation targets in this paper were deliberately chosen to review the practicality of the proposed benchmark. We have evaluated and compared the functionality of the more prominent classical and logical IR models - Boolean, naïve (i.e., zero-threshold and binary) vector space, threshold vector space (multi-valued), probabilistic, situation theory based, naïve (i.e., zero-threshold and binary)

possible world based and threshold possible world based (multi-valued) IR models. The advantages and disadvantages of the properties inherent to these models and how these properties affect effectiveness are analysed. Furthermore, some important experimental results could be explained theoretically via the benchmarking. This will hopefully shed light on existing IR models and help further research towards more effective IR models.

The rest of the paper is organized as follows. In the next section (i.e. Section 2), the definition of the functional benchmark is outlined. The benchmark is based on the aboutness framework proposed by Bruza et al. [Bruza and Huibers 1994; Bruza and Huibers 1996; Bruza, Song and Wong 2000]. A formal functional benchmarking methodology is also proposed in this section. Sections 3 and 4 then present the evaluation of some of classical [Van Rijsbergen 1979; Salton 1988; etc.] and logical IR models [Bruza and Lalmas 1996; Lalmas 1998; Lalmas and Bruza 1998], respectively, using the proposed benchmark. Finally, a conclusion including a summary on the evaluation results is given in Section 5.

2. Defining the Functional Benchmark Suite

Our approach in defining the functional benchmark suite is performed in two stages. (a) We first identify a set of aboutness properties, which will be used to analyze matching functions. They will be used as the evaluation criteria for the functional benchmark. (b) We then define a formal methodology outlining the steps to perform inductive evaluation.

2.1 Properties of Aboutness

Despite several research studies devoted to aboutness, there is still no consensus on the desirable properties of aboutness relation. Nonetheless, a number of properties are commonly discussed in the literature, e.g., reflexivity, transitivity, symmetry, simplification, supraclassicality, equivalence, and, right weakening and left (right) monotonicity [Lalmas and Bruza 1998]. The primary reason for the lack of consensus is the fact that the logic-based framework chosen has some influence on the associated aboutness properties. One would think that reflexivity, i.e., the assumption that an information carrier (such as a document) is about itself, would not generate any difference in opinion. However, reflexivity is a property *not* supported by Hunter’s default logic-based aboutness framework [Hunter 1996], but *is* supported by Huibers’ situation-theoretic framework [Huibers 1996]. In addition, a substantial body of work on defining aboutness properties has been inspired by symbolic characterizations of the preferential entailment relation¹ found in nonmonotonic reasoning. This has slanted the corresponding characterizations of aboutness [Bruza and Huibers 1994; Bruza and Huibers 1996; Amati and Georgatos 1996; Bruza and Van Linder 1998]. Recent work has argued that the aboutness relationship goes beyond the notion of preferential entailment [Bruza, Song and Wong 2000].

The attempts in the literature to characterize the aboutness relationship have been useful to stimulate investigation into what “aboutness” really is without being burdened by the baggage of a particular retrieval model. An unfortunate consequence of this freedom has been a lack of connection with commonly accepted notions of IR

¹ The term “migration” preferentially entails “salmon” if and only if all preferred documents on migration are also about salmon. That is, the user’s information need is assumed to impose a preferential ordering on the set of underlying documents

performance. We argue that aboutness properties selected for the purposes of functional benchmarking should be able to be related to the traditional IR performance criteria: Precision² and Recall³. This allows theoretical insights provided by the inductive evaluation to be correlated with insights gleaned via experimental evaluation.

The inductive evaluation paradigm requires that the aboutness properties be expressed symbolically. This requires that a conceptual framework be established which provides a sufficient diversity of concepts with which useful aboutness properties can be expressed. In this regard, Lalmas and Bruza [1998] have stated: “The framework should not be biased towards any given model, i.e., it should be neutral. Moreover, it should be sufficiently abstract to filter away unnecessary details of the various IR models. In such an abstract and neutral setting, IR models can be inductively compared”.

In this paper, we will employ the framework proposed by Bruza et al. [Bruza and Huibers 1994; Bruza and Huibers 1996; Bruza, Song and Wong 2000]. This framework is abstract and not biased towards any given IR retrieval model, and is parsimonious with respect to the number of underlying concepts. Moreover, it is based on notions drawn from information-based logic. It would seem reasonable to build on research from this area if one accepts that determining whether a document is about query or not, involves an information-based reasoning process.

In the framework, descriptors, documents and queries share the same notion of information carriers. Given two information carriers i and j , the aboutness between i and j , i.e., i is about j , is denoted by a binary relation \models , i.e., $i \models j$. On the other hand, $i \not\models j$ denotes “ i is not about j ”. For example, assuming an animal context, “penguin” is about “birds”, but “penguin” is not about “flying”.

Information carriers can be composed. The composition of information is denoted by $i \oplus j$, which contains the information carried by both i and j . It can be conceived of as a form of informational “meet”. Viewed from a situation-theoretic perspective [Lalmas 1996], the information composition represents the intersection between the situations supporting i and the situations supporting j . For example, $flying \oplus bird$ represents the intersection of “flying” situations and “bird” situations, that is the situations which support the information “A bird is flying”.

Information carriers are ordered. For example, we can say “an information carrier i contains at least the same information that another carrier j does”. In the literature, several authors have proposed that information can be ordered with respect to containment [Barwise and Etchemendy, 1990; Landman, 1986]. Information containment, denoted by $i \rightarrow j$, is a relation over the information carriers formalizing the intuition that information is fundamentally “nested” (see also [Van Rijsbergen 1989]). This nesting may simply be a product of the syntax of the information carriers, e.g., in a Boolean language, $i \wedge j \rightarrow i$. Information containment also embodies how information is sometimes *implicitly* nested. For example, the information conveyed by “salmon” also carries the information “fish”. The former we refer to as *surface containment*, and the latter *deep containment*. In general, information containment (either *surface* or *deep*) will be denoted by the symbol \rightarrow , whereby \rightarrow is the union of the relations surface containment (\xrightarrow{s}) and deep containment (\xrightarrow{d}). It is important to make this distinction as some IR models only support surface containment, whereas others support a notion approximating deep

² Precision is defined as the ratio of relevant retrieved documents to retrieved documents

³ Recall is the ratio of relevant retrieved documents to relevant documents

containment. Moreover, related to the information composition, there are $i \oplus j \rightarrow i$ and $i \oplus j \rightarrow j$.

Information carriers i and j are said to preclude each other, denoted $i \perp j$, if the information carried by i clashes, or contradicts, with the information carried by j . It is acceptable to assume that an information carrier precludes its own negation. However, information preclusion is a subtler notion than contradiction in logic. Information carriers may clash due to underlying natural language semantics, or convention. For example, *swimming* \oplus *crocodile* is acceptable, but *flying* \oplus *crocodile* is meaningless in most contexts. It has also been suggested that information preclusion arises in IR as a consequence of information needs [Bruza and Van Linder, 1998]. For example, when searching for documents about *wind surfing*, terms such as *internet*, *web*, *net* etc. may be precluded as the user is not interested in *web surfing*. In some accounts, (e.g. [Landman, 1986; Bruza and Huibers, 1994]), the composition of clashing information is formalized as the “meaningless” information carrier, denoted by 0. It is attributed with properties similar to *falsum* in propositional logic, e.g. $A \perp B \Leftrightarrow A \oplus B = 0$. The *meaningless information carrier* contains all the information carriers used in an application.

Furthermore, the concept of Information Field is defined. It provides the necessary building blocks to express the properties of aboutness. An Information Field is a structure $(\mathcal{I}, \rightarrow, \oplus, \perp, 0)$ where

- \mathcal{I} is a non-empty set of information carriers
- $(\mathcal{I}, \rightarrow)$ is a poset (partially ordered set)
- $0 \in \mathcal{I}$ and for all $i \in \mathcal{I}$, $0 \rightarrow i$
- If $i, j \in \mathcal{I}$ then $i \oplus j \in \mathcal{I}$, where $i \oplus j$ is the largest information carrier such that $i \oplus j \rightarrow i$ and $i \oplus j \rightarrow j$
- $\perp \subseteq \mathcal{I} \times \mathcal{I}$

A set of postulates⁴ determining the aboutness properties is given in terms of concepts from the Information Field. IR models can be mapped to the aboutness framework. Based on these postulates, the properties they satisfy can be reflected. Moreover, different IR models can be compared according to the postulates they support.

Postulate 1: Reflexivity (R) $i \models i$

An information carrier is about itself.

Postulate 2: Containment (C) $\frac{i \rightarrow j}{i \models j}$

An information carrier is about the information it contains (surface or deep). Deep containment models the transformation of information. For example, assuming that “penguin” has the information “bird” nested within it i.e., $penguin \rightarrow bird$, then the Containment postulate permits the conclusion that “penguin” is about “bird(s)”. As a consequence, a document about “penguin” is also about “bird”. This postulate is recall oriented.

On the other hand, exact match IR models, which attempts to promote precision, can be defined in terms of surface Containment: $D \models Q$ only if $D \xrightarrow{s} Q$. In other

⁴ The notion “postulate” is intended to characterize the assumptions inherent within a given retrieval mechanism with regard to aboutness.

words, document D is not about query Q if D does not include Q (completely). This can be modeled by the following postulate:

Postulate 3: Closed World Aboutness Assumption (CWAA)

$$\frac{i \not\rightarrow^s j}{i \not\models k}$$

If an information carrier i is present in another carrier j , we sometimes infer that i is not about j . Exact match IR models, such as Boolean retrieval, are based on the CWAA. For example, if query Q is not contained in a document D , it is assumed that D is not about Q . CWAA helps improve precision but degrade the recall, because it ignores the partial matching and the possible information transformation, which could establish the aboutness relationship between D and Q . The negative impact of Closed World Assumption has been known for some time [Van Rijsbergen 1986b].

Postulate 4: Right Containment Monotonicity (RCM)

$$\frac{k \models i, i \rightarrow j}{k \models j}$$

This postulate allows transitivity of aboutness relation with respect to information containment. More implicit aboutness relationships can be derived via this postulate. Thus, it is recalled oriented. For example, given a document d is about “penguin” and “penguin” contains the information “bird”, we can conclude that d is also about “bird(s)”. From an IR perspective, RCM models term based query expansion whereby the term i is replaced by the broader term j .

Postulate 5: Left Compositional Monotonicity (LM)

$$\frac{i \models k}{i \oplus j \models k}$$

Postulate 6: Right Compositional Monotonicity (RM)

$$\frac{i \models k}{i \models k \oplus j}$$

LM and RM are used to an underlying assumption of some overlap-based IR models: aboutness is preserved under composition. Therefore, they are recall-oriented postulates and they could negatively affect the precision (see [Bruza, Song and Wong 2000] for an extended discussion on this topic). By way of illustration, consider a document d about “emperor penguins” ($d \models \text{emperor} \oplus \text{penguin}$), so d is also about “penguins” (via RCM: $d \models \text{penguin}$). Right Compositional Monotonicity allows us to compose arbitrary information to the right hand side. Thus, $d \models \text{publisher} \oplus \text{penguin}$ would be permitted, which is an example of an unsound aboutness inference that would lead to a loss of precision in the retrieval mechanism. Query expansion is an example of an IR process that is not monotonic with respect to information composition. The terms selected to expand a query must be carefully chosen. This suggests that a conservatively monotonic process is involved.

The postulates LM and RM can be more clearly related to IR in the following way. LM models the case whereby aboutness is preserved when information j is added to a document:

$$\frac{d \models q}{d \oplus j \models q}$$

A retrieval function satisfying this property makes aboutness judgment insensitive to a document's length. In this way, the issue of document length normalization⁵ can be characterized at the symbolic level.

RM, on the other hand, can be envisaged as query expansion, or any process that attempts to improve a query by composing information to it (e.g., pseudo-relevance feedback [Xu and Croft, 1996]). We have just shown that this is unsound:

$$\frac{d \models q}{d \models q \oplus j}$$

Next, we give some conservative forms of mononicity to constrain how information is composed in various ways in order to promote precision.

Postulate 7: Mix (M)

$$\frac{i \models k, j \models k}{i \oplus j \models k}$$

For example, from “penguin \models bird” and “tweety \models bird”, we can derive “tweety \oplus penguin \models bird”, meaning “penguin” is about “bird(s)”, “tweety” is about a “bird”, so “Tweety, the penguin” is about a “bird”.

Postulate 8: Context-Free And (C-FA)

$$\frac{k \models i, k \models j}{k \models i \oplus j}$$

Boolean retrieval is founded on this postulate. For example, if a document is about “computer software” and the same document is about “computer hardware”, it is also about both “computer software and hardware”.

Postulate 9: Guarded Left Compositional Monotonicity (GLM)

$$\frac{i \models k, i \perp j}{i \oplus j \models k}$$

Postulate 10: Guarded Right Compositional Monotonicity (GRM)

$$\frac{i \models k, k \perp j}{i \models k \oplus j}$$

GLM and GRM are conservative forms of LM and RM. An information carrier can only be composed to another one when no preclusion relationships are violated. For example, suppose “penguin” precludes “flying” and “penguin” is about “bird”. According to GLM, “flying” cannot be composed to “penguin” so that “flying \oplus penguin \models bird” (flying penguin is about a bird) cannot be derived.

Postulate 11: Qualified Left Monotonicity (QLM)

⁵ Document length normalization improves the effectiveness of retrieval; more sophisticated matching functions normalize according to document length.

$$\frac{i \models k, k \perp j}{i \oplus j \models k}$$

Postulate 12: Qualified Right Monotonicity (QRM)

$$\frac{i \models k, i \perp j}{i \models k \oplus j}$$

QLM and QRM are other conservative forms of LM and RM. LM allows “bird \oplus tweety \models flying” (Tweety, which is a bird, is about flying) to be inferred from “bird \models flying” (A bird is about flying). QLM prevents this via the qualifying preclusion “tweety \perp flying”. QRM works in the similar way.

The next postulate expresses a principle based on the preservation of “non-aboutness”.

Postulate 13: Negation Rational (NR)

$$\frac{i \not\models k}{i \not\models k \oplus j}$$

If a document is not about *bird*, it is impossible to be about *flying bird*. This is the intuition behind the postulate NR. Thus it is precision oriented.

The above thirteen postulates could be classified into *recall-oriented* and *precision oriented* according to their effects to IR. Postulate R can be considered a starting point of aboutness inference. Postulates C (deep), RCM, LM, RM and CWA are mainly recall-oriented because they tend to produce more aboutness relations than exact match. Postulates C-FA, M, GLM, GRM, QLM, QRM and NR, on the other hand, intend to prevent undesirable aboutness inferences by employing some kinds of guarded conditions. This is closely related to the conservative monotonicity of IR, which will be discussed later in Section 5. The Containment (surface) postulate characterizes exact match IR models meaning the query must be fully contained in the document.

2.2. Formal Evaluation Methodology

Functional benchmark for IR is based on a formal methodology for inductive evaluation. It is conducted in the following steps:

Step A For each IR model, perform the following:

- (A.1): Define the background of the IR model to be evaluated.
- (A.2): Map the IR model to the aboutness framework. This includes the representations of document, query, aboutness decision, containment, composition, and preclusion.
- (A.3): Inductive evaluation. Determine which aboutness postulates the IR model supports. With respect to an aboutness postulate, the IR model could fall into one of the following four categories:
 - It *fully supports* the postulate.

- It does *not support* the postulate.
- It *conditionally supports* the postulate: The model does not support the postulate in every situation. Under certain conditions, which are determined extraneously, however, it would be supported. In this paper, “conditionally supporting” is applicable to models, which involve settings or estimations outside the models themselves. For example, whether the threshold vector space model, threshold possible world based model and the classical probabilistic model support certain postulates depends on the threshold settings or the estimations. Note that the notion of “Conditionally support” is inapplicable to IR models not involving extraneous factors.
- The postulate is *inapplicable* to the model: Some operators involved in the postulate may be foreign (i.e. inapplicable) to the model. Thus we are unable to evaluate the model using that postulate. For example, the preclusion operator is foreign to the vector space model. This in turn implies that postulates involving the preclusion operator are inapplicable to the vector space model. Practically, this is the same as “not supported”. This category is separated out in order to provide additional information on why a model fails to support the postulate.

Step B Collect the evaluation results of the different IR models and compare their functionality.

In the following sections, we use the above-defined functional benchmarking suite to evaluate and analyze various classical and, logical IR models. We only show the formal proofs of postulates Left Monotonicity (LM) and Right Monotonicity (RM) for illustration. The other postulates can be proven similarly (Refer to [Song 2000] for details).

3. Inductive Evaluation of Classical IR Models

The common classical IR models are the Boolean, vector space, and probabilistic models. In particular, the vector space model is divided into two types, zero-threshold (binary) and threshold (multi-valued) vector space models. The former is referred to as naïve vector space model.

3.1 Boolean Model

3.1.1 Background

The Boolean model is based on set theory and Boolean algebra. This model has been adopted by many early retrieval systems due to its simplicity. In Boolean retrieval, a document D is represented by a set of characterization terms $X(D) = \{t_1, t_2, \dots, t_n\}$, a query Q is expressed in term of index terms combined by Boolean logical connectives AND, OR, and NOT. A document is retrieved if and only if the query Q can be deduced from $X(D)$ according to the following set of inference rules.

Rule 1: if $t_i \in X(D)$ then $X(D) \vdash t_i$, where \vdash denotes the logical consequence.

Rule 2: if $X(D) \vdash t_i$ and $X(D) \vdash t_j$, then $X(D) \vdash t_i \wedge t_j$.

Rule 3: if $X(D) \vdash t_i$ or $X(D) \vdash t_j$, then $X(D) \vdash t_i \vee t_j$.

Rule 4: if $X(D) \not\vdash t_i$ then $X(D) \vdash \neg t_i$.

To generalize, Boolean expressions are assumed to be in CNF (Conjunctive Normal Form) or DNFs (Disjunctive Normal Form), e.g. $(t_1 \vee t_2) \wedge (t_3 \vee t_4) \wedge (t_5 \vee t_6)$.

3.1.2 Boolean Aboutness (\models_{BL})

Let U be the set of all documents, and T be the set of index terms. Let D be a document (i.e., $D \in U$), and Q a query. Suppose $t_i \in T$, $X(D) = \{t_1, t_2, \dots, t_n\}$ denotes the set of characterization terms of D . Let BL_{OR} be the Boolean Language defined on T in DNF of t_i (or $\neg t_i$). Furthermore, let $Q = q_1 \wedge q_2 \wedge \dots \wedge q_m$ be a formula in CNF, where $q_i \in BL_{OR}$, i.e., $q_i = t_{i1} \vee t_{i2} \vee \dots \vee t_{ik}$. Thus, aboutness in the Boolean model is characterized by the following definition:

- $D \models_{BL} Q$ iff $X(D) \vdash Q$ (Aboutness)
 $X(D) \vdash Q$ iff $(\forall q_i) (X(D) \vdash q_i)$
 $X(D) \vdash q_i$ iff $(\exists t_{ij}) (X(D) \vdash t_{ij})$
- If $D \not\models_{BL} Q$ then $D \models_{BL} \neg Q$ (Close World assumption)
- $D \xrightarrow{s} Q$ iff $X(D) \vdash Q$ (Surface Containment)
- Deep Containment is inapplicable.
- Let $Q1 = q_{11} \wedge q_{12} \wedge \dots \wedge q_{1m}$ and $Q2 = q_{21} \wedge q_{22} \wedge \dots \wedge q_{2l}$;
 $Q1 \rightarrow Q2$ iff $Cl(\{q_{11}, q_{12}, \dots, q_{1m}\}) \supseteq \{q_{21}, q_{22}, \dots, q_{2l}\}$ where $Cl(Q1)$ is defined as the set of DNF formulas which are logical consequences of q_{11}, q_{12}, \dots , and q_{1m} .
- $Q1 \oplus Q2 \Leftrightarrow Q1 \wedge Q2$ (Query Composition)
- $D1 \oplus D2 \Leftrightarrow D1 \cup D2$ (Document Composition)
- Suppose D is considered as formula $t_1 \wedge t_2 \wedge \dots \wedge t_n$, then
 $D \perp Q \Leftrightarrow D = \neg Q$ (Preclusion)
- $Q \perp \neg Q$

3.1.3 Inductive Evaluation

Theorem 1 Boolean model supports the Postulates R, C (Surface), C-FA, RCM (Surface), LM, M, GLM, QLM, NR, and CWAA⁶. Deep Containment is inapplicable to this model.

Proofs of LM and RM are shown as below:

- LM: Left Compositional Monotonicity is supported.

⁶ Note that postulates Mix, GLM and QLM are trivially supported, as LM is supported.

$$\begin{aligned}
& \text{Given } D1 \models_{BL} Q \\
& \Rightarrow X(D1) \vdash Q \\
& \Rightarrow X(D1 \oplus D2) = X(D1 \cup D2) \vdash Q \\
& \therefore D1 \oplus D2 \models_{BL} Q
\end{aligned}$$

- RM: Right Compositional Monotonicity is not supported.

$$\begin{aligned}
& \text{Given } D \models_{BL} Q1 \text{ and } Q = Q1 \oplus Q2 \\
& \Rightarrow X(D) \vdash Q1 \text{ and } Q1 \oplus Q2 \Leftrightarrow Q1 \wedge Q2 \\
& \text{But } X(D) \vdash Q1 \wedge Q2 \text{ cannot be concluded} \\
& \therefore D \models_{BL} Q1 \oplus Q2 \text{ cannot be concluded.}
\end{aligned}$$

3.1.4 Remarks

- The Boolean model is an exact match IR model, thereby promoting precision.
- The Boolean model is left monotonic, rendering it insensitive to document length.
- The Boolean model supports the closed world assumption, which would negatively affect recall.
- RM is not supported by the Boolean model. Instead, a conservative form, C-FA, is supported. This would promote precision.

In general, the Boolean model supports a fair degree of precision and weak in recall. Its insensitivity to document length makes it less effective than models whose matching functions support document length normalization.

3.2 Vector Space model

3.2.1 Background

In the Vector Space model, both queries and documents are represented as a vector of weighted or binary index terms. Practically, each index term is treated as an axis in a n-dimensional space. The documents are ranked by the similarity between the document D and the query Q. There are a number of measures of vector similarity, such as Inner product, Dice coefficient, Cosine coefficient, etc. The commonly used form is the cosine function:

$$\text{Cos}(D, Q) = \frac{\sum_i x_i y_i}{\sqrt{\sum_i x_i^2 \times \sum_i y_i^2}} \text{ where } D = \{x_1, x_2, \dots, x_n\}, Q = \{y_1, y_2, \dots, y_n\}.$$

A threshold value is always employed to determine relevance. In the following discussions, we first consider the naïve and simplest case of the model. For this case, the aboutness between D and Q is equivalent to simple overlapping, i.e. if D and Q share at least one index terms, they are about each other. We then investigate the more general case of non-zero multi-valued threshold. Note that the threshold value is extraneously controlled. To simplify, we just consider the case where index terms are un-weighted. The case of weighted terms could be investigated similarly.

3.2.2 Naïve Vector Space Aboutness ($\models_{VS-NAIVE}$)

Let U be the set of all documents, and T be the set of index terms. Let $D \in U$ be a document, and Q a query. Both D and Q are represented as vectors.

- $D = D^+ \cup D^-$
 $D^+ = \{t_1^+, t_2^+, \dots, t_f^+\}$
 $D^- = \{t_1^-, t_2^-, \dots, t_g^-\}$
 $Q = Q^+ \cup Q^-$
 $Q^+ = \{t_1^+, t_2^+, \dots, t_k^+\}$
 $Q^- = \{t_1^-, t_2^-, \dots, t_h^-\}$
 $f+g = k+h = n$ (dimension of the vector).
where $t_i \in T$, t_i^+ is the i -th non-zero term in the vector, and t_j^- is the j -th zero term in the vector.

Based on the above D and Q vectors, the following definitions of naive vector space aboutness are defined:

- $D \models_{VS-NAIVE} Q$ iff $D^+ \cap Q^+ \neq \emptyset$ (Aboutness)
 $D \not\models_{VS-NAIVE} Q$ iff $D^+ \cap Q^+ = \emptyset$
- $D \xrightarrow{s} Q$ iff $D^+ \supseteq Q^+$ (Surface Containment)
 $Q1 \xrightarrow{s} Q2$ iff $Q1^+ \supseteq Q2^+$
- Deep Containment is inapplicable.
- $Q = Q1 \oplus Q2 \Leftrightarrow Q^+ = Q1^+ \cup Q2^+$ and $Q^- = (Q1^- - Q2^+) \cup (Q2^- - Q1^+)$ (Query Composition)
- $D = D1 \oplus D2 \Leftrightarrow D^+ = D1^+ \cup D2^+$ and $D^- = (D1^- - D2^+) \cup (D2^- - D1^+)$ (Document Composition)
- \perp is inapplicable, as it is not supported in the naive vector space model.

3.2.3 Inductive Evaluation

Theorem 2 Naive vector space model supports R, C (surface), C-FA, LM and RM⁷. Deep containment is inapplicable to this model. The postulates GLM, GRM, QLM and QRM are inapplicable, as preclusion is inapplicable.

Proofs of LM and RM are shown as below:

- LM: Left Compositional Monotonicity is supported.
Given $D1 \models_{VS-NAIVE} Q$, $D = D1 \oplus D2$
 $\Rightarrow D1^+ \cap Q^+ \neq \emptyset$, $D = D1 \oplus D2$
 $\Rightarrow (\exists t_i) (t_i \in D1^+ \wedge t_i \in Q^+)$, and
by the definition of composition, $D = D1 \oplus D2 \Leftrightarrow D^+ = D1^+ \cup D2^+$

⁷ Note that postulates Mix is trivially supported, as LM is supported. The postulate C-FA is trivially supported, as RM is supported.

$$\begin{aligned}
&\Rightarrow t_i \in D^+ \text{ and } t_i \in Q^+ \\
&\Rightarrow D^+ \cap Q^+ \neq \emptyset \\
&\therefore D1 \oplus D2 \models_{VS-NAIVE} Q
\end{aligned}$$

- RM: Right Compositional Monotonicity is supported.

$$\begin{aligned}
&\text{Given } D \models_{VS-NAIVE} Q1, Q = Q1 \oplus Q2 \\
&\Rightarrow D^+ \cap Q1^+ \neq \emptyset \text{ and } Q = Q1 \oplus Q2 \\
&\Rightarrow (\exists t_i) (t_i \in D^+ \wedge t_i \in Q1^+), \text{ and} \\
&\quad \text{by the definition of composition, i.e., } Q = Q1 \oplus Q2 \Leftrightarrow Q^+ = Q1^+ \cup Q2^+ \\
&\Rightarrow t_i \in D^+ \text{ and } t_i \in Q^+ \\
&\Rightarrow D^+ \cap Q^+ \neq \emptyset \\
&\Rightarrow D \models_{VS-NAIVE} Q1 \oplus Q2
\end{aligned}$$

3.2.4 Threshold Vector Space Aboutness(\models_{VS-T})

Let U be the set of all documents, and T be the set of index terms. Let $D \in U$ be a document, and Q a query. Both D and Q are represented as vectors. Based on these, the following definitions of threshold vector space aboutness are given:

- $D \models_{VS-T} Q$ iff $COS(D, Q) \geq \partial$, where $\partial \in (0, 1]$. (Aboutness)
- $D \not\models_{VS-T} Q$ iff $COS(D, Q) < \partial$
- The mappings of containment, composition and preclusion are same as those in Section 3.2.2.

3.2.5 Inductive Evaluation

Theorem 3 Threshold vector space model supports R, and conditionally supports C (surface), CWAA, RCM (surface), LM, RM, M, C-FA and NR. Deep containment is inapplicable to this model. The postulates GLM, GRM, QLM and QRM are inapplicable, as preclusion is inapplicable.

The proof of LM and RM are as follows:

- LM: Left Compositional Monotonicity is conditionally supported.
Let $|D1^+| = f1$, $|D2^+| = f2$, $|Q^+| = k$, $|D1^+ \cap Q^+| = c1$, $|D2^+ \cap Q^+| = c2$ and $|D1^+ \cap D2^+| = l$.
Then there are $|Q^+ \cap (D1 \oplus D2)^+| = c1 + c2 - l$ and $|(D1 \oplus D2)^+| = f1 + f2 - l$.
Given $D1 \models_{VS-T} Q$, $D = D1 \oplus D2$

$$\Rightarrow COS(D1, Q) = \frac{c1}{\sqrt{f1 + k}} \geq \partial, D = D1 \oplus D2$$

This cannot imply $COS(D1 \oplus D2, Q) \geq \partial$. Consider the case where $D2^+$ is much larger than $D1^+$. $COS(D1 \oplus D2, Q)$ may be reduced to a very small value, even less than ∂ .

$\therefore D1 \oplus D2 \models_{VS-T} Q$ cannot be guaranteed.

To ensure $D1 \oplus D2 \models_{VS-T} Q$, $COS(D1 \oplus D2, Q) = \frac{c1+c2-l}{\sqrt{(f1+f2-l)+k}}$ must not be less than ∂ .

Thus, given $D1 \models_{VS-T} Q$, i.e. $COS(D1, Q) = \frac{c1}{\sqrt{f1+k}} \geq \partial$, the LM postulate is supported only under the condition of $\partial \leq \frac{c1+c2-l}{\sqrt{(f1+f2-l)+n}}$.

- RM: Right Compositional Monotonicity is conditionally supported.
Let $|D^+| = f$, $|Q1^+| = k1$, $|Q2^+| = k2$, $|D^+ \cap Q1^+| = c1$, $|D^+ \cap Q2^+| = c2$ and $|Q1^+ \cap Q2^+| = l$.
Then there are $|D^+ \cap (Q1^+ \oplus Q2^+)| = c1+c2-l$ and $|((Q1 \oplus Q2)^+)| = k1+k2-l$.
Following the similar way of the proof for LM, we can get the conclusion that, given $D \models_{VS-T} Q1$, i.e. $COS(D, Q1) = \frac{c1}{\sqrt{f+k1}} \geq \partial$, the RM postulate is supported only under the condition of $\partial \leq \frac{c1+c2-l}{\sqrt{f+(k1+k2-l)}}$.

3.2.6 Remarks

- The naïve vector space model is both left and right monotonic. As these properties degrade precision, this model would be imprecise in practice.
- We argue that IR is *conservatively monotonic* in nature, rather than fully monotonic or non-monotonic. Conservative monotonicity means that when new information is composed to either left or right hand side, the aboutness relationship should be preserved only under certain guarding conditions. For example, consider the query expansion process. When a query is expanded using additional terms, the terms added are not arbitrary. They must be chosen carefully, i.e., conservative monotonicity is at work here. In terms of aboutness, such models embody properties such as QLM, QRM, etc. without also supporting LM and RM.
- Threshold vector space model only supports R. The monotonic properties such as LM and RM are conditionally supported depending on the threshold. This means that by adjusting the threshold value, users could adjust the degree of nonmonotonicity. In this way, the threshold vector space model mimics conservative monotonicity by conditionally supporting LM and RM. For example, the condition of the threshold vector space model supporting LM can be conceived in the following terms: Consider a set of terms Q (the query) and the set of terms D (the document). For reasons of clarity, assume that $Q \subset D$. The decision whether $D \models_{VS-T} Q$ holds can be analysed in terms of LM: Starting with Q, terms are composed to Q until the set D has been constructed. Observe that as the number of terms in D increases, the cosine normalization will increase. There will be a point where the cosine between D and Q will become less than the threshold value δ . In other words, LM is more likely to be preserved for short documents, which in a practical sense means that the threshold vector space model

will favour the retrieval of short documents. Observe the nonmonotonicity of the threshold vector space model is not determined by the model itself, but by external settings. This is undesirable from a theoretical point of view.

3.3 Probabilistic Model

3.3.1 Background

In the probabilistic model, the probability of relevance of a document D subjected to a query Q is given by $P(\text{rel}|D)$. To simplify, D is assumed to be a vector-valued random variable (t_1, t_2, \dots, t_n) , and t_1, t_2, \dots, t_n are assumed to be stochastically independent of each other. $P(D)$ is then given by:

$$P(D) = P(D|\text{rel})P(\text{rel}) + P(D|\text{nrel})P(\text{nrel})$$

$P(\text{rel}|D)$ is computed as follows:

$$P(\text{rel}|D) = \frac{P(D|\text{rel})P(\text{rel})}{P(D)}$$

$$P(\text{nrel}|D) = \frac{P(D|\text{nrel})P(\text{nrel})}{P(D)}$$

$$P(D|\text{rel}) = \prod_{i=1}^n P(t_i|\text{rel})^{t_i}$$

$$P(D|\text{nrel}) = \prod_{i=1}^n P(t_i|\text{nrel})^{t_i}$$

$t_i = 0$ iff term i is absent in D

$t_i = 1$ iff term i is present in D

$P(\text{rel})$ and $P(\text{nrel})$ are the priori probabilities of relevance and non-relevance, respectively.

$P(t_i|\text{rel})$ and $P(t_i|\text{nrel})$ could be estimated if we have complete information about the relevant and non-relevant documents in the collection.

The Bayes' Decision Rule is used to make the decision for or against relevance: D is relevant if and only if $P(\text{rel}|D) > P(\text{nrel}|D)$, i.e. $P(D|\text{rel})P(\text{rel}) > P(D|\text{nrel})P(\text{nrel})$. This leads to a discriminant function:

$$g(D) = \frac{P(D|\text{rel})P(\text{rel})}{P(D|\text{nrel})P(\text{nrel})} = \frac{P(\text{rel}) * \prod_{i=1}^n P(t_i|\text{rel})^{t_i}}{P(\text{nrel}) * \prod_{i=1}^n P(t_i|\text{nrel})^{t_i}} . \text{ The document } D \text{ is retrieved}$$

if and only if $g(D) > 1$.

Note that $P(\text{rel})/P(\text{nrel})$ is constant for a given query and document base, and is independent of any particular document.

3.3.2 Probabilistic Aboutness (\models_{PB})

Let U be the set of all documents, and T be the set of index terms. Let $D \in U$ be a document, and Q a query. D is represented as a vector of index terms, as described in the last section. The representation of a query is not specified in the model. In this paper, we just assume the representation of Q is the same as that of D . Based on these, the following definitions of probabilistic aboutness are defined:

- The representations of D and Q are the same as those of the vector space model.
- $D \models_{PB} Q$ iff $g(D) > 1$. (Aboutness)
 $D \not\models_{PB} Q$ iff $g(D) \leq 1$
- The mappings of containment, composition and preclusion are the same as those in Section 3.2.2.

3.3.3 Inductive Evaluation

Theorem 4 Probabilistic model conditionally supports R, C (surface), CWAA, RCM (surface), LM, RM, C-FA, M and NR. Deep containment is inapplicable to this model. The postulates GLM, GRM, QLM and QRM are inapplicable, as preclusion is inapplicable.

The proofs of LM and RM are shown as follows:

- LM is conditionally supported.

Given $D1 \models_{PB} Q$, $D = D1 \oplus D2$

$$\Rightarrow g(D1) = \frac{P(rel) * \prod_{i=1}^n P(t_i | rel)^{t_i}}{P(nrel) * \prod_{i=1}^n P(t_i | nrel)^{t_i}} > 1 \text{ with respect to } Q1, D^+ = D1^+ \cup D2^+$$

Suppose the terms $\{t_j, \dots, t_k\}$ in D^+ but not in $D1^+$

$$\Rightarrow g(D) = g(D1) \times \frac{\prod_{i=j}^k P(t_i | rel)}{\prod_{i=j}^k P(t_i | nrel)}$$

Whether $g(D) > 1$ depends on $\frac{\prod_{i=j}^k P(t_i | rel)}{\prod_{i=j}^k P(t_i | nrel)}$. Only if the new composed terms

from $D2$ have higher probability of occurring in the relevant set than the non-relevant set, then LM is supported (i.e. $g(D) > 1$).

- RM is conditionally supported.

Given $D \models_{PB} Q1$

$$\Rightarrow g(D) = \frac{P(rel) * \prod_{i=1}^n P(t_i | rel)^{t_i}}{P(nrel) * \prod_{i=1}^n P(t_i | nrel)^{t_i}} > 1 \text{ with respect to } Q1,$$

With respect to $Q1 \oplus Q2$, however, the above estimations may change. Thus $g(D) > 1$ could not be guaranteed any more.

Therefore, with respect to $Q1 \oplus Q2$, only when the estimations of the priori probability of relevance and the probability of index terms in D occurring in the relevant set are stronger than those of non-relevance, $g(D) > 1$ could be obtained.

3.3.4 Remarks

- The classical probabilistic model conditionally supports R, LM and RM. This shows that it is fully nonmonotonic. The non-monotonicity is achieved by the estimation of relevance and non-relevance and the probability of occurrence of index terms in the relevant and non-relevant sets via a training process. This leads to good performance for the probabilistic model in practice.
- The properties supported by the threshold vector space and probabilistic models are almost the same. These models are generally most effective in practice. The key here is that LM, RM are conditionally supported (i.e. they mimic conservative monotonicity). For example, the condition of probabilistic model supporting RM is that new terms composed to a document must have higher probability of occurrence in the relevant set than the non-relevant set. This is consistent with the nature of conservative monotonicity.
- The advantage of probabilistic model over threshold vector space model is that its decision rule is included within the model, while the threshold value in the threshold vector space model is not determined by itself. On the other hand, however, the probabilistic model does not directly deal with the matching between documents and queries. Instead, as we have shown in the proofs of its properties, the estimations are conducted on the whole document set with respect to a query. Moreover, the model itself does not specify the criteria of the estimation. This means it may vary from one query to another. This explains why the probabilistic model does not fully support R (i.e. even if a document is identical to query, the probabilistic model could not determine that they are relevant).

3.4 Discussion of Extended Boolean and Inference Network Models

A well-known alternative Boolean model is the extended Boolean model [Salton 1988], also called p -norm model. On the other hand, the inference network model [Turtle and Croft 1992] is an alternative probabilistic model. Both of them can simulate from conventional Boolean model to inner-product vector space model by tuning certain parameters between their top and bottom margins (e.g. $1 \leq p \leq \infty$ for the extended Boolean model; $n \leq c \leq \infty$ for the inference network model, where n is the number of parents at a given node in the inference network). It has been proven by Turtle and Croft [Turtle and Croft 1992] that when the extended Boolean and inference network models are adjusted to simulate Boolean and inner-product vector space models respectively, they produce the same results. They are similar to each

other when they produce the intermediate systems between Boolean and inner-product vector space models for $1 < p < \infty$ and $n < c < \infty$ respectively. For this reason, we only give the detailed discussion on the extended Boolean model in this paper. The inference network model can be analyzed similarly. Moreover, the treatment of this model is a bit different from the others. We focus on showing how the most important property, left and right monotonicity, of the extended Boolean model changes from Boolean to vector space models with the change of p -value.

The extended Boolean model [Salton 1988] provides term weighting and ranking of the answer set. The similarity between a document and a query is adjusted by a special parameter, namely p -value. Different p -values lead to different document output values. In this model, a query is the conjunction or disjunction of n terms, and a document is represented as a vector $D = (t_1, t_2, \dots, t_n)$. For the purpose of this paper, we assume terms in the query are binary. The similarity between a document and a query is given by:

$$Sim(D, Q_{and}) = 1 - \left[\frac{(1-t_1)^p + (1-t_2)^p + \dots + (1-t_n)^p}{n} \right]^{\frac{1}{p}}$$

$$Sim(D, Q_{or}) = \left[\frac{t_1^p + t_2^p + \dots + t_n^p}{n} \right]^{\frac{1}{p}}, \text{ where } 1 \leq p \leq \infty.$$

When $p = \infty$, the extended Boolean model simulates normal Boolean logic, i.e. $sim(D, Q_{and}) = \min(t_i)$ and $sim(D, Q_{or}) = \max(t_i)$; For $p = 1$, it behaves like a simple normalized inner-product vector space model, i.e. $sim(D, Q_{and}) = sim(D, Q_{or}) = \frac{\sum t_i}{n}$.

For intermediate p -values, this model generates “soft” Boolean systems whose properties are between the Boolean and vector space models. We then show this by analyzing how the monotonicity of extended Boolean model changes from Boolean to inner-product vector models with respect to the p -value. We first define the extended Boolean aboutness ($=_{EB}$) as below:

- $D =_{EB} Q$ iff $sim(D, Q) \geq \partial$, where $\partial \in (0, 1]$.

We suppose the query is represented in Conjunction Normalized Form (CNF). To simplify the analysis, we use the representation of $sim(D, Q_{and})$ for the computing of complex queries in CNF, since both $sim(D, Q_{or})$ and d_i are in the interval $[0, 1]$. Information composition (\oplus) between two queries are modeled as logical AND, while composition between two documents is modeled as $D = D1 \oplus D2 \Leftrightarrow D^+ = D1^+ \cup D2^+$. The left and right monotonicity of extended boolean aboutness can then be analyzed:

- Left Monotonicity is supported:

Given $DI \models_{EB} Q$

$$\Rightarrow \text{Sim}(DI, Q) = 1 - \left[\frac{(1-t_1)^p + (1-t_2)^p + \dots + (1-t_n)^p}{n} \right]^{\frac{1}{p}} \geq \partial$$

$D = DI \oplus D2 \Leftrightarrow D^+ = D_1^+ \cup D_2^+$; Suppose $D = (t'_1, t'_2, \dots, t'_n)$

$$\Rightarrow \text{Sim}(D, Q) = 1 - \left[\frac{(1-t'_1)^p + (1-t'_2)^p + \dots + (1-t'_n)^p}{n} \right]^{\frac{1}{p}} \geq \text{Sim}(DI, Q) \geq \partial$$

$\Rightarrow D \models_{EB} Q$

The above proof shows that the extended Boolean model is left monotonic no matter what the p-value is. This is consistent with the conventional Boolean model (see Section 3.1). Compared with the threshold vector space model using the cosine function (see Section 3.2.5), which conditionally supports left monotonicity, the similarity function of extended Boolean model is normalized using only the query terms, without considering the expansion of document space. Thus, it is not as effective as cosine vector space system with respect to left monotonicity. That is, it remains insensitive to document length.

- Right Monotonicity:

Given $D \models_{EB} Q1$

$$\Rightarrow \text{sim}(D, Q1) = 1 - \left[\frac{(1-t_1)^p + (1-t_2)^p + \dots + (1-t_n)^p}{n} \right]^{\frac{1}{p}} \geq \partial$$

Suppose $Q2$ is a conjunction of k components.

$$\Rightarrow \text{sim}(D, Q1 \oplus Q2) = 1 - \left[\frac{(1-t_1)^p + (1-t_2)^p + \dots + (1-t_n)^p + \dots + (1-t_{n+k})^p}{n+k} \right]^{\frac{1}{p}}$$

It is not necessary that $\text{sim}(D, Q1 \oplus Q2) \geq \partial$. Thus, RM is conditionally supported depending on the values of p and ∂ .

Now, let's consider how the change of p leads to the change of the degree of right monotonicity of the model. Suppose $\text{sim}(D, Q1 \oplus Q2) < \partial$. p being increased

implies $1/p$ being decreased. Due to $\left[\frac{(1-t_1)^p + (1-t_2)^p + \dots + (1-t_n)^p}{n} \right] \leq 1$,

$\left[\frac{(1-t_1)^p + (1-t_2)^p + \dots + (1-t_n)^p}{n} \right]^{\frac{1}{p}}$ would be increased and in turn

$1 - \left[\frac{(1-t_1)^p + (1-t_2)^p + \dots + (1-t_n)^p}{n} \right]^{\frac{1}{p}}$ should be decreased. Thus, larger p

implies larger distance between $\text{sim}(D, Q1 \oplus Q2)$ and ∂ , i.e. higher degree of right non-monotonicity. For $p = \infty$ and binary document terms, the extended Boolean

model reduces to conventional Boolean model, which has the highest degree of non-monotonicity (i.e. right monotonicity is not supported (see Section 3.1)). Only if all the new terms composed to the query are true in the document, the original aboutness relation can be preserved. This condition is too strict, i.e. many documents even with high possibility of relevance could not be retrieved. For $p = [1, \infty)$, smooth decrease of p means smooth decrease of the degree of non-monotonicity. When p is reduced to 1, the extended Boolean model becomes the inner-product vector space model, which has the most relaxed condition for conditionally supporting right monotonicity. As a consequence, this model would not be ideal for supporting query expansion, or pseudo-relevance feedback. Following this way, the other aboutness properties can be analyzed similarly.

3.5 Summary

In summary, the probabilistic model has potentially the highest degree of precision, followed by the threshold vector space model, then the Boolean model and the naïve vector space model. This conclusion is consistent with the experimental results. The motivation for this judgment lies in the varying degrees to which they respectively support (or do not support) conservative monotonicity.

4. Inductive Evaluation of Logical IR Models

In the past decade, a number of logic based IR models have been proposed (see [Bruza and Lalmas 1996; Lalmas 1998; Lalmas and Bruza 1998] for detailed surveys). These models can be generally classified into three types: Situation Theory based, Possible World based, and other types. In what follows, we investigate two well-known logic IR models.

In the following analyses, the fact of a document D consisting of information carrier i is represented by $D \rightsquigarrow i$. For example, Guarded Left Compositional Monotonicity (i.e., postulate 7) means that if *a document consisting of i is about k (i.e. $i \models k$), under the guarded condition that i doesn't preclude j ($i \perp j$), we can conclude that *a document consisting of $i \oplus j$ is about k ($i \oplus j \models k$)*. In the following benchmarking exercise, we adopt this interpretation for logical IR models for reasons of simplicity. For the classical models, we treated the document and the query as information carriers directly, for there are no term semantic relationships involved in classical models.*

4.1 Situation theory based model

4.1.1 Background

Van Rijsbergen and Lalmas developed a situation theory based model [Lalmas 1996; Van Rijsbergen and Lalmas 1996]. In their model, a document and the information it contains are modeled as a situation and types. A situation s supports the type ϕ , denoted by $s \models \phi$, means that ϕ is a part of the information content of the situation. The flow of information is modeled by constraints (\rightarrow). Here, we assume $\phi \rightarrow \phi$. A query is one type (single type query) or a set of types (complex query), e.g., a query $\phi = \{\phi, \psi\}$.

For a situation s and a set of types ϕ , there are two methods to determine whether d supports ϕ . The first is that d supports ϕ if and only if s supports ϕ for all types $\phi \in \phi$ [Barwise 1989]. Later Lalmas relaxed the condition to represent partial relevance: any situation supports ϕ if it supports at least one type in ϕ [Lalmas 1996].

IR system is to determine to which extent a document d supports the query ϕ , denoted by $d|\models\phi$. If $d|\models\phi$, then the document is relevant to the query with certainty. Otherwise, constraints from the knowledge set will be used to find the flow that lead to the information ϕ . The uncertainty attached to this flow is used to compute the degree of relevance.

A channel is to link situations. The flow of information circulates in the channel, where the combination of constraints in sequence $(c_1; c_2)$ and in parallel $(c_1 || c_2)$ can be represented. Given two situations $s1, s2$, $s1| \xrightarrow{c} s2$ means that $s1$ contains the information about $s2$ due to the existence of the channel c . A channel c supports constraint $\phi \rightarrow \psi$, denoted $c|\models\phi \rightarrow \psi$, if and only if for all situations $s1$ and $s2$, if $s1|\models\phi$, $s1| \rightarrow s2$, and $\phi \rightarrow \psi$, then $s2|\models\psi$. The notation $s1|\models\phi | \xrightarrow{c} s2|\models\psi$ stands for $c|\models\phi \rightarrow \psi$ and $s1| \rightarrow s2$, which means that $s1|\models\phi$ carries the information that $s2|\models\psi$, due to channel c . If $s1|\models\phi | \xrightarrow{c} s2|\models\psi$ and $s1=s2$, then c is replaced by a special channel 1 , and ϕ logically entails ψ .

4.1.2 Situation Theory Based Aboutness ($|\models_{ST}$)

Let U be the set of documents, S be the set of situations, T be the set of types, C be the set of channels. Furthermore, let $D \in U$ be a document, and Q a query. Then,

- D is modeled as a situation.
- Q is modeled as a set of types
- Given two set of types $\phi1$ and $\phi2$:
 - $D \rightsquigarrow \phi1$ iff $(\forall \phi \in \phi1)(D|\models\phi)$.
 - $\phi1 |\models_{ST} \phi2$ iff $(\exists c \in C) (\forall D/D \rightsquigarrow \phi1) (\exists \phi \in \phi1) (\exists \psi \in \phi2) (D |\models\phi | \xrightarrow{c} D' |\models\psi)$. Note that D' could be D itself, i.e. $c=1$. A more special case is $D |\models\psi | \xrightarrow{1} D |\models\psi$. (Aboutness)
 - $\phi1 \not\models_{ST} \phi2$ iff $(\exists c \in C) (\forall D/D \rightsquigarrow \phi1) (\exists \phi \in \phi1) (\exists \psi \in \phi2) (D |\models\phi | \xrightarrow{c} D' |\models\psi)$.
 - $\phi1 \xrightarrow{s} \phi2$ iff $\phi1 \supseteq \phi2$ (Surface Containment)
 - $\phi1 \xrightarrow{d} \phi2$ iff $(\exists \psi1 \in \phi1) (\exists \psi2 \in \phi2) (\phi \rightarrow \psi)$. (Deep Containment)
 - $\phi1 \oplus \phi2 \Leftrightarrow \phi1 \cup \phi2$ (Composition)
- A type precludes its negation, e.g., $(s/ s|\models\langle\langle hit, john, x; 1 \rangle\rangle) \perp (s/ s|\models\langle\langle hit, john, x; 0 \rangle\rangle)$. (Prelusion)
- Suppose the negation of a set of types Q is the set of the negations of every component type, then $Q \perp \neg Q$.

4.1.3 Inductive Evaluation

Theorem 5 Situation theory based IR model supports R, C, LM, RM, M, C-FA, GLM, GRM, QLM and QRM⁸.

⁸ Note that postulates Mix, GLM and QLM are trivially supported, as LM is supported. Postulates C-FA, GRM and QRM are trivially supported, as RM is supported.

The proofs of LM and RM are provided as follows:

- LM: Left Compositional Monotonicity is supported.

Given $\phi 1 \models_{ST} \phi 2$

$\Rightarrow (\exists c1 \in C) (\forall D/D \rightsquigarrow \phi 1) (\exists \psi 1 \in \phi 1) (\exists \psi 2 \in \phi 2) (D \models \psi 1 \xrightarrow{c1} D' \models \psi 2), \phi 1 \oplus \phi 3 \Leftrightarrow \phi 1 \cup \phi 3, \text{ and } \{ \forall D/D \rightsquigarrow \phi 1 \oplus \phi 3 \} \subseteq \{ \forall D/D \rightsquigarrow \phi 1 \}$
 $\Rightarrow (\forall D/D \rightsquigarrow \phi 1 \oplus \phi 3) (\exists \psi 1 \in \phi 1 \oplus \phi 3) (\exists \psi 2 \in \phi 2) (D \models \psi 1 \xrightarrow{c1} D' \models \psi 2),$
 $\therefore \phi 1 \oplus \phi 3 \models_{ST} \phi 2$

- RM: Right Compositional Monotonicity is supported.

Given $\phi 1 \models_{ST} \phi 2$

$\Rightarrow (\exists c1 \in C) (\forall D/D \rightsquigarrow \phi 1) (\exists \psi 1 \in \phi 1) (\exists \psi 2 \in \phi 2) (D \models \psi 1 \xrightarrow{c1} D' \models \psi 2), \phi 2 \oplus \phi 3 \Leftrightarrow \phi 2 \cup \phi 3, \text{ and } \{ \forall D/D \rightsquigarrow \phi 2 \oplus \phi 3 \} \subseteq \{ \forall D/D \rightsquigarrow \phi 2 \}$
 $\Rightarrow (\forall D/D \rightsquigarrow \phi 1) (\exists \psi 1 \in \phi 1) (\exists \psi 2 \in \phi 2 \oplus \phi 3) (D \models \psi 1 \xrightarrow{c1} D' \models \psi 2),$
 $\therefore \phi 1 \models_{ST} \phi 2 \oplus \phi 3$

4.2 Possible world based model

4.2.1 Background

A number of possible world based logical IR models have been proposed. As stated in [Lalmas and Bruza 1998], these systems are founded on a structure $\langle W, R \rangle$, where W is the set of worlds and $R \subseteq W \times W$ is the accessibility relation. They can be classified according to the choice made for the worlds $w \in W$ and accessibility relation R . For example, w can be a document (or its variation) and R is the similarity between two documents $w1$ and $w2$ [Nie 1989; Nie 1992], or w is a term and R is the similarity between two terms $w1$ and $w2$ [Crestani and van Rijsbergen 1995(a); Crestani and van Rijsbergen 1995(b); Crestani and Van Rijsbergen 1998], or w is the “retrieval situation” and R is the similarity between two situations $w1$ and $w2$ [Nie et al. 1995], etc.

Most of these systems use a technique called imaging. To obtain $P(D \rightarrow Q)$, where the connective \rightarrow represents conditional, we can move the probability from non-D-world to D-world by a shift from the original probability distribution P of the world w to a new probability distribution P_D of its closest world w_D where D is true. This process is called deriving P_D from P by imaging on D . The truth of $D \rightarrow Q$ at w will then be measured by the truth of Q at w_D . To simplify the analysis, let’s suppose that the truth of Q in a world is binary⁹ and the closest world of a world w is unique¹⁰.

$P(d \rightarrow q)$ can be computed as follows:

$$P(D \rightarrow Q) = \sum_{w \in W} P(w) w_D(Q) = \sum_{w \in W} P_D(w) w(Q) \quad (1)$$

⁹ Actually, it can be multi-valued in an interval.

¹⁰ There is also an approach called General Logical Imaging that does not rely on this assumption.

$$\sum_w P(w) = 1 \quad (2)$$

$$w(Q) = \begin{cases} 1, & \text{if } Q \text{ is true in } w \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

$$P_D(w) = \sum_{w' \in W} P(w') I(w, w') \quad (4)$$

$$I(w, w') = \begin{cases} 1, & \text{if } w = w'_D \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

$$w_D \text{ is the closest world of } w \text{ where } D \text{ is true} \quad (6)$$

Now, we study in detail Crestani and van Rijsbergen's model which models the terms as possible worlds to see some properties of the possible world based approach. In this model, term is considered as vector of documents, while the document and query are vectors of terms. The accessibility relations between terms are estimated by the co-occurrence of terms. $P(D \rightarrow Q)$ can be computed as:

$$P(D \rightarrow Q) = \sum_{t \in T} P(t) t_D(Q) = \sum_{t \in T} P_D(t) t(Q) \quad (7)$$

$$\sum_{t \in T} P(t) = 1 \quad (8)$$

$$t(Q) = \begin{cases} 1, & \text{if } t \text{ occurs in } Q \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

$$P_D(t) = \sum_{t' \in T} P(t') I(t, t') \quad (10)$$

$$I(t, t') = \begin{cases} 1, & \text{if } t = t'_D \\ 0, & \text{otherwise} \end{cases} \quad (11)$$

$$t_D \text{ is the closest term of } t \text{ where } d \text{ is true} (t_D \text{ occurs in } D) \quad (12)$$

Generally, D is deemed relevant to Q when $P(D \rightarrow Q)$ is greater than a threshold value, e.g., a positive real number ∂ . Similar to the vector space model (see section 3.3.2), the simplest case is that at least one term which occurs in both D and Q , or it is also the closest term of some other terms occurring in D and Q . This case is referred to as naïve possible world based model and the general case as threshold possible world based model.

4.2.2 Naïve Possible World Aboutness Based on Crestani and van Rijsbergen's Model ($\models_{NAIVE-PW-CV}$)

Let U be the set of all the documents, T be the set of all the index terms, Furthermore, let $D \in U$ be a document, Q be a query, and t be a term. The aboutness in the naïve Possible World based models is defined as follows:

- D and Q are sets of terms
- $D \models_{NAIVE-PW-CV} Q$ iff $P(D \rightarrow Q) > 0$ (aboutness)
- $D \not\models_{NAIVE-PW-CV} Q$ iff $P(D \rightarrow Q) = 0$
- $D \rightarrow Q$ iff $D \supseteq Q$ (Surface containment)
- $Q1 \rightarrow Q2$ iff $Q1 \supseteq Q2$

- $t1 \rightarrow t2$ iff $t1$ is the closest term of $t2$ (Deep containment)
- $D1 \oplus D2 \Leftrightarrow D1 \cup D2$ (Composition)
- $Q1 \oplus Q2 \Leftrightarrow Q1 \cup Q2$
- Preclusion is foreign to this model.

4.2.3 Inductive evaluation

Theorem 6 The Naïve Possible World based model supports R, C (surface), LM, RM, M and C-FA¹¹. Postulates GLM, GRM, QLM and QRM are inapplicable as preclusion is inapplicable.

Proofs of LM and RM are given as follows:

- LM: Left Compositional Monotonicity is supported.
 Given $D1 \models_{NAIVE-PW-CV} Q$, and $D = D1 \oplus D2$
 $\Rightarrow P(D1 \rightarrow Q) = \sum_t P_{D1}(t)t(Q) > 0$, $D1 \oplus D2 = D1 \cup D2$
 \Rightarrow At least one term t_i is the closest term of some terms where
 $D1$ is true and $t_i \in Q$, and $D1 \oplus D2 = D1 \cup D2$
 $\Rightarrow t_i$ is also true in $D1 \oplus D2$, and $t_i \in Q$
 $\Rightarrow P(D1 \oplus D2 \rightarrow Q) = \sum_t P_{D1 \oplus D2}(t)t(Q) > 0$
 $\therefore D1 \oplus D2 \models_{NAIVE-PW-CV} Q$
- RM: Right Compositional Monotonicity is supported.
 Given $D \models_{NAIVE-PW-CV} Q1$, and $Q = Q1 \oplus Q2$
 $\Rightarrow P(D \rightarrow Q1) = \sum_t P_D(t)t(Q1) > 0$, and $Q = Q1 \oplus Q2 = Q1 \cup Q2$,
 $\Rightarrow (\exists t_i \in Q1) (\exists t'_i \in T) (I(t_i, t'_i) = 1) \text{ and } t_i \in Q$
 $\Rightarrow P(D \rightarrow Q1 \oplus Q2) = \sum_t P_D(t)t(Q1 \oplus Q2) > 0$.
 $\therefore D \models_{NAIVE-PW-CV} Q1 \oplus Q2$

4.2.4 Threshold Possible World Aboutness Based on Crestani and van Rijsbergen's Model ($\models_{T-PW-CV}$)

Let U be the set of all documents, T be the set of all index terms, Furthermore, let $D \in U$ be a document, Q be a query, and t be a term. The aboutness in this models is then defined as follows:

- D and Q are sets of terms
- $D \models_{T-PW-CV} Q$ iff $P(D \rightarrow Q) \geq \partial$,
 where ∂ is a positive real number in the interval $(0, 1]$. (aboutness)
- $D \not\models_{T-PW-CV} Q$ iff $P(D \rightarrow Q) < \partial$

¹¹ Note that postulates Mix is trivially supported, as LM is supported. Postulate C-FA is trivially supported, as RM is supported.

- The mappings of containment, composition and preclusion are same as those in Section 4.3.2.

4.2.5 Inductive Evaluation

Theorem 7 The Threshold Possible World based model supports R, LM, RM, M, C-FA, and conditionally supports C, CWAA, RCM and NR. Postulates GLM, GRM, QLM and QRM are inapplicable as preclusion is inapplicable.

Proofs of LM and RM are given as follows:

- LM: Left Compositional Monotonicity is supported.
 Given $D1 \models_{T-PW-CV} Q$, and $D = D1 \oplus D2$
 $\Rightarrow P(D1 \rightarrow Q) = \sum_t P_{D1}(t)t(Q) \geq \partial$, $D1 \oplus D2 = D1 \cup D2$
 \Rightarrow The number of index terms which are the closest terms of certain terms where $D1 \oplus D2$ is true must be not less than that of index terms which are the closest terms of certain terms where $D1$ is true. This implies that $P_{D1 \oplus D2}(t) \geq P_{D1}(t)$.
 $\Rightarrow P(D1 \oplus D2 \rightarrow Q) = \sum_t P_{D1 \oplus D2}(t)t(Q) \geq \sum_t P_{D1}(t)t(Q) \geq \partial$
 $\therefore D1 \oplus D2 \models_{T-PW-CV} Q$
- RM: Right Compositional Monotonicity is supported.
 Given $D \models_{T-PW-CV} Q1$ and $Q = Q1 \oplus Q2$
 $\Rightarrow P(D \rightarrow Q1) = \sum_t P_D(t)t(Q1) \geq \partial$ and $Q = Q1 \oplus Q2 = Q1 \cup Q2$ (i.e. $Q1 \subseteq Q$ and $Q2 \subseteq Q$),
 $\Rightarrow P(D \rightarrow Q) = \sum_t P_D(t)t(Q1 \oplus Q2) \geq \sum_t P_D(t)t(Q1)$
 $\Rightarrow P(D \rightarrow Q1 \oplus Q2) = \sum_t P_D(t)t(Q1 \oplus Q2) \geq \partial$.
 $\therefore D \models_{T-PW-CV} Q1 \oplus Q2$

4.3 Discussion

- Deep containment is irrelevant to classical models, unless they are augmented by thesauri and the like from which deep containment relationships like *penguin* \rightarrow *bird* can be extracted. Logical models, by their very nature, can directly handle deep containment relationships. This means logical models support information transformation e.g., logical imaging in the possible world models. This is a major advantage of logical models. Moreover, they provide stronger expressive power, e.g. concepts such as situation, type and channel, etc. in situation theory based model make it more flexible.
- The properties of an IR model are largely determined by the matching function it supports. Two classes of matching function are widely used: *exact match* and *overlapping (naïve and non-zero threshold)*. The Boolean model is an example of exact match model, which requires that all the information of the query must be contained in or can be transformed to the information of the document. The naïve

vector space model and naïve possible world based model have similar properties (except that deep containment is applicable to possible world based model only) due to their simple overlapping retrieval mechanism (i.e., a document is judged to be relevant if it shares at least one term with the query). Compared with Boolean model, the naïve vector space and the naïve possible world based model support Left and Right Compositional Monotonicity, which causes imprecision. The Boolean model supports Right Containment Monotonicity, which promotes recall, at the expense of precision. They also support the Negation Rationale, which can improve precision. For the naïve vector space and possible world based models, Right Containment Monotonicity and Negation Rational are not supported. In summary, it is evident that the Boolean model is more effective than the naïve vector space and the naïve possible worlds based models.

- The naïve possible worlds model uses imaging (i.e., imaging from non-D world to D-world) besides simple overlapping. Even though there may exist a containment relation between a term $t1$ in the document and another term $t2$ in the query, if $t1$ is not shared by the document and the query, then this transformation from $t2$ to $t1$ is ineffective to establish the relevance. This explains why naïve possible world model does not support Containment (deep). The mechanics of imaging is dependent on a notion of similarity between worlds. Experimental evidence shows a relation between retrieval performance and the way in which the relationship between worlds is defined [Crestani and Van Rijsbergen 1998]. As the underlying framework for inductive evaluation presented in this paper does not explicitly support a concept of similarity, the mapping of the possible worlds based model into the inductive framework is incomplete. More will be said about this point in the conclusions.
- The threshold possible worlds model is both left and right monotonic. As a consequence there are some grounds to conclude that this model would be imprecise in practice, and also be insensitive to document length. As mentioned in the previous point, retrieval performance depends on how the similarity between worlds is defined. As both LM and RM are supported, it can be hypothesized that the baseline performance for the threshold possible world model would be similar to the naïve overlap model. More sophisticated similarity metrics between worlds would improve performance above this baseline. Crestani and Van Rijsbergen allude to this point as follows: “.. it is possible to obtain higher levels of retrieval effectiveness by taking into consideration the similarity between the objects involved in the transfer of probability. However, the similarity information should not be used too drastically since similarity is often based on cooccurrence and such a source of similarity information is itself uncertain” [Crestani and Van Rijsbergen 1998]. When the threshold possible world model judges a document D relevant to the query Q , this implies that D shares a number of terms with Q or a number of terms can be transformed to the shared terms so that $P(D \rightarrow Q)$ is not less than the threshold ∂ . The expansion of D or Q can only increase $P(D \rightarrow Q)$. This judgment is not true for threshold vector space model, for after the expansion of D (or Q), the increase of the space of D (or Q), i.e. number of terms in D and Q , may be much more than the increase of the shared terms. Thus the degree of overlapping may be decreased.
- The threshold possible worlds model and situation theory using Lalmas’ relaxed condition support LM and RM. This suggests that these models would be less precise than probabilistic and threshold vector space models. This in turn reflects

the likely possibility that despite their previously mentioned expressive power, this power does not necessarily translate into precision. The scant experimental evidence available bears this out [Crestani et al. 1995].

5. Results Summary and Conclusions

5.1 Results Summary

Table1: Summary of the results of the evaluation.

Models Postulates	Boolean	Naïve Vector Space	Threshold Vector Space	Probabi- listic Model	Situation Theory Based	Naïve Possible World	Threshold Possible World
R	√	√	√	CS	√	√	√
C (Surface)	√	√	CS	CS	√	√	CS
C (Deep)	NA	NA	NA	NA	√	×	CS
RCM (Surface)	√	×	CS	CS	×	×	CS
RCM (Deep)	NA	NA	NA	NA	×	×	CS
CWAA	√	×	CS	CS	×	×	CS
LM	√	√	CS	CS	√	√	√
RM	×	√	CS	CS	√	√	√
M	√	√	CS	CS	√	√	√
C-FA	√	√	CS	CS	√	√	√
GLM	√	NA	NA	NA	√	NA	NA
GRM	×	NA	NA	NA	√	NA	NA
QLM	√	NA	NA	NA	√	NA	NA
QRM	×	NA	NA	NA	√	NA	NA
NR	√	×	CS	CS	×	×	CS

Note: NA means *not applicable*, CS means *conditionally support*, √ means *support*; and × means *not supported*.

5.2 Conclusion

The functional benchmarking exercise presented in this paper indicates that functional benchmarking is both feasible and useful. It has been used to analyze and compare the functionality of various classical and logical IR models. Through functional benchmarking, some phenomena encountered in experimental IR research can be explained from a theoretical point of view using a symbolic perspective. The theoretical analysis could in turn help us better understand IR and provide guideline to investigate more effective IR models.

A major point to be drawn is that IR is conservatively monotonic in nature. It is important that conservatively monotonic models be studied and developed, as these would help achieve a better understanding of the tradeoff between precision and recall. The postulates GLM, GRM, QLM, QRM, etc. guarantee the conservatively monotonic properties, but they are foreign to some models. Even in those models, which support some of the conservatively monotonic properties, preclusion is only based on the assumption that an information carrier precludes its negation. Moreover, GLM, QLM and MIX are the special cases of LM, and GRM, QRM and C-FA are the special case of RM. As such, if a model supports LM, GLM is vacuously supported. Therefore, a model supporting conservative monotonicity should embody

conservatively monotonic properties without supporting LM and RM. The probabilistic model and threshold vector space model show good performance in practice as they mimic conservative monotonicity.

Current logical IR models have advantage of modeling information transformation and their expressive power. However, they are still insufficient to model conservative monotonicity. A primary reason is that important concepts, such as (deep and surface) containment, information preclusion, etc., upon which conservative monotonicity is based, are not sufficiently modeled. For example, semantics of information preclusion is not explicitly defined in current logical models. We just simply assume that an information carrier precludes its negation during the benchmarking. It is interesting to show that if we add some kind of semantics of preclusion to the logical IR models, the conservative monotonicity could be partially realized. For example, we could add the following definition to the model:

Preclusion:

Given two types $\phi1$ and $\phi2$, $\phi1 \perp \phi2$, $s1 \models \phi1$ and $s2 \models \phi2$, there does not exist any channel between $s1$ and $s2$.

The Left composition monotonicity (LM) is no longer supported:

Given $\phi1 \models_{ST} \phi2$

$$\Rightarrow (\exists c1 \in C) (\forall D/D \xrightarrow{\sim} \phi1) (\exists \psi1 \in \phi1) (\exists \psi2 \in \phi2) (D \models \psi1 \xrightarrow{c1} D' \models \psi2),$$

$$\phi1 \oplus \phi3 \Leftrightarrow \phi1 \cup \phi3$$

Assume LM is supported, i.e. $(\forall D/D \xrightarrow{\sim} \phi1 \oplus \phi3) (\exists \psi1 \in \phi1 \oplus \phi3) (\exists \psi2 \in \phi2) (D \models \psi1 \xrightarrow{c1} D' \models \psi2)$.

Consider the case of $\phi2 \perp \phi3$. This implies for $D \models \phi3$ and $D' \models \phi2$, there does not exist a channel between D and D' . This contradicts the above assumption, because $\{\forall D/D \xrightarrow{\sim} \phi1 \oplus \phi3\} \subseteq \{\forall D/D \models \phi3\}$.

\therefore It is not necessary that $\phi1 \oplus \phi2 \models_{ST} \phi2$.

On the other hand, RM is not supported for the similar reason of LM. However, by applying the conservative forms of monotonicity, QLM and QRM, with the qualifying non-preclusion conditions, the above-like counter example will no longer exist.

The above definition of preclusion is simply for the purposes of illustration. It is true that current IR systems are not explicitly defined in terms of concepts such as preclusion, information containment, etc. However, such informational concepts are in the background. Preclusion relationships can be derived via relevance feedback [Amati and Georgatos 1996, Bruza *et al.* 1998]. For restricted domains, information containment relationships can be derived from ontologies, and the like. For example, we have been investigating the automatic extraction of deep containment relationships based on Barwise and Seligman's theory of information flow [Barwise and Seligman 1997, Bruza and Song 2001; Song and Bruza 2001]. When language processing tools have advanced further, the concepts under the aboutness theory could be applied to IR more easily and more directly. More sensitive IR systems would then result; in particular those which are conservatively monotonic with respect to composition. Therefore, more investigations about how to achieve conservative monotonicity in current logical IR models are necessary.

Finally, we reflect on the strengths and weaknesses of the inductive theory of information retrieval evaluation. The strengths are summarized below:

- *Enhanced perspective:* Matching functions can be characterized qualitatively in terms of aboutness properties that are, or are not implied, by the matching function in question. It may not be obvious what the implications are of a given numeric formulation of a matching function. The inductive analysis allows some of these implications to be teased out. By way of illustration, models based on overlap may imply monotonicity (left or right), which is precision degrading. In addition, inductive analysis allows one to compute under what conditions a particular aboutness property is supported. It has been argued that a conservatively monotonic aboutness relationship promotes effective retrieval. The analysis in this paper revealed that although both of the threshold vector space and probabilistic models mimic conservative monotonicity, the fundamentals of this support are very different: the thresholded vector space model support for conservative monotonicity depends on overlap between document and query terms modulo the size of the document. Support for conservative monotonicity in the probabilistic model depends on whether the terms being added have a high enough probability of occurring in relevant documents. From an intuitive point of view, the latter condition would seem a more sound basis for support because it is directly tied to relevance.
- *Transparency:* One may disagree with a given functional benchmark (as represented by a set of aboutness properties), or with how a given matching function has been mapped into the inductive framework, however, the assumptions made have been explicitly stated. This differs from some experimental studies where the underlying assumptions (e.g., the import of certain constants) are not, or insufficiently, motivated.
- *New insights:* The use of an abstract framework allows new insights to be gleaned. Inductive evaluation has highlighted the import of monotonicity in retrieval functions, and its affect on retrieval performance. Designers of new matching functions should provide functions that are conservatively monotonic with respect to the composition of information. More sensitive IR systems would then result. The lack of such systems currently can be attributed in part to the inability to effectively "operationalize" information preclusion. Most common IR models are either monotonic or non-monotonic - another class of IR models, namely those that are explicitly conservatively monotonic is missing. For this reason, the inductive analyses reported in this paper revealed no distinctions based on conservatively monotonic rules such as MIX and CF-A. Conservatively monotonic models are interesting for purposes of producing a symbolic inference foundation to query expansion and perhaps even relevance feedback.

The weaknesses of an inductive theory for evaluation are:

- *Difficulty in dealing with weights:* Much of the subtlety of IR models remains buried in different weighting schemes. Due to its symbolic nature, the inductive approach can abstract "too much", thereby losing sensitivity in the final analysis. For example, the nuances of document length normalization [Singhal et al. 1996], term independence assumptions, probabilistic weighting schemes are difficult, if not impossible, to map faithfully into a symbolic, inductive framework.

- Difficulties with mapping: For an arbitrary model, it may not be obvious how to map the model into an inductive framework. This is particularly true for heavily numeric models such as probabilistic models. It is often the case that such models do not support many symbolic properties – they are like black holes defying analysis [Bruza, Song and Wong 2000]. However, by analysing the conditions under which given properties are supported allow us to “peak at the edges of the black hole”.
- Incompleteness of framework: In order to pursue functional benchmarking, a sufficiently expressive framework is necessary in order to represent salient aspects of the model in question. This is an issue of completeness. In the inductive analysis of the possible worlds based models presented in this paper, we have seen that the notion of similarity inherent to these models cannot be directly translated into the underlying inductive framework. This suggests that the framework presented in this paper should be extended. One could also argue that not all salient aspects of aboutness have been captured by the properties used for the benchmark. These are not criticisms of inductive evaluation, but of the expressiveness of the underlying informational framework, in this case information fields.

It is noteworthy that conventional experimental IR evaluation approaches are reasonably solid but some times fail to address deeper issues. Functional benchmarking is a framework and methodology that can help fill this gap. It is not intended to replace the former, but to complement it.

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Appendix List of Notations

- Information carrier (IC)
- Information composition (\oplus)
- Information containment (\rightarrow)
- Surface containment (\xrightarrow{s})
- Deep containment (\xrightarrow{d})
- Information preclusion (\perp)
- Aboutness ($|=$)
- Non-aboutness ($| \neq$)
- A document D (or a query Q) consisting of information carrier i ($D \rightsquigarrow i$ or $Q \rightsquigarrow i$)