Analytical Investigation of the Dynamic and Diffusive Behaviour of Building Envelopes

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Summary

A one-dimensional model describing the steady state dynamic and diffusive behaviour of a three-layer building envelope element is developed with the objective of elucidating the physics of simultaneous heat and vapour transport through dynamically insulated building envelopes. The equations are simple to programme on a spreadsheet enabling architects to build tools which will enable them design "breathing" envelope constructions. The variables at the designers disposal are air and vapour permeabilities, thermal conductivities and thicknesses of the layers comprising the envelope. Users wishing to consider more complicated constructions will find that they can readily extend, by inspection, the equations from the three layers presented here to any number of layers. Whilst mass transfer has been discussed in terms of water vapour transport the equations are very general and can be applied to the transport of any gas through a permeable wall.

Notation

A_j	length parameter, relative measure of convective and conductive heat fluxes (m^{-1})
B_j	length parameter, relative measure of convective and conductive mass fluxes (m^{-1})
c_a	specific heat of air (J/kg.C)
C	mass concentration of gas or vapour per unit volume of dry air (kg/m^3)
C_j	concentration at interface between j^{th} and $(j+1)^{th}$ component of the envelope (kg/m^3)

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D_j	diffusion coefficient of binary gas mixture through jth component of the envelope (m^2/s)
G	dimensionless parameter, measure of the total thermal resistance of the envelope
H	dimensionless parameter, measure of the total mass transfer resistance of the envelope
h	heat transfer coefficient ($W/m^2.K$)
j	ordinal number of component in building envelope, j = 1, 2, 3
L_j	distance from outer surface of interface between jth and (j+1)th component (m)
m	total mass flux ($kg/m^2.s$)
m_{d0}	diffusion mass flux at outer surface ($kg/m^2.s$)
m_d	diffusion mass flux ($kg/m^2.s$)
m_u	convective or bulk flow mass flux ($kg/m^2.s$)
q	total heat flux (W/m^2)
q_{c0}	conduction heat flux at outer surface (W/m^2)
q_c	conduction heat flux (W/m^2)
q_u	convective or bulk flow heat flux (W/m^2)
R	total thermal resistance of building element ($m^2.K/W$)
R_d	total diffusion resistance of building element (s/m)
T	temperature of the air at a point within the envelope (C)
T_j	temperature of the air at the interface between jth and (j+1)th component (C)
x	distance through building element from outer surface (m)
u	air velocity (m/s)
u_c	critical air velocity (m/s)
λ_j	thermal conductivity of jth component (W/m.K)
ρ_a	density of air (kg/m^3)

1. Introduction

Recent interest in providing adequate natural ventilation in dwellings while minimising heat loss, condensation and the accumulation of harmful indoor pollutants has focused on vapour and gas permeable walls and ceilings. This type of building construction is colloquially known as the "breathing envelope". The main characteristic of this form of building envelope construction is that it is intended to control the flow of air and moisture through the envelope rather than stop them by means of air and vapour barriers. Of the two types of breathing envelope that have been described in the literature, "diffusive insulation" and "dynamic insulation" envelopes, the former is the version of the breathing envelope construction most widely encountered in the UK today. The essential difference between them is that in "dynamic insulation" envelopes air is drawn through the insulation material by a pressure difference created either by a fan or stack effect whereas in "diffusive insulation" it is assumed that the only driving forces are concentration gradients. Otherwise, both types of breathing wall are superficially very similar (Fig 1).

The analytical expressions for the temperature and water vapour concentration profiles for single layers of dynamic insulation are well established [1,2]:

$$\frac{T - T_o}{T_i - T_o} = \frac{\exp\left(\frac{u\rho_a c_a x}{\lambda}\right) - 1}{\exp\left(\frac{u\rho_a c_a L}{\lambda}\right) - 1} \tag{1}$$

$$\frac{C - C_o}{C_i - C_o} = \frac{\exp\left(\frac{ux}{D}\right) - 1}{\exp\left(\frac{uL}{D}\right) - 1} \tag{2}$$

The temperature profiles have been verified experimentally in the laboratory and in field trials many times [3 - 5] while Bartussek is the only researcher known to the author who has measured the humidity profiles. The temperature profile has been frequently used to detect and indeed measure the air flow rate through porous media [3, 6, 7] because the air velocities (typically 0.3 mm/s) are too small to measure directly.

This paper derives the temperature and humidity profiles for a three layer permeable wall envelope for the special case of one-dimensional, steady heat and mass flow. The practical application of this is to develop a tool to enable the designer to explore the potential of possible "breathing" envelope constructions. The variables at the designers disposal are air and vapour permeabilities, thermal conductivities and thicknesses of the layers comprising the envelope. The majority of dynamic building envelopes to date have consisted of a three layer core with ventilated air gaps separating the core from the outer cladding and the inner wearing surface (Fig 1). Users wishing to consider more complicated constructions will find that they can readily extend, by inspection, the equations from three layers presented here to any number of layers.

2. Heat Transfer through a 3-layer Dynamic Building Element.

2.1 Temperature Profiles

In the three layer construction shown in Fig 1 each layer has thermal conductivity, λ_j , and the governing equation for steady state heat transfer in one dimension through each layer is :

$$\lambda_j \frac{d^2T(x)}{dx^2} - u\rho_a c_a \frac{dT(x)}{dx} = 0, \quad j = 1, 2, 3 \tag{3}$$

The air flow u is taken to be positive in the direction of increasing x. The boundary conditions

at x_j, are $T = T_j$, for j = 0, 1, 2, 3 where j = 0 corresponds to the outer surface of the three layer element at x = 0 and j = 3 is the inner surface at $x = L_3$. There is the additional requirement that the conductive heat flux is continuous across each interface between the layers:

$$\lambda_j \left.\frac{dT}{dx}\right|_{x=L_j} = \lambda_{j+1} \left.\frac{dT}{dx}\right|_{x=L_j} \tag{4}$$

The steady state solutions for the air temperature in each layer are:

$$\frac{T-T_o}{T_3-T_o} = \begin{cases} \dfrac{\exp(A_1 x)-1}{G-1} & 0 \le x \le L_1 \\ \dfrac{\exp\left(A_1L_1 + A_2(x-L_1)\right)-1}{G-1} & L_1 \le x \le L_2 \\ \dfrac{\exp\left(A_1L_1 + A_2(L_2-L_1) + A_3(x-L_2)\right)}{G-1} & L_2 \le x \le L_3 \end{cases} \tag{5}$$

where the parameters A_j, with dimensions of (length)$^{-1}$, and G are defined by:

$$A_j = \frac{u\rho_a c_a}{\lambda_j} \tag{6}$$

$$G = \exp\left(\sum_{j=1}^{3} A_j\left(L_j - L_{j-1}\right)\right) \tag{7}$$

Using the total thermal resistance, R, of the envelope

$$R = \sum_{j=1}^{3} \frac{L_j - L_{j-1}}{\lambda_j} \tag{8}$$

G can be written more concisely as

$$G = \exp(u\rho_a c_a R) \tag{9}$$

The form of this equation has the advantage that the dimensionless parameter G can be interpreted as the total thermal resistance of the dynamic wall element.

2.2 Heat Flux and Heat Transfer Coefficient

The conductive heat flux q_{cj} at any point in the jth layer of the building element is

$$q_{cj} = -\lambda_j \frac{dT}{dx} \tag{10}$$

Differentiating each of the three layers in equation (5) the respective conductive heat fluxes are given by

$$q_c = \begin{cases} -q_{c0}\exp\left(A_1 x\right) & 0 \le x \le L_1 \\ -q_{c0}\exp\left(A_1 L_1 + A_2\left(x - L_1\right)\right) & L_1 \le x \le L_2 \\ -q_{c0}\exp\left(A_1 L_1 + A_2\left(L_2 - L_1\right) + A_3(x - L_2)\right) & L_2 \le x \le L_3 \end{cases} \quad (11)$$

where the conductive heat flux at x=0, q_{c0}, is

$$q_{c0} = \frac{\ln G}{G-1}\frac{\left(T_3 - T_0\right)}{R} \quad (12)$$

The convective heat flux, q_u, is defined by

$$q_u = u\rho_a c_a\left(T - T_o\right) \quad (13)$$

Substituting equation (5) into equation (13) and then simplifying using (7) and (12) readily gives the convective heat flow for each layer.

$$q_u = \begin{cases} q_{c0}\left(\exp\left(A_1 x\right) - 1\right) & 0 \le x \le L_1 \\ q_{c0}\left(\exp\left(A_1 L_1 + A_2\left(x - L_1\right)\right) - 1\right) & L_1 \le x \le L_2 \\ q_{c0}\left(\exp\left(A_1 L_1 + A_2\left(L_2 - L_1\right) + A_3(x - L_2)\right) - 1\ \right) & L_2 \le x \le L_3 \end{cases} \quad (14)$$

Summing the conductive and convective fluxes in their respective layers shows that the total heat flux is constant and is equal to q_{c0}

To draw out the physical meaning of the above equations a typical temperature profile through a 3-layer envelope with total thermal resistance of 5.43 m^2K/W for the case where air is flowing at 1.0 m3/m2h from outside to inside against the conductive heat flux (contra-flux case) is plotted in Fig 2. The corresponding conductive, convective and total heat fluxes are shown in Fig 3. This clearly shows the similarity and differences between the conductive and convective fluxes and that at any given point in the wall the net or total heat flux equals the heat flux at the outer surface. Fig 3 also clearly bring out that in order to minimise the heat flow to the outside the total heat flux, q_{co}, needs to be made as small as is practicable.

The heat transfer coefficient for the three layer wall element is defined appropriately, using the total heat flux, q, as

$$h = \frac{q}{T_3 - T_o} \tag{15}$$

which, using equation (12) for dynamic behaviour, simplifies to

$$h = \frac{\ln G}{(G-1)R} \tag{16}$$

The ratio of the heat transfer coefficients in the dynamic to the static case is easily seen to be

$$\frac{h_d}{h_s} = \frac{\ln G}{G-1} \tag{17}$$

where the heat transfer coefficient for conventional static or diffusive behaviour, h_s, is simply 1/R. In a similar fashion, the ratio of the total heat flux in the dynamic case, q_{co}, to the static case, q_s, can be shown to be

$$\frac{q_{c0}}{q_s} = \frac{\ln G}{G-1} \tag{18}$$

Equations 17 and 18 encapsulate the essential features of dynamic behaviour (Fig 4). From q_{co}/q_s the change in total heat flux relative to the static case is readily calculated from a knowledge of the thermal resistance, R, and the air flow, permitting a ready estimate of the energy saved when the envelope behaves dynamically.

From Fig 4 some general conclusions can be drawn

1. In general the pro-flux and contra-flux behaviour is asymmetrical. It is only in the special case of low thermal resistance R that they are approximately symmetrical.
2. Pro-flux increases the heat losses from the envelope.

3. The potential of contra-flux to reduce the heat loss through the envelope increases dramatically with thermal resistance R.

These conclusions are valid even though the actual details of construction of the building envelope such as choice of materials and thicknesses may very. Very different physical envelopes having the same thermal resistance R will have the same thermodynamic performance for the same air flow. This represents a major simplification in the design and analysis of dynamic envelopes. The final choice of envelope construction will depend on other factors such as cost, environmental sustainability and aesthetics as well as physical characteristics such as hygroscopicity, thermal capacity.

3. Mass Transfer through a 3-layer Dynamic Building Element.

The governing equation for steady state mass transfer in one dimension through each layer has a similar form to the heat transfer equation discussed above:

$$D_j \frac{d^2C(x)}{dx^2} - u\frac{dC(x)}{dx} = 0, \quad j = 1, 2, 3 \tag{19}$$

where C is the concentration (kg per m^3 of dry air) of the gas or vapour that is diffusing through the wall and D_j is the diffusion coefficient for that gas or vapour in air through material j. These diffusion coefficients are essentially the diffusion coefficients for the binary gas combination, as calculated from Chapman-Enskog theory or empirical correlations [8] modified by the porosity (effective flow area) and tortuisity (length) of the channels of the porous media. For building envelopes the diffusing component that is of crucial importance is water vapour because of the risks of condensation and all its attendant ill effects. Whilst in the following discussion, attention will focus on the movement of water vapour it should not be overlooked that the above governing equation and its solutions are generally applicable to any diffusing gas.

3.1 Water Vapour Concentration Profiles

Because of the similarity of the governing equations for heat and mass transfer and continuity of mass flux at the interfaces the solutions to equation (19) for the three layer model Fig 1 can be written down immediately.

$$\frac{C-C_o}{C_3-C_o}=\begin{cases}\dfrac{\exp(B_1x)-1}{H-1} & 0\le x\le L_1\\ \dfrac{\exp\left(B_1L_1+B_2(x-L_1)\right)-1}{H-1} & L_1\le x\le L_2\\ \dfrac{\exp\left(B_1L_1+B_2(L_2-L_1)+B_3(x-L_2)\right)-1}{H-1} & L_2\le x\le L_3\end{cases} \tag{20}$$

where parameters B_j and H are defined as:

$$B_j=\frac{u}{D_j} \tag{21}$$

$$H=\exp\left(\sum_{j=1}^{3}B_j\left(L_j-L_{j-1}\right)\right) \tag{22}$$

The parameter H can be interpreted as being the total vapour diffusion resistance of the dynamic wall element as exemplified by the following equivalent form of equation (22)

$$H=\exp\left(uR_d\right) \tag{23}$$

where the diffusion resistance R_d of the three layer wall element is

$$R_d=\sum_{j=1}^{3}\frac{L_j-L_{j-1}}{D_j} \tag{24}$$

3.2 Convective and Diffusive Vapour Fluxes

The diffusive vapour flux m_{dj} at any point in the j^{th} layer of the building element is

$$m_{dj}=-D_j\frac{dC}{dx}\quad j=1,2,3 \tag{25}$$

For the three layers the diffusive vapour flux becomes by differentiating equation (20)

$$m_d = \begin{cases} -m_{d0}\exp\left(B_1 x\right) & 0 \le x \le L_1 \\ -m_{d0}\exp\left(B_1 L_1 + B_2\left(x - L_1\right)\right) & L_1 \le x \le L_2 \\ -m_{d0}\exp\left(B_1 L_1 + B_2\left(L_2 - L_1\right) + B_3(x - L_2)\right) & L_2 \le x \le L_3 \end{cases} \tag{26}$$

where the magnitude of the diffusive vapour flux at x=0 , m_{do} is

$$m_{do} = \frac{u\left(C_3 - C_o\right)}{H - 1} \tag{27}$$

The convective heat flux at any plane in the envelope, m_u is defined simply by

$$m_u = C\,u \tag{28}$$

For the record the convective vapour fluxes for each layer are then easily shown to be

$$m_u = \begin{cases} m_{d0}\left(\exp\left(B_1 x\right) - 1\right) + uC_o & 0 \le x \le L_1 \\ m_{d0}\left(\exp\left(B_1 L_1 + B_2\left(x - L_1\right)\right) - 1\right) + uC_o & L_1 \le x \le L_2 \\ m_{d0}\left(\exp\left(B_1 L_1 + B_2\left(L_2 - L_1\right) + B_3(x - L_2)\right) - 1\ \right) + uC_o & L_2 \le x \le L_3 \end{cases} \tag{29}$$

The total vapour flux, the sum of the diffusive and convective fluxes, is

$$m = -m_{do} + u\,C_o \tag{30}$$

Substituting for m_{do} (equation 27) gives the following result for the net mass flux

$$m = \frac{u\left(H\,C_o - C_3\right)}{H - 1} \tag{31}$$

Using the fact that the mass flux without air flow, m_s, is simply

$$m_s = -\frac{C_3 - C_o}{R_d} \tag{32}$$

the ratio of the net mass flux under dynamic conditions to the purely diffusive or static case is easily seen to be

$$\frac{m}{m_s} = \frac{\left(C_3 - H\,C_o\right)\ln H}{\left(C_3 - C_o\right)\left(H - 1\right)} \tag{33}$$

If it as assumed that conditions are such that condensation does not occur then the total vapour flux will be constant through the wall. Up to this point there is an exact parallel between heat and mass transfer, however, equations (29) and (33) now highlight a very crucial difference. There is a value of the air flow, u, for which the total vapour flux is zero. In contrast, for heat transfer, the bulk air flow cannot stop air molecules passing on to each other the kinetic energy of their random motion. This critical air velocity is found by setting either equation (30) or (33) to zero and solving for u

$$u_c = \frac{\ln\left(C_3 / C_o \right)}{R_d} \qquad (34)$$

The implication of this is that, in contra-flux, if the air velocity is greater than u_C then water vapour will be carried from outside to inside despite their being a higher water vapour concentration on the inside. For a typical timber frame insulated wall construction with total thermal resistance of 6.434 m^2K/W (as in Fig 4) and the indoor and outdoor temperature and humidity conditions of 15C, 85% RH and 5C, 95% RH respectively as specified in BS 5250 [9], this critical air velocity is very low at 0.063 m^3/m^2h. This, is very much lower than the air flows of 0.5 to 1.5 m^3/m^2h recommended by Dalehaug [2]. The partial vapour pressure difference corresponding to the standard internal and external conditions, stated above, is 621 Pa. The authors have measured the air permeability of a variety of insulating materials and the air permeance of 200 mm of cellulose is found to be 1.5 m^3/m^2hPa. and that for 12 mm thick softboard was 0.116 m^3/m^2hPa. The controlling resistance to air flow in a wall construction comprising of wood wool board (air permeance too high to measure), 200 mm cellulose, 12 mm softboard is the softboard. The pressure drop across the wall at the critical air flow corresponds to a difference in air pressure of only 0.054 Pa Thus water vapour cannot flow from inside to out through a wall operating in contra-flux mode. It behaves, as has been verified experimentally by Bartussek, as if it contained a vapour barrier on the warm side. There is then a conflict between the air flow requirements to minimise heat losses and that necessary to maximise the removal of water vapour or other indoor pollutants.

The temperature and water vapour concentration profiles (Fig 2 and Fig 5) for the same air flow through the same envelope are very different due to the envelope having very different heat and water vapour transport properties. Fig 5 clearly demonstrates how air flowing from outside to inside creates an effective vapour barrier as reported by Bartussek. Fig 6 shows how the ratio of the net mass flux to the purely diffusive flux (equation 33) varies with air flow for envelope water vapour diffusion resistances that will encompass those found in building envelopes using cellulose insulation. The water vapour concentration on the warm and cold sides correspond to 15°C, 85% RH and 5°C, 95% RH respectively. The larger diffusion resistance corresponds to 200 mm of insulation and the lower to an envelope containing only 40 mm. It is seen that, for a given air flow, the magnitude of the mass flux relative to the purely diffusive flux is greater for envelopes with a larger diffusion resistance. On the other hand the critical velocity is greater for envelopes of low diffusion resistance.

4. Discussion

The steady-state one-dimensional model presented above enables some general observations to be drawn about the dynamic and diffusive behaviour of building envelopes. The first is to emphasis that any building envelope will exhibit both dynamic and diffusive behaviour to a greater or lessor extent. The distinction between dynamic and diffusive insulation envelopes is perhaps a misleading. It suggests that there are two different types of wall construction. It might be more accurate instead to refer to the dynamic and diffusive behaviour of the building envelope. The dominant mechanism depends as much on the internal and external boundary conditions as on the type of construction.

In order to minimise the heat losses the air flow should be in the opposite direction of the conductive heat flux (contra-flux) (Fig 4). However to maximise the venting of indoor pollutants (water and organic vapours) then the bulk air flow should be in the same direction as the concentration gradient (pro-flux). This conflict can be resolved by arranging the air flow to be contra-flux and venting the moist exhaust air to atmosphere via a heat recovery system. In this way pre-heated and filtered outdoor air will dilute the concentrations of indoor pollutants. There would appear to be little merit in trying to remove indoor pollutants via the building fabric since it will lead to their accumulation within the fabric with possible attendant health risks in addition to increased heat losses. There is a possible exception to this. The hygroscopic properties of wood based products and cellulose fibre in particular may permit useful water vapour and heat storage. This is the subject of further investigation.

In hot climates where the objective is to cool the interior then a pro-flux air flow would increase the flow of both heat and indoor pollutants to outside. However, this has the disadvantage of drawing in warm, unfiltered air from outside. A source of cool air created by a natural or engineered external micro-climate may enable the dynamic behaviour of a porous envelope to increase the cooling effect.

Since in cold climates contra-flux is to be preferred to pro-flux there is the problem of ensuring that a building is always operating in this mode under fluctuating and extreme wind conditions. The building envelope and its ventilation system must be designed to achieve an adequate depressurisation under all likely wind loadings [2].

5. Conclusions

The one-dimensional model describing the steady state dynamic and diffusive behaviour of a three-layer building envelope element presented above is a very useful aid to understanding the physics of these concepts in real building envelopes. Mass transfer has been discussed in terms of water vapour transport since it is usually of primary importance in building design however the equations themselves are very general and can be applied to any binary gas or vapour combination.

The model as presented in this paper has limitations. The main one being that water vapour is treated as a non-condensing gas and so the above equations apply only up to the point at which condensation will occur but can say nothing about what happens through or beyond the condensation region. Nevertheless it can provide a guide as to whether condensation is likely occur and where it will occur. Another limitation, however one that is easily corrected, is the omission of the internal and external film resistances of the air. Including these resistances would raise the outer surface temperature above the outdoor ambient temperature and lower the inner surface temperature below the room temperature.

The equations can readily be programmed on a spreadsheet to provide a simple but powerful tool for assessing the dynamic and diffusive behaviour of building envelope elements under a variety of temperature and vapour/gas boundary conditions. The authors have found such a spreadsheet model to be very useful for quickly assessing the merits of outline designs for solving architectural and building problems. Equation (18) provides a direct link into the First Law of Thermodynamics this enabling a complete energy and mass balance for the building envelope to be carried out.

Acknowledgements

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References

1. H. Bartussek, *Poren-Luftung (Porous Ceilings),* Osterreichisches Kuratorium fur Landtechnik, Vienna, 1981
2. A. Dalehaug, Dynamic Insulation in Walls, Research Report No 53, Hokkaido Prefectural Cold Region Housing and Urban Research Institute, 1993
3. R. R. G. Crowther, Breathing Walls: Fact and Fiction, 3rd European Conf. on Architecture “Solar Energy in Architecture and Urban Design”, Florence, 1993
4. P. Roots, Skanterra Optima - a Wall with Dynamic Insulation, LUTVDG/(TVBH-7133)?1-33(1992), Dept of Building Physics, University of Lund, Sweden, 1992
5. J. Timusk, Design, Construction and Performance of a Dynamic Wall House, 8th Air Infiltration Centre Conference, Sept 1987, Uberlingen
6. P. Roots, Skanterra - a Wall with Dynamic Insulation, Proceedings of the 3rd Symposium Building Physics in Nordic Countries, Vol 1, Copenhagen, 13-15 Sept., 1993
7. P. Wallenten, OPTIMAT Field Measurements of Dynamic Insulation, Proceedings of the 3rd Symposium Building Physics in Nordic Countries, Vol 1, Copenhagen, 13-15 Sept., 1993
8. E. L. Cussler, *Diffusion: Mass Transfer in Fluid Systems*, CUP, 1984
9. BS 5250:1989, Control of Condensation in Buildings, British Standards Institution, London, 1989

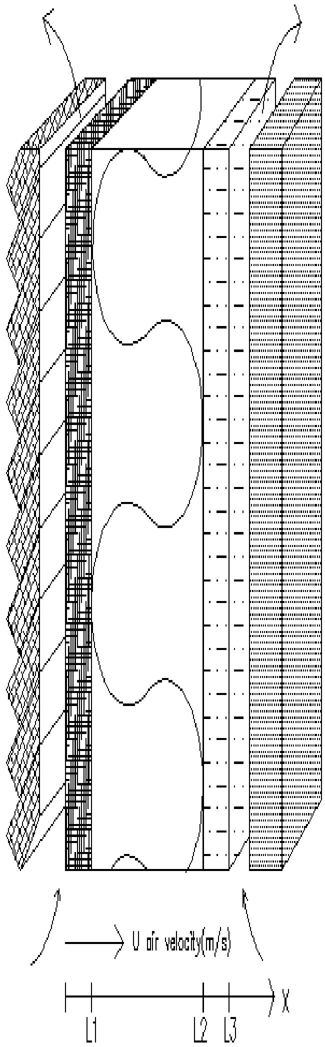
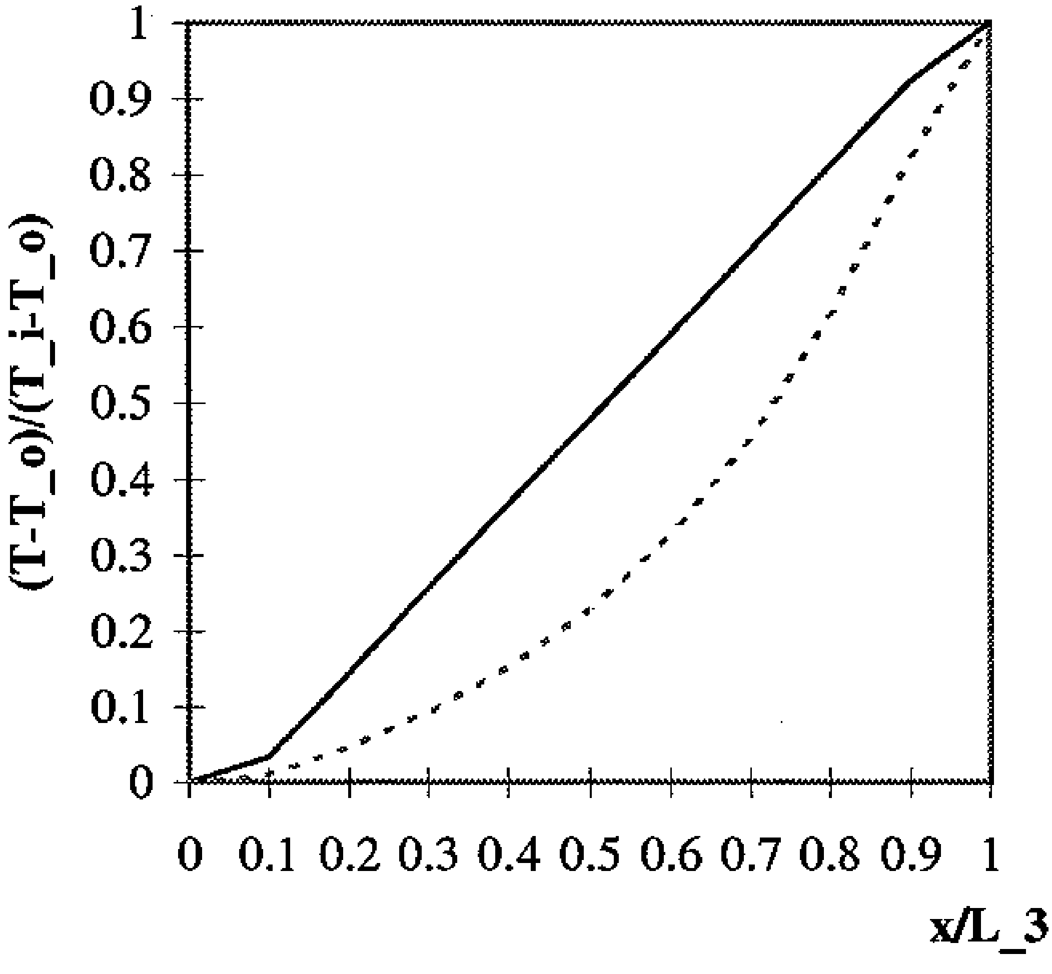


Fig 1 Three Layer Dynamic Wall Element



Envelope Thermal Resistance = 6.434 m^2 / W.K

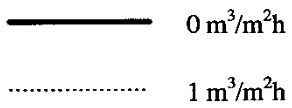
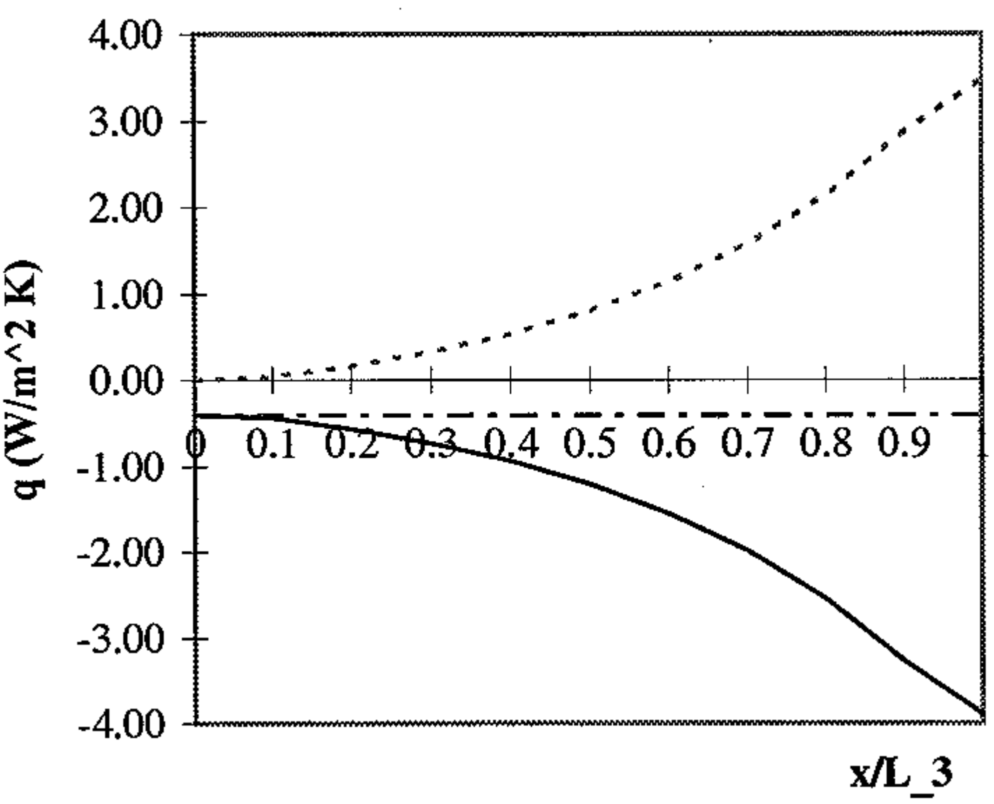


Fig 2 Temperature Profile



Air flow = 1m^3/m^{2}h
Envelope Thermal Resistance = 6.434 m^2/W.K

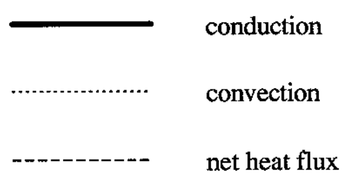


Fig 3 Heat Flux Profiles

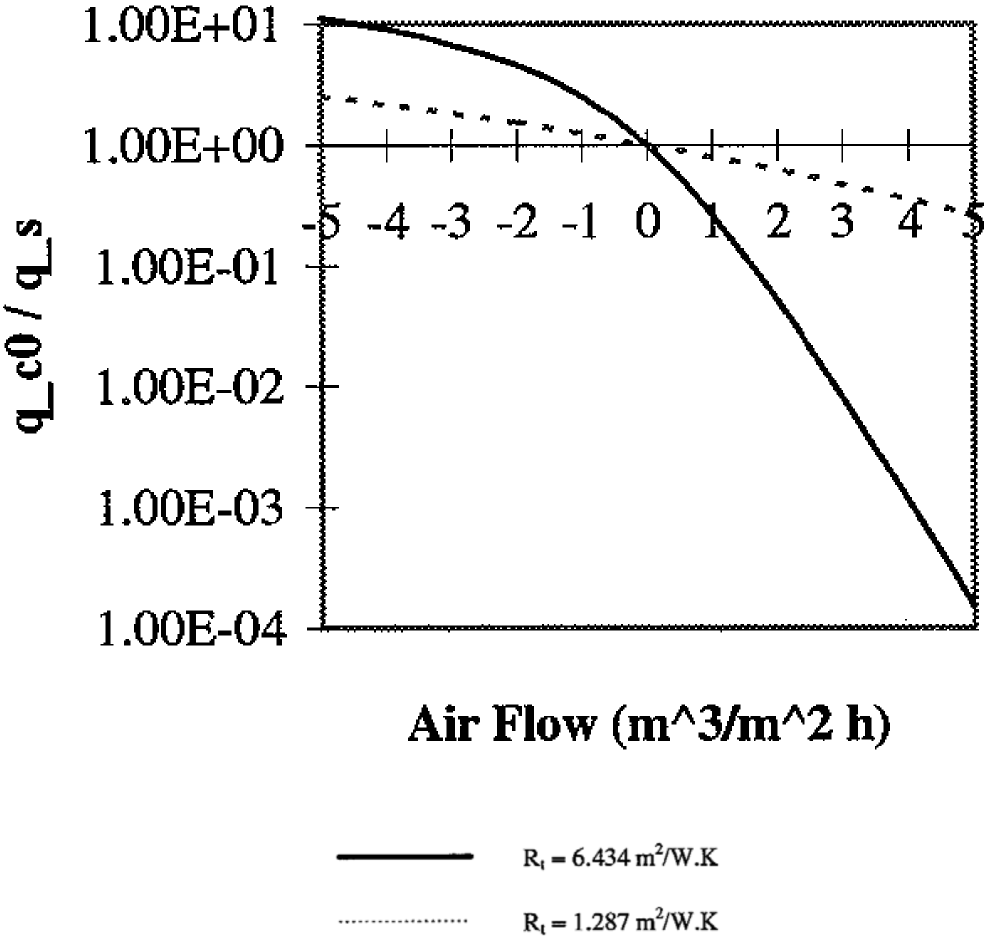
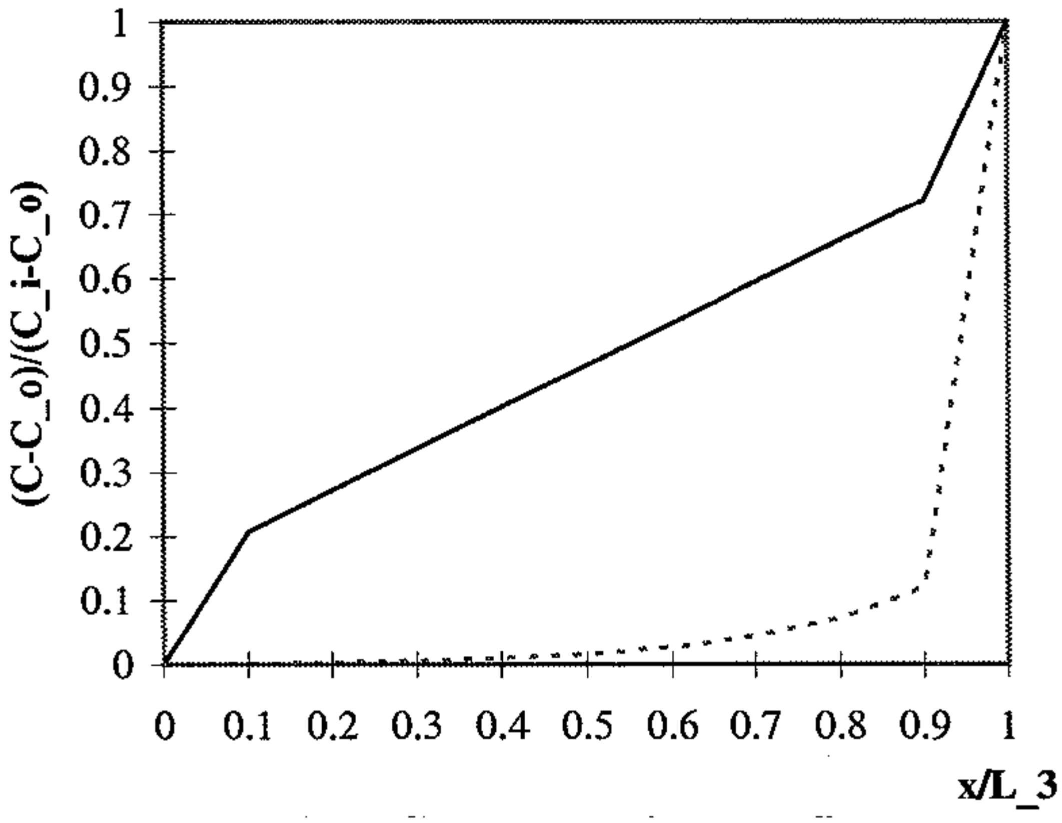


Fig 4 Ratio of dynamic to diffusive heat fluxes



Envelope Diffusion Resistance = 2.765 x 10^4 s/m

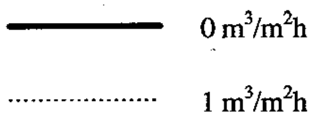


Fig 5 Concentration Profiles

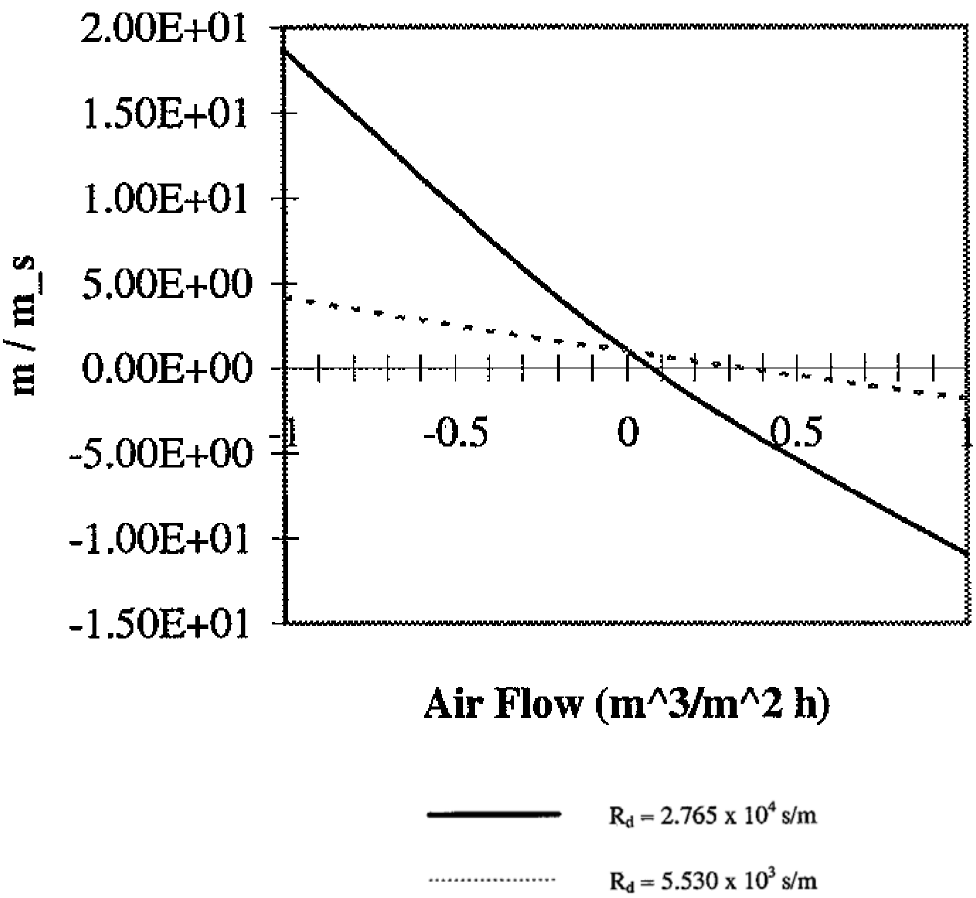


Fig 6 Ratio of dynamic to diffusive mass fluxes