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Parametric sensitivity analysis to maximise auxetic effect of polymeric fibre based helical yarn

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Abstract

Studies on designing polymeric fibres based helical auxetic yarn (HAY) to maximise their auxetic effect are yet to propose optimised design configurations for general impact mitigation applications. This study therefore presents optimal design parameters through analytical calculations and finite element (FE) method. Three main design parameters were considered which includes Poisson's ratio, core/wrap diameter ratio, and starting wrap angle. The Poisson's ratio of the HAY was calculated by measuring its total diameter at a given rate of strain. The investigation found here to be a starting wrap angle of a HAY (critical angle) that resulted in the highest possible exhibiting of the auxetic effect. The critical angle was determined to be 7°, and a maximum NPR of -12.04 was achieved with this design.

Keywords: helical auxetic yarn (HAY), design parameters, negative Poisson's ratio (NPR), sensitivity, parametric analysis, safety applications.

Nomenclature

NPR – Negative Poisson's Ratio

FEA – Finite Element Analysis

HAY – Helical Auxetic Yarn

Θ – Wrap angle (°)

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v_{xy} – Poisson's ratio of yarn

L – Length of yarn (mm)

L_{c0} – Initial core length (mm)

L_{w0} – Initial wrap length (mm)

d_c – Core diameter (µm)

d_w – Wrap diameter (µm)

d_{c0} – Initial core diameter (µm)

d_{w0} – Initial wrap diameter (µm)

ε_x – Longitudinal engineering strain

ε_y – Contractile engineering strain

1. Introduction

It has been suggested that helical auxetic yarn (HAY) can be woven into technical auxetic textiles and placed in a composite for body armour and blast mitigation applications [1]. A material that is auxetic exhibits a negative Poisson's ratio (NPR). This means that the cross-section of the material will become larger when a tensile force is applied in the transverse direction, and smaller when a compressive force is applied in the transverse direction. The opposite is true in conventional materials [2]. Auxetic materials have been found to possess a range of unconventional mechanical properties. These include increased indentation hardness [3], fracture toughness [4], strain energy dissipation [5], and shear toughness [6, 7]. These unusual properties make auxetic structure ideal for use in many applications in many different disciplines, including defence, fashion, medicine and sport [8-10].

The helical auxetic yarn (HAY) presented itself as the most fitting structure to be chosen for the study; as well as having the greatest potential to maximise the auxetic effect

[11-19]. The HAY has been shown to be capable of achieving a NPR of -6.8 [17], which is the highest published value of any auxetic material to date. An array of HAYs that make up a fabric sheet all exhibiting a NPR would amplify the auxetic effect, thus increasing the desirable enhanced mechanical characteristics. Work published by previous authors up to now, has identified key parameters involved in manipulating the behaviour of the HAY, such as the starting wrap angle [13-15,17], tensile moduli of component materials [13, 15-17], and fibre diameter sizes [13-14]. However, contradictory claims made by various authors as to which design configuration increases the auxetic effect have made it evident that further investigation is required to determine the optimal design parameters. Whilst previous studies have shown that lowering the starting wrap angle tends to result in a higher maximum NPR [13-14, 17]; none have identified at which angle that assumption becomes untrue. This study aims to determine the optimal design configuration for maximising the auxetic effect by finding the starting wrap angle which results in the greatest NPR using finite element analysis (FEA). The scope of the study was extended to investigate the effect of changing core and wrap diameter sizes through FEA.

Papers published on HAYs since they were first designed by Hook [7] have studied the design characteristics of the structure and focused on identifying the defining factors that contribute to the auxetic behaviour in the material [13-17]. This has been done mainly through simulated or experimental tensile testing; or by comparisons of both tests. The structure of the HAY can be defined by various geometrical parameters that all have an effect on its auxeticity to varying degrees. These were the factors considered in the modelling of the yarn. The factors are: wrap diameter, core diameter and starting wrap angle [13]. The material properties that affect a HAY's performance are: Young's modulus of both core and wrap; and Poisson's ratio of core and wrap. Various studies of HAYs [11-12, 15] showed that the wrap angle has a profound effect on the NPR value. For example, different values for

NPR obtained by Miller *et al.* [11] based on three different HAY designs. The lowest wrap angle of 10° resulted in the largest NPR value of -5.8, indicating that a low starting wrap angle is desired for maximising the auxetic behaviour of the HAY. The results of Sloan *et al.* [15] study generally agreed with these findings. The HAY with the lowest angle of 13° achieved a maximum NPR of around -2.5. The HAYs with wrap angles of 30° and 38° did not ever have a negative Poisson's ratio when put in tension. This revealed that the influence of the starting angle of the wrap on NPR is so prominent, that an angle too large will prevent the auxetic effect from taking place at all [15].

Wright *et al.* [14] observed that the selection of boundary conditions at the end of the yarn have a minimal effect on the yarn behaviour after 10 cycles. A model made up of 10 cycles displayed results within 1.5% accuracy of a model of 50 cycles – showing that the end effects did not have a profound effect on result accuracy at that many cycles. The findings indicated that designing of the HAY model did not require an excessive number of cycles. Wright *et al.* [14] also claimed that the stiffness of the fibre components were the major influences on the performance of the HAY. They alleged that a HAY with a low stiffness ratio is not fit-for-purpose, and it was shown that a lower core/wrap diameter ratio results in a higher NPR value. The study also highlighted the effect that increasing the Young's modulus of the wrap had on the NPR value. Study by Sibil and Rawan [13] claimed that by increasing the core/wrap diameter ratio – and therefore lowering the stiffness ratio – the value of NPR can be increased. They showed that a HAY with a stiffness ratio of 0.003 (achieved by virtually reducing the wrap diameter by three orders of magnitude) displayed a higher magnitude of NPR when compared to a HAY with a stiffness ratio of 7.4. These contrasting results made it difficult to predict the ideal design parameters the HAY should have had, but elucidated the need for further study to determine the ideal design configuration to maximise the auxetic effect. To further develop the design of the HAY, it was important to identify the optimum configuration of core and wrap that this previous work had not shown. However, both studies showed that the core/wrap diameter ratio is inversely proportional to the core/wrap stiffness ratio.

For the arrangement of wrap and core to work as intended; the wrap should be composed of material with a relatively high Young's modulus, and the core should be composed of material with a Young's modulus lower than that of its wrap counterpart [18]. It is also important to note that the behaviour of HAY fabrics is heavily affected by the inter-yarn contact friction and shear, and in single HAYs this is compounded by the friction and contact between the core and the wrap. However, in this study, the model does not consider contact friction between the core and the wrap, and therefore, the purpose of this study is to optimise the design features of the auxetic behaviour under deformation mechanisms and enhanced properties because of having a negative Poisson's ratio.

2. Methodology

The model in this study was analysed under the axial tensile load test conditions. The tensile testing method is the only procedure to date, that has been used to determine the NPR of a HAY [11-19]. The load was applied until a maximum deformation of 1 mm per cycle of HAY had been reached [17], and the model does not consider contact friction between the core and the wrap.

2.1 Poisson's Ratio

To discover how prominent, the effect of changing the component Poisson's ratios of the HAY, a sensitivity test was carried out. This would involve the comparison of NPR values achieved by the HAY used in Sibal and Rawan's study [13], with a new design. As previously mentioned, the study assumed a Poisson's ratio value of 0.3 for both core and wrap components. These components were made of *nylon* and *carbon fibre*, respectively. However, the real Poisson's ratio for these materials can be sourced from data sheets or manufacturers. A data sheet provided by DuPont confirmed that the Poisson's ratio for nylon 66 is 0.4 [20]. These values were selected as the material properties for the new design of HAY. The remaining input parameters of each model, i.e. Young's modulus, component diameter and wrap angle, are shown in **Table A** (*sensitivity test*). The reason for changing only component Poisson's ratio values was to confirm from the test that a different NPR would be attributed to Poisson's ratio, and prove the new model results to be more authentic.

2.2 Wrap Diameter

Sibal and Rawan's study [13] attempted to prove that a greater core/wrap diameter ratio increases the auxetic effect, but Wright *et al.* [14] study contradicts their results, by claiming the opposite to be true. These inconclusive results indicated that further investigation was required to verify which of those claims were correct. To do so, models with varied core and wrap diameters were designed and analysed. Using a base model with the same parameters as Model A2 from the previous section, variant designs were produced for the test. Model A2 was used as the base design for this test because the material properties of that model were a better representation of the component materials than Model A1. The variant models' input parameters are shown in **Table A** (section 2: core-to-wrap diameter ratios).

The first variant, Model B1, was designed with a new core diameter of 600 µm (as used in the Wright *et al.* [14] study). All other parameters remained unchanged so that the results would prove that a change in NPR was due to the reduced core diameter. To further investigate the effect of changing diameter size, a second variant, B2 was designed. This model had an increased diameter, proportional to the size difference in B1. This was done so that a trend in behaviour of the HAYs can be attributed to changing the diameter size of the core, and to make explicit the effect it has on the auxetic effect.

Table A. Input parameters for models used in: (1) sensitivity test, (2) to determine the effect of varied core-to-wrap diameter ratios, and (3) critical angle test.

	Models	**Wrap Young's modulus (GPa)**	**Core Young's modulus (GPa)**	**Wrap Poisson's ratio**	**Core Poisson's ratio**	**Wrap Diameter (μm)**	**Core Diameter (μm)**	**Wrap Angle (°)**
(1) Sensitivity test	A1 (Sibal & Rawan [13])	143	1.6	0.3	0.3	200	700	10
	A2 (base model)	143	1.6	0.3	0.4	200	700	10
(2) Core-to-wrap diameter ratios	B1	143	1.6	0.3	0.4	200	600	10
	B2	143	1.6	0.3	0.4	200	800	10
	C1	143	1.6	0.3	0.4	171.4	700	10
	C2	143	1.6	0.3	0.4	228.6	700	10
(3) Critical angle test	D1	143	1.6	0.3	0.4	200	700	9
	D2	143	1.6	0.3	0.4	200	700	8
	D3	143	1.6	0.3	0.4	200	700	7
	D4	143	1.6	0.3	0.4	200	700	6

The third variant, C1, was designed with a reduction in wrap diameter. It was decided that the proportion the wrap was reduced should also be equivalent to the percentage reduction in the core of B1. This was calculated using the following equations.

$$\%\,Reduction\ in\ core = \left(1 - \frac{New\ diameter}{Original\ Diameter}\right) \times 100\% \qquad \text{(Eq. 1)}$$

$$\%\,Reduction\ in\ core = \left(1 - \frac{600}{700}\right) \times 100\% = 14.3\%$$

The new diameter of wrap can then be calculated by using the original wrap diameter in place of original core diameter, and rearranging.

$$New\ diameter\ of\ wrap = Original\ Diameter \times (1 - \%\ Reduction) \quad \text{(Eq. 2)}$$

$$New\ diameter\ of\ wrap = 200 \times (1 - 0.143) = 171.4\ \mu m$$

Variants C1 and C2 were included to further clarify results, and to see which component diameter size has a greater influence on the behaviour of the HAY. By using the same percentage reduction for the core diameter of B1 and wrap diameter of Model C1, the influence each component had on maximum NPR can be distinguished. As before, all other input parameters were unchanged. Similarly, a final variant, C2 was designed with an increased wrap diameter. As with models B1 and B2, the increase in wrap size was proportional to the reduction.

2.3 Wrap Angle

To determine the exact angle, the wrap should make with the axis of the core, models of varying wrap angle size were designed. Like before, Model A2 was used as the base design for the input parameters. Each model in this test was designed with a wrap angle 1° lower than the previous design, starting at 10°. It was expected that eventually a 'critical' angle would be reached, meaning that the maximum NPR value that was achieved cannot be surpassed by lowering the starting angle by another degree. Due to the gradual straightening of the wrap under strain, it would be expected that lowering the angle past the critical angle would cause the maximum NPR to start to decrease. For this reason, the critical angle can be considered the desired wrap angle for optimal performance. The **Table A** (section 3: critical angle test) shows material properties and parameters of the wrap and core components of HAY for each model. Again, each model designed consisted of the same material properties and design parameters; so, that the results of this test could be verified.

3. Analytical Analysis

The initial geometrical parameters of each design of HAY were determined using simple trigonometry. **Figure 1** shows the arrangement of wrap and core components for each cycle of HAY form a right-angled triangle, where the length of hypotenuse is equal to the length of the wrap if the wrap was unravelled and laid out straight. The initial length of the core, L_{c0} – and therefore the length one cycle of HAY – can be found by calculating the length of the bottom leg of the triangle. The known input parameters: core diameter, d_{cO}, wrap diameter, d_{wO}, and wrap angle, θ can be inserted into the following equations to calculate core and wrap lengths, respectively [14].

$$L_{c0} = \frac{\pi(d_{cO}+d_{wO})}{tan\theta} \quad \text{(Eq. 3)}$$

$$L_{w0} = \frac{\pi(d_{cO}+d_{wO})}{sin\theta} \quad \text{(Eq. 4)}$$

To calculate the total initial length of the yarn, L_{c0} can just be multiplied by the number of cycles in the yarn. In each instance, the design used 10 cycles, so the total length of the yarn can be found by increasing the core length value by one order of magnitude. In this work, the focus was on the HAY's elastic properties, and engineering strain was used. Since strain is the ratio of change in length over initial length, the following equation can be formed and used to calculate the length of core and wrap at a given strain [13]:

$$L = L_0 + \varepsilon_x L_0 \quad \text{(Eq. 5)}$$

where ε_x is the level of axial tensile strain. This is also the type of strain that induces auxetic behaviour in the yarn. Once this had been computed, the changing diameters of both core and wrap can be determined. The Poisson's ratio, v_{xy} is defined as the negative ratio of lateral contractile strain, ε_y to longitudinal strain, ε_x [21]. This can be applied to each component materials' respective Poisson's ratios and geometrical parameters. Thus, the diameters of the wrap and core under a given longitudinal strain can be found using the following equation:

$$v_{xy} = -\frac{\varepsilon_y}{\varepsilon_x} \quad \text{(Eq. 6)}$$

where ε_y is the change in diameter divided by the initial diameter of the component. In accordance with the geometrical configuration of the HAY – which consists of elastic core with a smaller diameter fibre wrapped in a helical manner around it – the Poisson's ratio of the HAY can be calculated from first principles. This was under the assumption that friction between the yarn's components could be ignored, and that they were always in perpetual radial contact. It was considered that a repeating cycle of HAY must have a total diameter, d_t, which could be defined as a "virtual cylinder" that exactly contained the wrap and core, under a given level of tensile strain.

The initial total diameter, d_{t0} can be determined by computing the total diameter that would enclose the cross-section of the yarn under zero strain. Since the wrap makes half a revolution of the core in one cycle of HAY, this means it is present at both the top and bottom of the core's cross-sectional area. This means that the initial total diameter is given by:

$$d_{t0} = d_c + 2d_w \qquad \text{(Eq. 7)}$$

Using equation 6, the Poisson's ratio of the yarn, v_{xy} can be computed by using d_t and the initial total diameter, d_{t0} to find ε_y. The yarn Poisson's ratio is the most important value to the study, as it indicates the auxetic effect taking place.

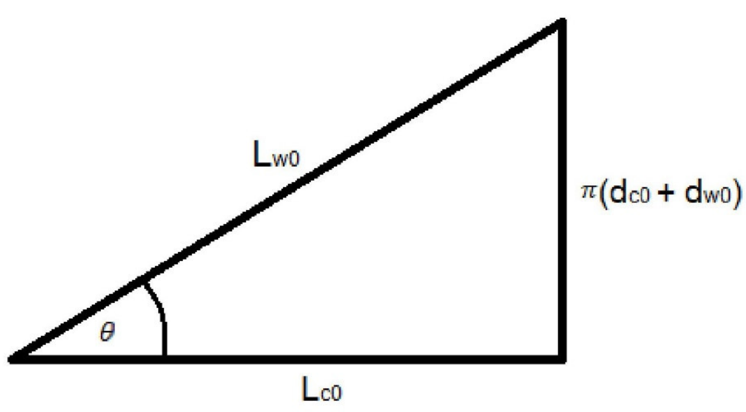

Figure 1. *Right-angled triangle formed if the wrap was unravelled from the core (adapted from ref. [14]).*

4. Finite Element Analysis

HAY's computational design in this study were modelled using SOLIDWORKS® 3D CAD software. The helical arrangement of the wrap component around the core was simply modelled using the 'helix' feature and by extruding the base of a concentric 2D core with a centre point aligned with the centre of the helix. The model is illustrated in **Fig. 2(a)**. Since all the models consisted of the same two components wound together in a helical manner, the design parameters can be easily edited using a configuration **Table A**, so that several models were produced. The configuration software did not allow for the wrap angle to be a predetermined input parameter into the configuration table, so the angle had to be calculated using equation 4. The models were then individually imported into ANSYS® Workbench for analysis. This was modelled using only half the geometry with symmetry constraints due to the symmetrical nature of the design; although since ANSYS®'s parametric optimisation features were employed it was easier to just use the full model. As previously mentioned, the HAY designs modelled each consisted of ten cycles, to increase the accuracy of the results [14].

Once a model was generated in SOLIDWORKS® and imported into ANSYS® Workbench, the respective material and its properties was then assigned to the wrap and core components. The ANSYS® simulation did not provide the component materials of the HAY in their database, so "Carbon Fibre" and "Nylon 66" were added as new materials. As mentioned above, the component material properties that affect the performance of a HAY are Young's modulus and Poisson's ratio. These were the parameters put into the Engineering Data part of the ANSYS® when adding new materials to the project. Conveniently, there is an option that allows for the linear isotropic elasticity of a newly added material to be derived solely from these two input values. This allowed for an accurate representation of the HAY components' material behaviour. These properties can be changed to allow for the sensitivity test to be carried out, so that Nylon 66 can have a Poisson's ratio of 0.3 for Model A, and 0.4 for Model B (**Table A**). Previous work on elastic-plastic modelling yielded inaccurate and unreliable results, caused by plastic deformation [19]. Essentially, the HAY must be able to recover from deformation to maintain its performance characteristics. For this reason, the study focussed solely on the effects of NPR within the elastic region of the HAY.

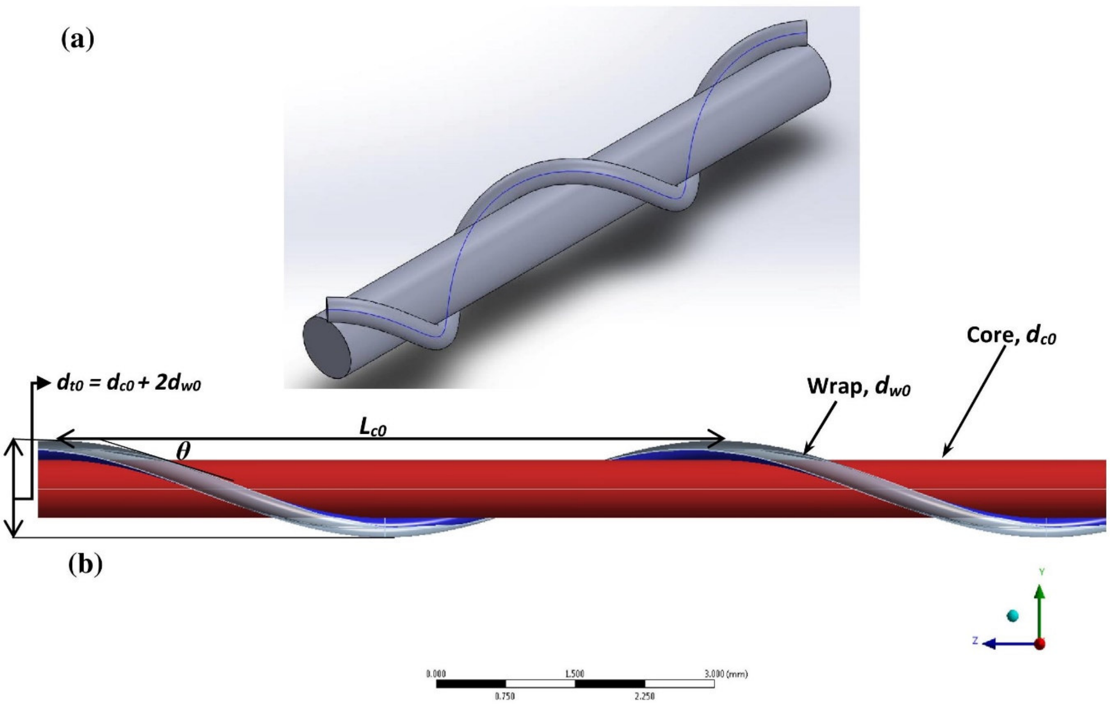

Figure 2. *(a) Two cycle model of HAY created in SOLIDWORKS® 3D CAD software, and (b) HAY connections showing the contact region between the wrap and core, highlighted in blue and red, respectively.*

To accurately portray the motion between wrap and core components, a contact region was applied (i.e. the model does not consider contact friction between the core and the wrap). As shown in **Fig. 2(b)**, the inner surface area of wrap, and outer surface area of the wrap were selected as the connected areas, so that the components would act as a single body during simulation. Had this connection not been applied, the wrap and core would have separated from each other as soon as a load was applied. Initially, the mesh sizing applied to each individual component of HAY was to be finalised by conducting a convergence test, to determine the coarsest mesh that could be applied to the HAY that gave the same level of accuracy in results. In FE modelling, a finer mesh typically results in a more accurate solution. However, as a mesh is made finer, the computation time increases. Solution iterations were carried out until convergence was achieved. The mesh was converged down to 0.1 mm for the core and 0.05 mm for the wrap.

Tetrahedral elements were used (are robust for large deformations, and as any 3D volume, regardless of shape or topology, can be meshed with tetrahedral) for the wrap for mesh refinement to be applied. Quadrilateral elements were used for the core mesh, as quadrilateral might have a slight edge in terms of accuracy for the primary variable (displacements). Face sizing was applied to the largest surface areas of the wrap and core, as shown in **Fig. 3(a)**. The element sizing was 0.05 mm and 0.1 mm for wrap and core respectively. Originally, a refinement of 2 was assigned to the inner face of the wrap (where it is in contact with the core), as can be seen in **Fig. 3(b)**. This arrangement is demonstrated in **Fig. 3(c)**. There were various ways in which the boundary conditions and loading could be applied to simulate a tensile test. A force or pressure could have been applied to the end faces of the wrap and core, however this method was not used due to the assumption that the cross-sectional area is constant – which is untrue [10]. The method used by Shen [19] employed a displacement at either end of the HAY, and simulated a pulling motion from both sides. This method was adopted and modified so that the HAY was instead fixed at one end by using a fixed support; and was stretched at the other by applying a displacement. As the displacement was going from left to right, the value applied was negative and in the Z-direction. The displacements in the X and Y directions were set as “free” to allow the auxetic behaviour to take place. The resultant deformation is illustrated in **Fig. 3(d)**. The deformation velocity was 1mm/cycle of HAY per second.

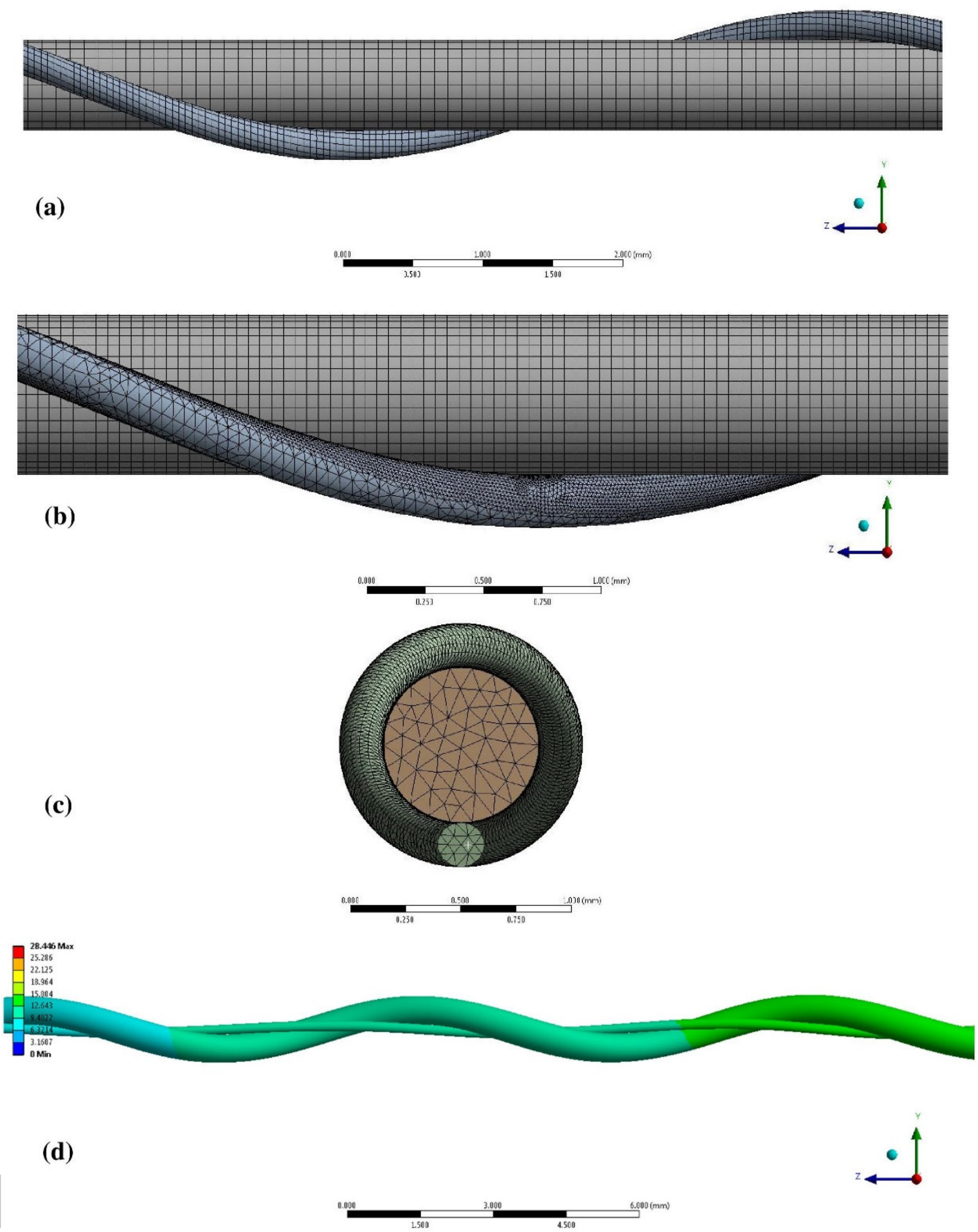

Figure 3. *(a) Final mesh sizing of largest surface areas of HAY components, (b) invalid model with refined mesh on contact region of wrap, (c) cross-sectional area of mapped face meshing applied to model, and (d) fully deformed HAY after tensile test had been simulated.*

Utilising the deformation probe ANSYS® provides, the maximum deformation of the core in the Y-direction can be measured by selecting the top face of the core. As the HAY is symmetrical, this value was just doubled to give the distance from peak to trough. Once the total diameter was calculated for the given rate of strain, equation 7 was used to calculate the Poisson's ratio of the HAY, as both the longitudinal strain, ε_x and lateral strain, ε_y can be calculated.

5. Results

5.1 Validation of Negative Poisson's Ratio

As can be seen in **Fig. 4**, the same kind of typical response was being induced by each model for a total deformation of 10 mm. This response was explained by the general pattern of each curve. When strain was first applied, it caused the wrap to tighten itself around the peripheral of the core – resulting in the total diameter shrinking below its initial size. This is shown by the sudden spike at the beginning of the graph, where the Poisson's ratio of the HAY shot up, then rapidly decreased due to the wrap starting to twist the core. The slope of each curve as it passes through the positive x-axis to negative represents how quickly the HAY became auxetic, and the minimum at the bottom of each curve represents the maximum NPR value of that HAY. Models with different wrap angles had different lengths, and so the rate of strain for the same change in length was not the same. Therefore, the curves on the graph do not stop at the same point. **Table B** summarise the maximum NPR value of each model during finite element simulation and the rate of strain at which it was achieved. The largest NPR value of any of the models was -12.04 (i.e. Model D3). The earliest rate of strain that the maximum NPR value was reached was at 1.65% (i.e. Model D4). The significance of these results and relevance to the study are discussed in detail in the following sections.

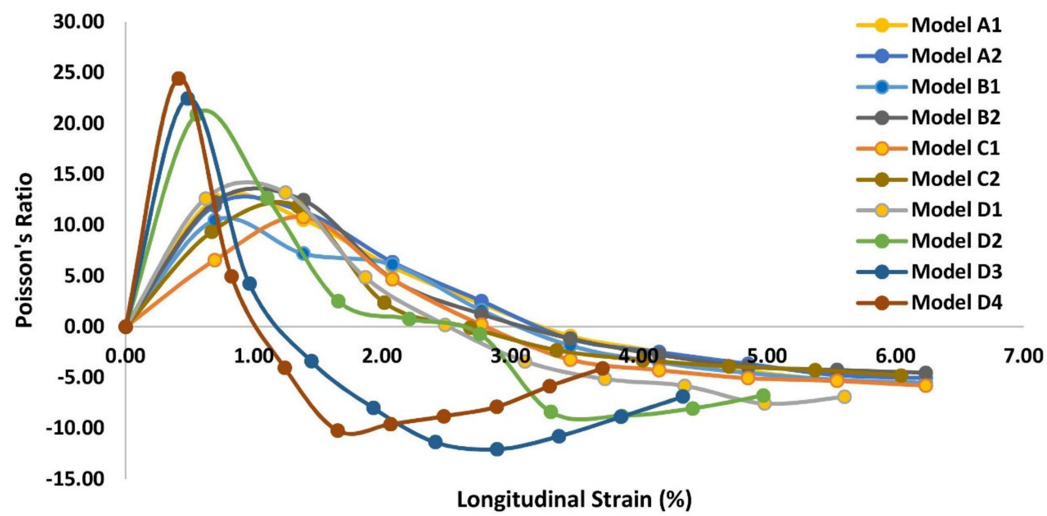

Figure 4. *Graph of longitudinal strain vs. Poisson's ratio of HAY for all finite element computational models tested using ANSYS® Workbench [figures appear in* colour *in the online version].*

Table B. ANSYS® Workbench finite element simulation results for each model tested, which shows the maximum NPR achieved and the percentage of longitudinal strain.

Model	Angle (°)	Max NPR	Strain rate at max NPR (%)
A1	10	-5.75	6.24
A2	10	-5.04	6.24
B1	10	-4.54	5.61
B2	10	-5.50	7.02
C1	10	-5.80	6.44
C2	10	-4.41	6.04
D1	9	-7.55	4.98
D2	8	-8.77	3.87
D3	7	-12.04	3.38
D4	6	-10.22	1.65

The results of Model A1 provided good validation of the simulation results. This study's maximum NPR of -5.75 for Model A1 was close to Sibal and Rawan's [13] maximum NPR of -5.7, demonstrating that the FEA results were successful. This means that the percentage difference between simulation results and theoretical results for Model A is less than 1%. Wright *et al.* [14] tested the same design in a practical experiment and measured a maximum NPR of -5.8, which further verifies the simulation results. As the same testing method used for Model A1 was used for every model, the data and results obtained from the simulations should be considered valid. It is possible that the mesh of the model on ANSYS® cannot be refined much to a point. This can explain the difference in simulation results from the theoretical [13] and experimental [14] results of previous studies. However, the results obtained are for demonstrating the different auxetic behaviours of different HAY designs; so, these slight inaccuracies are not detrimental to the goals this study set out to achieve. Wright *et al.* [14] found that the results acquired when using a medium density mesh for a HAY were within 1% accuracy of the high-density mesh results.

5.2 Sensitivity Test of Core Poisson's Ratio

The results of Model A1 and Model A2 were used for the sensitivity test. As mentioned earlier, the Model A1 was the same design used in the Sibal and Rawan study [13], with an assumed core Poisson's ratio of 0.3. The Model A2 (base model in this investigation) was a modified version of Model A1, but with a core Poisson's ratio of 0.4. As mentioned, this is a more authentic value for this material, sourced from a DuPont data sheet [20]. It was expected that by altering the Poisson's ratio of the core component, the HAY would respond to the axial tensile load differently and therefore give results that can be considered more reliable. The difference in results are shown in **Fig. 5(a)**. The Model A2, as expected, yielded a different set of results to Model A1. However, despite the difference in maximum NPR values caused by increasing the core Poisson's ratio, the general behaviour of the HAY was relatively unchanged. It can be seen from the graph that the Poisson's ratio peaks at roughly the same percentage of strain (0.9%); becomes auxetic at roughly the same percentage of strain (3.25%), and becomes most auxetic at the same percentage of strain (6.24%). This shows that making the core less compressible – by increasing its Poisson's ratio to 0.4 – does not greatly affect the behaviour of the HAY.

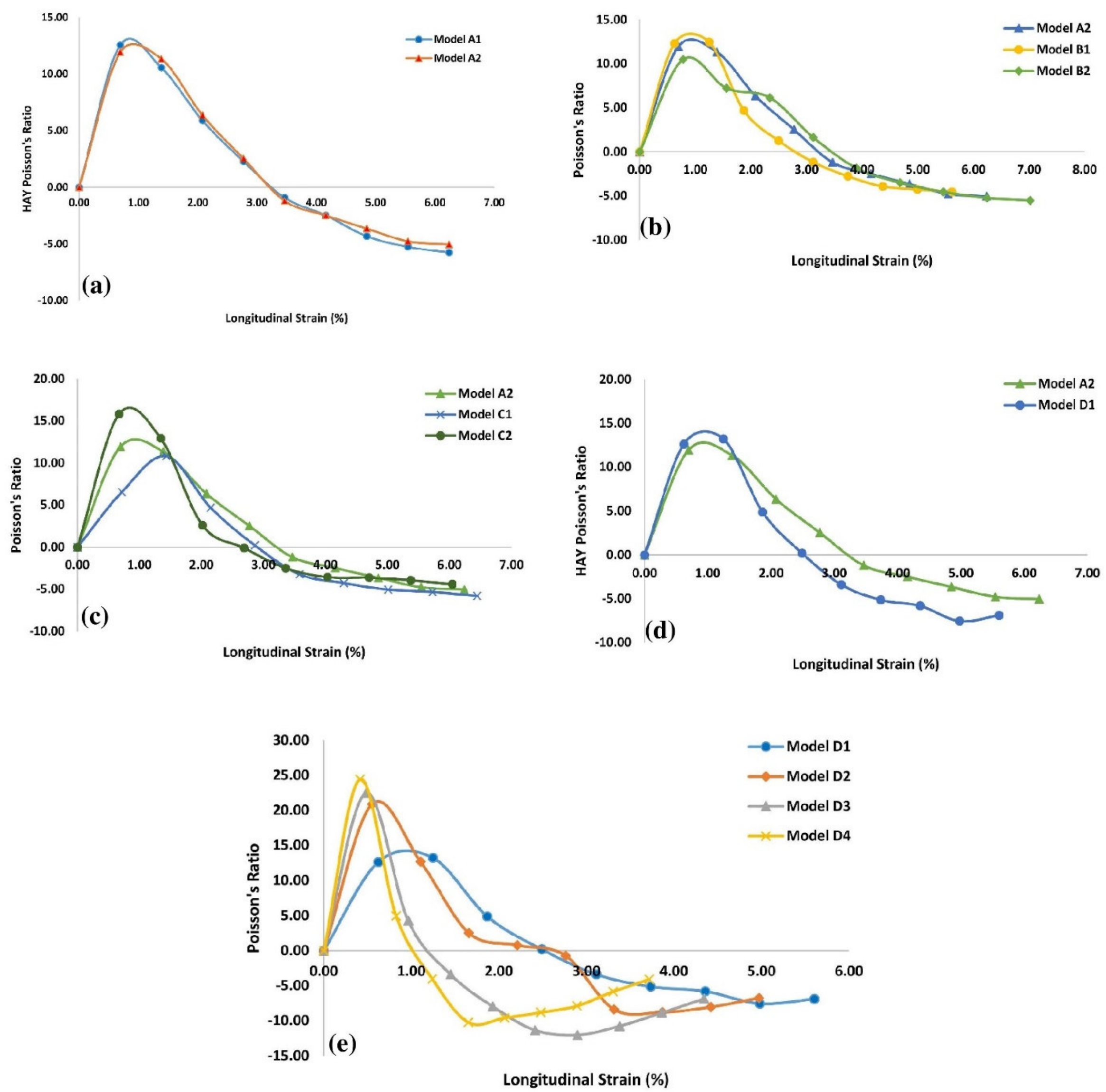

Figure 5. *Graph of HAY Poisson's ratio vs. longitudinal strain (finite element computational models using ANSYS® Workbench): (a) for Model A1 and Model A2, showing the point at which both designs' maximum NPR values were achieved, (b) for Model A1, B1 and B2, (c) for Model A2, C1 and C2, (d) for Model A2 and D1, and (e) for models used in critical angle test [figures appear in colour in the online version].*

The lower compressibility of Model A2's core explains the differences in results when compared to Model A1. For both models, the sharp spike at the beginning represents the tightening of the wrap around the core, causing the total diameter to drastically reduce –

therefore resulting in a large positive Poisson's ratio value. The higher peak value for Model A1 is due to the core being more easily compressed by the wrap, causing a reduction in its total diameter. Because of Model A2's larger core Poisson's ratio, its total diameter cannot be reduced by the same amount as Model A1. After the peak, the slope of Model A2's HAY Poisson's ratio declined more gradually and slower than the slope of model A1. This trend indicates that a HAY with a more compressible core will react quicker to the same tensile load.

5.3 Effect of Component Diameter Sizes

Another aim of the study was to investigate the effect of changing the size of component diameters on NPR. This was carried out by testing variants of Model A2 with core and wrap diameters both larger and smaller. Focusing on the variants, B1 and B2, the response of each design can be seen in **Fig. 5(b)**. Included on the graph is the base model, A2's response, so that differences caused by changing the size of the core's diameter can be interpreted. It was observed that Model B1 was more greatly affected by the wrap's initial tightening around the core, which was construed by the largest spike in the graph after the initial strain was applied. Conversely, Model B2's core was least susceptible to compression by the wrap and this was portrayed by the shortest spike in the graph once deformation had begun. When depicted graphically alongside Model A2's response, a pattern in the responses can be seen. This pattern indicated that by increasing the thickness of the core, the initial proportion of total diameter reduced by the tightening of the wrap is lessened. It was more difficult to recognise a pattern in **Fig. 5(c)**, which shows the responses of variants, C1 and C2 beside Model A2. No clear trend was observed after the initial strain was applied, which can be due to the small effect on total diameter that marginally changing the wrap thickness had.

Compared to changing the core diameter by 14.3%; changing the wrap diameter by the same proportion causes very little of the total diameter to increase or decrease at all.

It was found that Model B2 achieved a higher maximum NPR than the base model (Model A2); and Model B1 achieved a lower maximum NPR, as shown in **Fig. 5(b)**. The variants' relative increase and decrease in maximum NPR were around the same proportion in difference (-4.54 and -5.50) to Model A2's maximum of -5.04. This was a good indication that the results were coherent. It was found that increasing the ratio of core diameter thickness to wrap diameter thickness caused the HAY become more auxetic at around the same rate of strain. It also made clear that an increase in core diameter thickness caused the activation of auxetic behaviour to happen at a later rate of strain. This was shown by the pattern of the three curves crossing through the x-axis at different rates of strain in **Fig. 5(b)**. Similarly, the maximum NPR was reached at a later stage when the core thickness was increased. This means that the larger core takes longer to become its most auxetic. It can be expected then that changing the wrap diameter thickness also would influence the point at which the HAY became auxetic. This was shown to be true. This was observed in **Fig. 5(c)**, where the same pattern is displayed by the models. Unlike the core thickness, which made the HAY more auxetic when increased, increasing the wrap thickness was seen to result in a lower NPR value.

5.4 Effect of Wrap Angle

The critical angle test was devised to determine the optimal angle which the wrap should initially subtend the axis of the core for the HAY to achieve the highest NPR. The results were obtained by comparing the responses of identical models with different wrap angle sizes to each other. It was found that the wrap angle that gave the largest value of maximum NPR was 7°, making it the critical angle for a HAY. This was confirmed by the subsequent 6° model stopping the trend of a lower wrap angle equalling a higher maximum NPR – which was true of all angles above this value. This trend had been observed in previous studies [12-13, 15] which found that a lower starting wrap angle gave a higher NPR. This means that when designing a HAY for practical applications that wishes to exploit the maximum auxetic effect in the yarn, a starting wrap angle of 7° should be used.

It can be seen from the values in **Table B**, that changing the wrap angle by 1° had a substantial effect on the maximum NPR achieved by the HAY. When compared to other parameters discussed in this study, the starting wrap angle was the most influential design parameter in altering the auxetic effect exhibited by a HAY. **Figure 5(d)** shows the general change in behaviour when lowering the wrap angle, by comparing Model A2 and Model D1 – with starting wrap angles of 10° and 9°, respectively. It shows that reducing the wrap angle will tend to cause a greater reduction in total diameter when the wrap tightens around the core, shown by the higher peak of positive Poisson's ratio in Model D1. It also demonstrates the typical behaviour of a lower wrap angle causing the HAY to become auxetic at an earlier rate of strain, as Model D1 becomes negative at around 2.5% strain; compared to around 3.25% strain for Model A2. Furthermore, a lower wrap angle of a HAY can be seen in **Fig. 5(a)** to cause the maximum NPR to be achieved at an earlier rate of strain, whilst also giving a higher maximum value at the same time.

Changing the wrap angle has a radical effect on the overall auxetic behaviour of a HAY. This can be discerned when examining the completely different curves of the graph in **Fig. 5(b)**. The graph shows the behaviours of HAYs with wrap angles proceeding from 9° down to 6°. This continues to show that by lowering the starting wrap angle, the total diameter is reduced by the greatest proportion – and at an earlier rate. This then results in a steeper slope for HAYs with a lower wrap angle, which in turn causes the auxetic effect to be achieved faster. This is true even beyond the critical angle, where the auxetic effect in Model

D4 is activated at around 1% strain, and Model D3 – the design with the critical angle – becomes auxetic at around 1.2% strain. However, as can be seen in **Fig. 5(e)**, Model D4 does not achieve a higher maximum NPR than Model D3; which it would have been expected to, given the trend of previous results. Therefore, the critical angle was determined to be 7°.

Another behaviour characteristic of the models observed was the rate at which the auxetic effect reduced after the peak, shown in **Fig. 5(e)** by the decreasing NPR values that follow the maximum NPR value. All models can be seen to reduce in auxeticity after the maximum NPR is reached. This is shown graphically as a minimum turning point – the minimum representing the point which the maximum NPR occurred. The graph shows a more gradual reduction in auxetic behaviour exhibited by higher wrap angles; and conversely a much higher, faster rate of reduction of auxetic behaviour in HAYs with lower wrap angles. This rate of change is so significant, that it caused Model D3 – with the highest NPR value – to quickly become less auxetic than its predecessor, Model D2, at just 4% strain. This accelerated rate of reduction of auxetic behaviour means selecting the optimum design angle would depend on the specifications of the practical application that HAY would be needed for, and the operating conditions the HAY would be under.

6. Discussion

6.1 Sensitivity Analysis of Core Poisson's Ratio

The sensitivity test, whilst not vital to the aims of the study, provided a good means of obtaining more realistic results from the simulations. Had the model used by Sibal and Rawan [13] been used as the base model for all subsequent designs generated in this study, the behaviour and responses of the HAYs would not have been expected to vary by a great deal. This is evident from the similarity in results between Model A1 and Model A2. What it means for this study however, is that the results of the tests that followed the sensitivity test can be seen as more accurate; simply due to the fact that the core's Poisson's ratio value was taken from a data sheet [20]. It is important to stress that using the assumed values for component Poisson's ratios do not jeopardise the legitimacy of results from Sibal and Rawan's study [13], but rather proves the results of this study to be more realistic. Their study found that its results were close to the experimental results of the Wright *et al.* [14] study, but not the same. There is a possibility that this difference in results can be attributed to the incorrect assumption made, since the results for the same model tested in this study falls in between both studies' maximum value for NPR. When possible, the correct material property values should always be used.

The results of the sensitivity test did, however, highlight the effect Poisson's ratio plays in the auxetic response of a HAY. It can be seen from the graph that both HAYs crossover from positive to negative Poisson's ratio at the same rate of strain, but the steeper slope of Model A1 causes it to achieve a greater maximum NPR at 6.24% longitudinal strain. This suggests that a HAY with a more compressible core will become more auxetic, however it is difficult to make this claim based on only two sets of results. If this suggestion is true, a designer may find it useful to consider the compressibility of the core when selecting materials for an application. Based on the findings of the sensitivity test though, it is not thought to be a particularly defining factor for maximising the auxetic effect. For high velocity impact application, the shear modulus of the material (defined in equation 9) would be the desired property of the core to maximise; and so, a lower Poisson's ratio and higher tensile modulus should be aimed for in the selection of material [22].

6.2 Effect of Component Diameter Sizes

The main findings of this part of the study clarified uncertainties created by previously conflicting results. Until recently [e.g. 23], it was unclear whether increasing or decreasing the core/wrap diameter ratio would result in a higher maximum NPR. However, the current results agree with Sibal and Rawan's [13] claim that a nano-sized wrap would cause the HAY to exhibit a considerably larger NPR. These findings mean that when the core/wrap diameter ratio is increased, the maximum NPR will increase and amplify the auxetic effect exhibited by the HAY. Therefore, for future designs of HAYs, selecting a wrap with a large diameter should be avoided, as it has been recently demonstrated by Zhang *et al.* [23] that a larger core/wrap diameter ratio will offer a better and earlier auxetic performance of the sample.

Though Sibal and Rawan's findings [13] were based on theoretical results, this study backs up their claim and suggests that use of carbon nanotubes (CNTs) can maximise the auxetic effect in HAYs. Since the wrap was the greater influence on the HAY's auxetic response, it would be more sensible to focus on the material options and geometrical parameters of the wrap when designing a HAY. An investigation into CNTs was attempted during this study due to the exceptional mechanical qualities they possess. The tensile moduli of CNTs exceed 1 TPa – around ten times stronger than that of the carbon fibre material used in this study [24]. The advantage that using an even stronger wrap would give was clearly shown in the results. Changing the wrap diameter size had a greater influence on NPR, compared to changing the core diameter. As CNT yarns can be processed in several different ways – including spinning methods like HAYs – this indicates that manufacturing would not create additional practical constraints for a designer [25]. Decreasing the wrap thickness by 14.3% resulted in a maximum NPR of -5.8 (**Fig. 5(b)**); whereas a maximum NPR of -5.5 was achieved when the core thickness was increased by the same percentage. This means the wrap is the more useful component in manipulating the level of auxeticity in a HAY. This should be the biggest consideration when designing HAYs to maximise the auxetic effect, based on component geometrical parameters.

6.3 Wrap Angle Analysis

The results obtained from this test give an insight into which angle may be preferred to suit an application. If the application required a longer onset of auxetic behaviour, then raising the starting wrap angle of the HAY would be the best design option. As this would be at the expense of the maximum auxetic effect that could be reached by the HAY, it would be up to the designer to decide how much of each feature could be selected which incorporated the benefits of both attributes of the HAY into the design.

If the application of the HAY needed a greater expansion of the overall fabric, it can be put constantly under the rate of axial tensile strain that caused the maximum NPR to be achieved. This would result in the maximum auxeticity being maintained always. The need for this sort of application is unlikely though, as most applications of the HAY seek to exploit its changing nature – converting from a conventional material to an auxetic material with increasing axial tensile strain.

6.4 Future Directions: Design Parameters and Properties of HAYs

This paper presents a study of HAYs properties in terms of effect of various selected design parameters but does not consider any contact friction between the wrap and core. For any potential practical applications (e.g. resisting high velocity impact), the design would require the highest possible NPR to absorb a projectile's impact and dissipate its energy through the blanket's pores (created by the expansion of micro-sized HAYs). The early activation of the auxetic effect and high NPR inducing greater energy absorption would fit the criteria of the requirements of the blanket, especially when inter-yarn contact friction and shear are considered in design and analysis leading to industrial manufacturing. Very recently Bhattacharya *et al.* [26] experimentally investigated the effect of the interaction between the core and the wrap fibre on the auxetic behaviour of the HAY, including the effect of their relative moduli. It was revealed that an elevated difference in component moduli causes the wrap fibre embedding itself into the core fibre, thus decreasing the auxetic effect. Zhang *et al.* [23] studied 3-component auxetic yarn based on a stiff wrap fibre (the first component) helically wound around an elastomeric core fibre (the second component) coated by a sheath (the third component). It was suggested that the resultant structure can overcome problems such as slippage of the wrap and changes in wrapping angles previously encountered during the manufacture and utilisation of the two-component HAY. Zhang *et al.* [23] also showed that depending on the coating thickness of the third component, the 3-component auxetic system can demonstrate auxetic behaviour, and the coating thickness can be employed as a new design parameter to tailor both the Poisson's ratio and modulus of composite reinforcement for various applications. As mentioned earlier, the optimal design of HAYs would require the highest possible NPR to absorb a projectile's impact and dissipate its energy through the blanket's pores (e.g. conceptual layout in **Fig. 6**). This is possible, as the recent results by Zhang *et al.* [27] showed that a larger difference in component moduli, a higher core/wrap diameter ratio and a lower initial wrap angle can produce a larger maximum NPR value and thereby a better auxetic performance for HAYs.

Current work and above recent experimental propositions (e.g. [23, 26-27]) has considerable potential to improve HAYs properties and therefore provides a means of evaluating the auxetic quality with potential applications in resisting high velocity impact. Future work should consider further theoretical analysis and experimental verification [e.g. 28-29] and developing a composite wrap and core (where each fibre can be coated with thin graphene layers) for improved NPR properties, especially when single-layer graphene show some negative Poisson's ratio on its own [30].

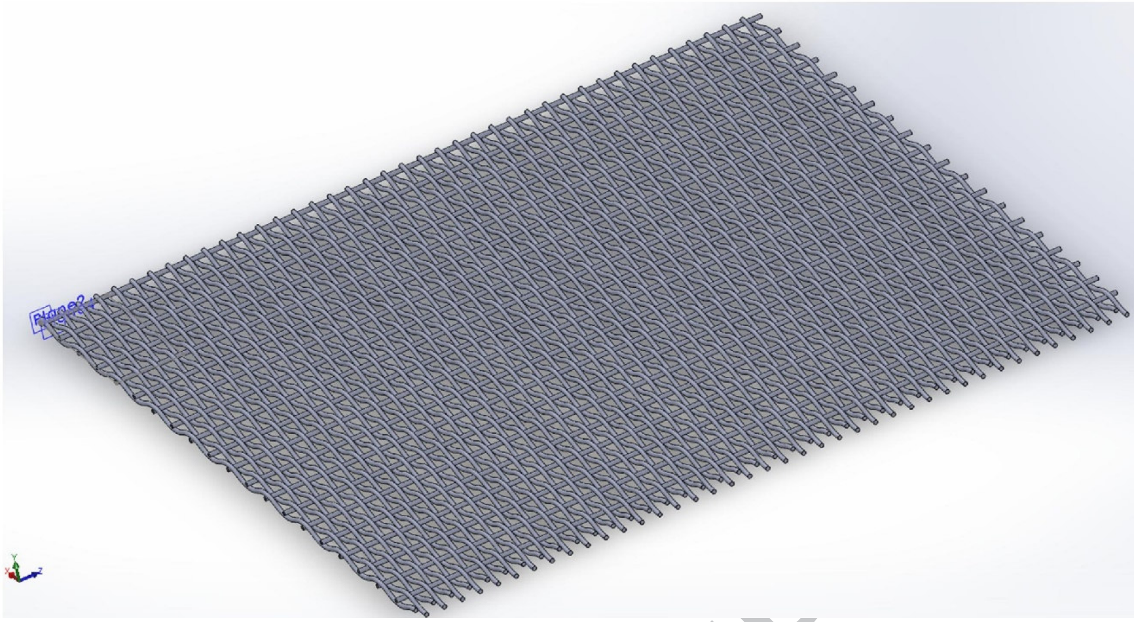

Figure 6. Unused conceptual model of high velocity impacts resistant blanket created in SOLIDWORKS® 3D CAD software.

7. Conclusion

Several designs of HAYs were generated so that their auxetic performance characteristics can be identified. The designs were tensile tested through finite element simulations to determine their maximum NPR and the rate of strain at which it was achieved. A maximum NPR of -12.04 was achieved by lowering the wrap starting angle. The findings show that a starting wrap angle of 7° will produce the highest NPR. This should be the design angle for optimised performance of the yarn. Additionally, the study proves that reducing the wrap diameter of a HAY, and hence increasing the core/wrap thickness ratio, causes it to become more auxetic, by achieving a higher NPR. Exploring the possibilities further, experiments using HAYs with CNT wraps can lead to a new understanding of how HAYs should be designed, with the probability that they will result in the highest NPRs ever exhibited, due to their exceptionally high tensile strength, and nano-sized diameter.

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References

[1] Zhang G, Ghita OR, Evans KE. Dynamic thermo-mechanical and impact properties of helical auxetic yarns. Composites Part B 2016;99:494-505.

[2] Lakes RS. Foam structures with a negative Poisson's ratio. Science 1987;235:1038–1040.

[3] Frii EA, Lakes RS, Park JB. Negative Poisson's ratio polymeric and metallic foams. J. Mater. Sci. 1988;23:4406-4414.

[4] Lim TK. Auxetic materials and structures. Engineering Materials 2015;1:156-158.

[5] Sadd, MH. Strain energy and related principles, Elasticity: Theor. Appl. & Numerics 2014;3:123-125.

[6] Alderson KL, Webber RS, Mohammed UF, Murphy E, Evans KE. An experimental study of ultrasonic attenuation in microporous polyethylene. Applied Acoustics 1999;50:23-33.

[7] Hook PB. Auxetic Mechanisms, Structures & Materials. PhD thesis, University of Exeter, 2003.

[8] Qiao JX, Chen CQ. Impact resistance of uniform and functionally graded auxetic double arrowhead honeycombs. Int. Journal of Impact Engineering 2015;83:47-58.

[9] Imbalzano G, Tran P, Ngo TD, Lee PVS. A numerical study of auxetic composite panels under blast loadings. Composite Structures 2016;135:339-352.

[10] https://www.advancedfabrictechnology.com/ (accessed, 10 Oct 2015).

[11] Miller W, Hoo PB, Smith CW, Wang X, Evans KE. The manufacture and characterisation of a novel, low modulus, negative Poisson's ratio composite. Composites Science and Technology 2009;69:651-655.

[12] Rant D, Rijavec T, Pavko-Cuden A. Auxetic textiles. Acta Chim. Slov. 2013;717:715–723.

[13] Sibal A, Rawal A. Design strategy for auxetic dual helix yarn systems. Materials Letters 2015;161:740–742.

[14] Wright JR, Sloan MR, Evans KE. Tensile properties of helical auxetic structures: A numerical study. Journal of Applied Physics 2010;108:art no. 044905.

[15] Sloan MR, Wright JR, Evans KE. The helical auxetic yarn – A novel structure for composites and textiles; geometry, manufacture and mechanical properties. Mechanics of Materials 2011;43:476-486.

[16] Wright JR, Burns MK, James E, Sloan, MR, Evans KE. On the design and characterisation of low-stiffness auxetic yarns and fabrics. Textile Research Journal 2012;82:645-654.

[17] Miller W, Ren Z, Smith CW, Evans KE. A negative Poisson's ratio carbon fibre composite using a negative Poisson's ratio yarn reinforcement. Composites Science and Technology 2012;72:761-766.

[18] Hook PB. Uses of Auxetic Fibres. U.S. Patent No. US2007210011, 13 September 2007.

[19] Shen Y. Modelling of Tensile Properties of Woven Fabrics and Auxetic Braided Structures by Multi-Scale Finite Element Method. PhD thesis, Auburn University, 2013.

[20] Microplastics, Inc. Micro Plastics Material Data. (2009) Retrieved from: https://secure.microplastics.com/MaterialData.aspx?mat=NYLON%2066.htm (accessed, 23 February 2016).

[21] Liu Q. Literature Review: Materials with Negative Poisson's Ratios and Potential Applications to Aerospace and Defence. Victoria: DSTO Defence Science and Technology Organisation, 2016.

[22] Rao MP, Duan Y, Keefe M, Powers BM, Bogetti T. Modelling the effects of yarn material properties and friction on the ballistic impact of a plain-weave fabric. Composite Structures 2009;89:556-566.

[23] Zhang GH, Ghita O, Evans KE. The fabrication and mechanical properties of a novel 3-component auxetic structure for composites. Composites Science and Technology 2015;117:257-267.

[24] Jorio A, Dresselhaus G, Dresselhaus MS. Carbon nanotubes: Advanced topics in the synthesis, structure, properties and applications. Applied Physics 2008;111:5-7.

[25] Jayasinghe C, Amstutz T, Schulz MJ, Shanov V. Improved processing of carbon nanotube yarn. Journal of Nanomaterials 2013;1:1-7.

[26] Bhattacharya S, Zhang GH, Ghita O, Evans KE. The variation in Poisson’s ratio caused by interactions between core and wrap in helical composite auxetic yarns. Composites Science and Technology 2014;102:87-93.

[27] Zhang GH, Ghita O, Lin C, Evans KE. Varying the performance of helical auxetic yarns by altering component properties and geometry. Composite Structures 2016;140:369-377.

[28] Du Z, Zhou M, Liu H, Linge He. Study on negative Poisson's ratio of auxetic yarn under tension: Part 1 – Theoretical analysis. Textile Research Journal 2015;85: 487-498.

[29] Du Z, Zhou M, He L, Liu H. Study on negative Poisson's ratio of auxetic yarn under tension: Part 2 – Experimental verification, Textile Research Journal 2015;85:768-774.

[30] Jiang JW, Park HS. Negative Poisson's ratio in single-layer graphene ribbons. Nano Letters 2016;16:2657-2662.

Table Captions

Table A. Input parameters for models used in: (1) sensitivity test, (2) to determine the effect of varied core-to-wrap diameter ratios, and (3) critical angle test.

Table B. Finite element results for each model tested, which shows the maximum NPR achieved and the percentage of longitudinal strain.

Figure Captions

Figure 1. Right-angled triangle formed if the wrap was unravelled from the core (adapted from ref. [14]).

Figure 2. (a) Two cycle model of HAY created in SOLIDWORKS® 3D CAD software, and (b) HAY connections showing the contact region between the wrap and core, highlighted in blue and red, respectively.

Figure 3. (a) Final mesh sizing of largest surface areas of HAY components, (b) invalid model with refined mesh on contact region of wrap, (c) cross-sectional area of mapped face meshing applied to model, and (d) fully deformed HAY after tensile test had been simulated.

Figure 4. Graph of longitudinal strain vs. Poisson's ratio of HAY for all finite element computational models tested using ANSYS® Workbench [figures appear in colour in the online version].

Figure 5. Graph of HAY Poisson's ratio vs. longitudinal strain (finite element computational models using ANSYS® Workbench): (a) for Model A1 and Model A2, showing the point at which both designs' maximum NPR values were achieved, (b) for Model A1, B1 and B2, (c) for Model A2, C1 and C2, (d) for Model A2 and D1, and (e) for models used in critical angle test [figures appear in colour in the online version].

Figure 6. Unused conceptual model of high velocity impacts resistant blanket created in SOLIDWORKS® 3D CAD software.

Appendix-I: *Example of NPR Calculation*

In the following section, the process used to calculate the NPR at 6.24% strain for Model A2 (base model, **Table A**) has been outlined.

Calculation of Unknown Geometrical Parameters:

The first step in the process is to calculate the various geometric parameters and create the model. Model A2 has the following geometric input parameters:

$$d_{cO} = 700\mu m$$

$$d_{wO} = 200\mu m$$

$$\theta = 10°$$

Therefore, the length of one cycle of core can be determined by inserting the known parameters into the equation:

$$L_{c0} = \frac{\pi(d_{cO} + d_{wO})}{tan\theta}$$

$$L_{c0} = \frac{\pi(700 + 200)}{tan10}$$

$$L_{c0} = 16.03517mm$$

The total length of the HAY was calculated by multiplying L_{c0} by the number of cycles, n:

$$L_{HAY} = L_{c0} \times n$$

$$L_{HAY} = 16.03517 \times 10$$

$$L_{HAY} = 160.3517mm$$

Similarly, the length of one cycle of wrap can be determined by inserting the known parameters into the equation:

$$L_{w0} = \frac{\pi(d_{cO} + d_{wO})}{sin\theta}$$

$$L_{w0} = \frac{\pi(700 + 200)}{sin10}$$

$$L_{c0} = 16.2825mm$$

The total length of the wrap was calculated by multiplying L_{w0} by the number of cycles, n:

$$L_{wrap} = L_{w0} \times n$$

$$L_{wrap} = 16.2825 \times 10$$

$$L_{wrap} = 162.825mm$$

The total extension – or change in length, ΔL – at this point was 10 mm, and so the longitudinal strain, ε_x can be calculated by rearranging the equation:

$$\varepsilon_x = \frac{\Delta L}{L_0}$$

$$\varepsilon_x = \frac{10}{160.3517}$$

$$\varepsilon_x = 0.062363$$

$$\varepsilon_x = 6.24\%$$

The initial total diameter, d_{t0} was calculated using equation:

$$d_{t0} = d_c + 2d_w$$

$$d_{t0} = 700 + 2(200)$$

$$d_{t0} = 1100\mu\text{m}$$

Calculation of NPR Using Simulation Data:

Once the analytical calculations had been carried out and the geometrical parameters of the model was known, the simulation can be carried out. The total diameter at 6.24% strain, d_t was found by applying a deformation probe to the maximum deformation in the Y-direction. The value given by ANSYS® Workbench was doubled, due to symmetry, to give the final value for d_t:

$$d_t = 722.725 \times 2$$

$$d_t = 1445.45\mu\text{m}$$

The lateral contractile strain of the HAY, ε_y was computed by taking the ratio of the change in total diameter, d_t over initial total diameter, d_{t0}:

$$\varepsilon_y = \frac{d_t - d_{t0}}{d_{t0}}$$

$$\varepsilon_y = \frac{1445.45 - 1100}{1100}$$

$$\varepsilon_y = 0.31405$$

Finally, the Poisson's ratio of the HAY, v_{xy} can be calculated using the equation:

$$v_{xy} = -\frac{\varepsilon_y}{\varepsilon_x}$$

$$v_{xy} = -\frac{0.31405}{0.062363}$$

$$v_{xy} = -5.03584$$

Therefore, the NPR of Model A2 at 6.24% axial tensile strain was -5.04.