AUTHOR(S): HUANG, G. and TU, W.

TITLE: Wireless information and energy transfer in nonregenerative OFDM AF relay systems.

YEAR: 2017

Publisher citation: HUANG, G. and TU, W. 2017. Wireless information and energy transfer in nonregenerative OFDM AF relay systems. Wireless personal communications [online], 94(4), pages 3131-3146. Available from: https://10.1007/s11277-016-3768-9.

OpenAIR citation: HUANG, G. and TU, W. 2017. Wireless information and energy transfer in nonregenerative OFDM AF relay systems. Wireless personal communications, 94(4), pages 3131-3146. Held on OpenAIR [online]. Available from: https://openair.rgu.ac.uk.

Noname manuscript No.
(will be inserted by the editor)

Wireless information and energy transfer in nonregenerative OFDM AF relay systems

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Received: date / Accepted: date

Abstract Energy harvesting (EH) is a promising strategy to prolong the operation of energy-constrained wireless systems. Simultaneous wireless information and energy transfer (SWIET) is a potential EH technique which has recently drawn significant attention. By employing SWIET at relay nodes in wireless relay systems, the relay nodes can harvest energy and receive information from their source nodes simultaneously as radio signals can carry energy as well as information at the same time, which solves the energy scarcity problem for wireless relay nodes. In this paper, we study SWIET for nonregenerative orthogonal-frequency-division multiplexing (OFDM) amplify-and-forward (AF) systems in order to maximize the end-to-end achievable rate by optimizing resource allocation. Firstly, we propose an optimal energy-transfer power allocation (EPA) policy which utilizes the diversity provided by OFDM modulation. We then validate that the ordered-SNR (signal-to-noise ratio) subcarrier pairing (SP) is the optimal SP scheme. After that, we investigate the information-transfer power allocation (IPA) and EH time optimization problem which is formulated as a non-convex optimization problem. By making the approximation at high SNR regime, we convert this non-convex optimization problem into a quasi-convex programming problem, where an algorithm is derived to jointly optimize the IPA and EH time. By analytical analysis, we validate that the proposed resource allocation scheme has much lower computational complexity than peer studies in the literature. Finally, simulation results verify the optimality of our proposed resource allocation scheme.

Keywords Energy harvesting (EH) · Simultaneous wireless information and energy transfer (SWIET) · Orthogonal-frequency-division multiplexing (OFDM) · Relay · Resource allocation · Quasi-convex optimization

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1 Introduction

In recent years, energy harvesting (EH) has attracted a great deal of attention in wireless communications [1]. As a promising EH technology to solve energy scarcity issues at energy-constrained wireless nodes, simultaneous wireless information and energy transfer (SWIET) has recently been proposed in [2]. Since then, various wireless systems have been studied to enable wireless nodes to harvest wireless energy and receive information simultaneously [3–6]. In wireless relay systems, by employing SWIET at source-to-relay links, a relay node equipped with EH devices is able to harvest energy from radio frequency (RF) signals while receiving information from its source. As such, the lifetime of energy-constrained relay nodes is prolonged, which is practically important for wireless relay systems. Therefore, in [7], the wireless relay system with SWIET was firstly studied and two protocols, time-switching relaying (TSR) and power-splitting relaying (PSR) protocols, were proposed for the EH relay to harvest energy from the source's RF signals. With the TSR protocol, the relay spends some time on harvesting energy and the remaining time on processing information; with the PSR protocol, the relay uses a portion of received power to harvest energy and the remaining power to process information. Under TSR and PSR protocols, the performance of a narrow-band single-carrier amplify-and-forward (AF) relay system was studied in [7].

It is well-known that resource allocation is critical for improving the performance of wireless networks and wire-lined networks. For instance, by taking into account the traffic type, the total available resources and the users' channel qualities, utility-based resource allocation algorithms were provided in [8] to optimize the performance of wireless networks. By casting the resource allocation problem into a network utility maximization model, the optimal algorithms were proposed in [9] to improve performance of wireless networks while meeting the various quality-of-service (QoS) requirements of users. Thus, in order to improve the performance of the SWIET-based wireless relay system, a great deal of research effort has devoted to how to properly allocate resource in the system recently. Under single-carrier narrow-band channels, the power allocation strategy in a PSR-based EH relay system with multiple source-destination pairs has been studied in [10, 11]. More recent studies on SWIET have focused on wireless broadband relay systems. This is because modern or future wireless communication systems operate in broadband channels in order to provide transmissions with high data rates. In [12], the resource allocation scheme in a PSR-based broadband orthogonal-frequency-division multiplexing (OFDM) system with decode-and-forward (DF) relaying has been investigated. In [13], both PSR-based and TSR-based resource allocations in a multi-antenna OFDM system with amplify-and-forward (AF) relaying have been studied.

In this paper, we consider a TSR-based OFDM system with a nonregenerative AF relay. We choose TSR instead of PSR because SWIET is comparatively simple to implement in TSR-based systems as current commercial circuits are usually designed to receive information and harvest energy separately [4]. Meanwhile, as compared to DF relaying, AF relaying has the big advantage that the relay needs no or only partly knowledge about the structure and coding scheme of the signal, which allows for the easy upgrading of a mobile communications system without also having to upgrade the relay stations [14]. As we mentioned above, TSR-based wireless relay systems have been investigated for narrow-band [7] and broadband [13] channels respectively. Since the end-to-end achievable rate is not a monotonic function of the EH time [7, 13], both studies have employed brute-force search to achieve the optimal EH time. At each search point for the optimal EH time, the transmission power constraint at the relay is determined. As such, a TSR-based wireless relay system can be equivalently regarded as a traditional wireless relay system in which the source and the relay have separate power constraints. Then, the problem of optimal resource allocation for the TSR-based wireless relay system becomes similar to that in the traditional wireless relay system without SWIET. Such a resource allocation scheme can also be applied to a TSR-based OFDM AF relay system - to optimize the EH time with brute-force search and then solve the resource allocation problem for an equivalent traditional OFDM AF relay system. From this point on we refer to such a resource allocation scheme as the BFS (brute-force searching) scheme. The BFS scheme has high computational complexity because it is not trivial to obtain the optimal EH time with brute-force search on a continuous space. Also, the calculation of the optimal resource allocation in a traditional OFDM AF relay system with separate power constraints at the source and the relay usually involves a two-dimensional grid search procedure to obtain two optimal Lagrange multipliers [15, 16].

Our objective in this paper is to maximize the end-to-end achievable rates for an OFDM AF relay system with SWIET by optimizing resource allocation with low computational complexity. Resource allocation in our AF relay system includes

- *EH time optimization* determines the time used to transfer energy from the source to the relay. It thus also determines the time used to transmit information.
- *Energy-transfer power allocation* (EPA) determines the power allocated to each subcarrier at the source when the source transfers energy to the relay.
- *The pairing of subcarriers* determines how the relay combines the subcarriers at the source-to-relay (SR) link and the relay-to-destination (RD) link to forward information to the destination.
- *Information-transfer power allocation* (IPA) determines the power allocated to each subcarrier at the SR and RD links when the source and the relay transmit information.

We first propose an EPA policy that can optimally utilize frequency diversity provided by OFDM modulation. Then, we validate that the ordered-SNR (signal-to-noise ratio) subcarrier pairing (SP) is optimal in terms of maximizing end-to-end transmission rates in AF relay systems. The ordered-SNR SP enables the relay to pair the subcarriers at the SR link and the RD link based on their SNR ordering. Finally, in order to optimize the EH time and IPA, we formulate a non-convex programming problem, and then convert it into a quasi-convex problem with high SNR approximation. By using the bisection search method to solve the quasi-convex problem, we derive an algorithm that achieves the optimal IPA as well as the optimal EH time. Our theoretical analysis shows the much lower computational complexity (as compared to the BFS scheme) achieved by the proposed new resource allocation scheme.

This paper is organized as follows. Section II proposes the system model. Section III presents the detailed studies on our optimal resource allocation scheme. Section IV demonstrates our simulation results which verify the optimality of our proposed resource allocation scheme. Section V concludes the paper.

2 System model and problem formulation

Consider an OFDM AF relay system with a source node (S), a destination node (D), and an EH relay node (R) as illustrated in Fig.1(a). The destination does not directly communicate with the source due

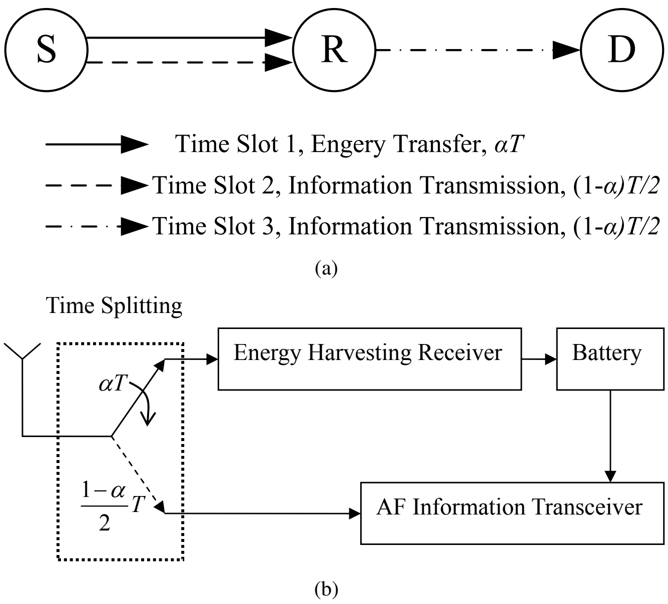

Fig. 1 System model and signal processing at the two-hop OFDM AF relay system with SWIET

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(a) System model; (b) signal processing at the energy harvesting relay node.

to distance or obstacles, thus the relay assists communication between the source and the destination [15–17]. The source has fixed energy supply while the relay is energy-constrained which needs to harvest energy from RF signal transmitted by the source and operates with the TSR protocol.

The transmission from the source to the destination is on a time-frame basis. Each time frame with equal duration, denoted as T, is divided into three time slots with one fraction of αT and two fractions of $(1-\alpha)T/2$. During the first time slot with a duration of αT, the relay harvests energy transferred by the source to charge its battery. During the second time slot with a duration of $(1-\alpha)T/2$, the relay receives the information signals transmitted by the source. The above processes are illustrated in Fig. 1(b). During the third time slot with a duration of $(1-\alpha)T/2$, the relay amplifies and forwards the received information signals to the destination by using the harvested energy at the first time slot. The information transmission during the second and third time slots is implemented on a SP basis, where the information transmitted by the source on one subcarrier at the first hop is forwarded by the relay to the destination on one designated subcarrier at the second hop [15–17]. The subcarrier pair set is denoted as $\mathcal{N} = \{1, 2, \cdots, N\}$, where N is the subcarrier number in the considered system.

The channel is assumed to be block fading, i.e., the channel gains are constant within the duration of one frame, but vary independently from one frame to another. Denote h_n^{SR} and h_n^{RD} as the channel responses of the source-relay (SR) link and relay-destination (RD) link on subcarrier n, respectively. The variances of the received additive white Gaussian noises (AWGN) at the relay and the destination are denoted as σ_{R}^2 and σ_{D}^2, which is uniformly distributed over all subcarriers. Then, $\gamma_n^{\mathrm{SR}} = \frac{|h_n^{\mathrm{SR}}|^2}{\sigma_{\mathrm{R}}^2/N}$ and $\gamma_n^{\mathrm{RD}} = \frac{|h_n^{\mathrm{RD}}|^2}{\sigma_{\mathrm{D}}^2/N}$ are the normalized channel gains.

Let $\mathbf{p}_{\mathrm{E}} = \left\{p_m^{\mathrm{S,E}} \geq 0, m = 1, 2, \cdots, N\right\}$ be the EPA policy, where $p_m^{\mathrm{S,E}}$ denotes the source's transmit power allocated to the mth subcarrier for the purpose of energy transfer. Then, the energy harvested at the relay can be expressed as [4]

$$E = \alpha T \sum_{m=1}^{N} \tau p_m^{\mathrm{S,E}} \left|h_m^{\mathrm{SR}}\right|^2 \tag{1}$$

where $0 < \tau < 1$ is the energy conversion efficiency which depends on the rectification process and the EH circuitry. Moreover, let $\mathbf{p}_{\mathrm{I}} = \left\{p_n^{\mathrm{S,I}} \geq 0, p_n^{\mathrm{R}} \geq 0, n \in \mathcal{N}\right\}$ be the IPA policy, where $p_n^{\mathrm{S,I}}$ and p_n^{R} denote the transmission power allocated on the subcarriers of subcarrier pair $n \in \mathcal{N}$ at the source and the relay respectively for information transmission purposes. It is assumed that the harvested energy at the relay is used for the relay's information transmission, and the energy consumed in transmission should not be larger than the harvested energy [7, 10–13]. Thus, p_n^{R} must satisfy

$$\frac{(1-\alpha)}{2} \sum_{n=1}^{N} p_n^{\mathrm{R}} \leq \alpha \sum_{m=1}^{N} \tau p_m^{\mathrm{S,E}} \left|h_m^{\mathrm{SR}}\right|^2 \tag{2}$$

For the OFDM AF relay system with SWIET, the end-to-end achievable rate can be expressed as [15]

$$R = \frac{(1-\alpha)B}{2N} \sum_{n=1}^{N} \log_2\left(1 + \frac{p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}} p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}}}{p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}} + p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}} + 1}\right) \tag{3}$$

where B is the total spectral bandwidth in the unit of Hz. Thus, we can formulate resource allocation problem to maximize the end-to-end achievable rate such as follows

$$\begin{aligned} &\max_{\alpha \in [0,1], \mathbf{p}_{\mathrm{E}}, \mathbf{p}_{\mathrm{I}}, \mathcal{N}} \; R \\ &\text{s.t. } (2), \; \sum_{m=1}^{N} p_m^{\mathrm{S,E}} \leq \mathcal{P}_{\mathrm{S}}, \; \sum_{n=1}^{N} p_n^{\mathrm{S,I}} \leq \mathcal{P}_{\mathrm{S}} \end{aligned} \tag{4}$$

where $\mathcal{P}_{\mathrm{S}}$ is the maximum allowable transmit power at the source.

3 Optimal resource allocation

3.1 Optimal power allocation for energy transfer

During the first time slot, the source transfers energy to the relay. Considering the transmission power constraint at the source and according to (1), we have

$$E = \alpha T \sum_{m=1}^{N} \tau p_m^{\mathrm{S,E}} \left|h_m^{\mathrm{SR}}\right|^2 \leq \alpha T \tau \mathcal{P}_{\mathrm{S}} \max\left(\mathcal{H}\right) \tag{5}$$

where $\mathcal{H} \triangleq \left\{\left|h_m^{\mathrm{SR}}\right|^2, m \in \mathcal{N}\right\}$. The inequality in (5) indicates, in order to maximize the harvested energy at the relay, the source should allocate all the available power to the subcarrier which has the maximum channel gain. Thus, we obtain the optimal EPA policy as follows:

Proposition 1 *For the OFDM AF relay system with SWIET, the optimal EPA* $\overline{p}_m^{\mathrm{S,E}}$ *for problem (4) is*

$$\overline{p}_m^{\mathrm{S,E}} = \begin{cases} \mathcal{P}_{\mathrm{S}}, & m = \arg\max_m \left\{ \left| h_m^{\mathrm{SR}} \right|^2 \right\}, \\ 0, & \text{otherwise}. \end{cases} \tag{6}$$

Based on proposition 1, (1) can be rewritten as

$$E = \alpha G T \tag{7}$$

where $G \triangleq \tau \mathcal{P}_{\mathrm{S}} \max(\mathcal{H})$ is a constant.

3.2 Optimal subcarrier pairing

As the optimal EPA is determined, problem (4) can be simplified as follows:

$$\max_{\alpha \in [0,1], \mathbf{p}_{\mathrm{I}}, \mathcal{N}} \; R \tag{8a}$$

$$\text{s.t.} \; \frac{(1-\alpha)}{2} \sum_{n=1}^{N} p_n^{\mathrm{R}} \leq \alpha G \tag{8b}$$

$$\sum_{n=1}^{N} p_n^{\mathrm{S,I}} \leq \mathcal{P}_{\mathrm{S}} \tag{8c}$$

It is noted that given any α, problem (8) is equivalent to the resource allocation problem for the traditional OFDM AF relay system with separate power constraints at the source and the relay. For such traditional OFDM AF relay system, it has been proved in [15] that the optimal SP to maximize the end-to-end achievable rate is to pair the SR and RD subcarriers at the relay with ordered-SNR SP, i.e., the SR subcarrier with the strongest channel gain is paired with the RD subcarrier with the strongest channel gain, the SR subcarrier with the second strongest channel gain is paired with the RD subcarrier with the second strongest channel gain, and so forth . Thus, we provide the following proposition.

Proposition 2 *The optimal subcarrier pair set* $\overline{\mathcal{N}}$ *for problem (4) can be obtained with the ordered-SNR SP.*

3.3 Optimal EH time and IPA

Based on the optimal ordered-SNR SP, we can reduce problem (8) to the optimization of EH time and IPA only. Nevertheless, the power constraint in (8b) is non-convex, thus problem (8) is a non-convex optimization problem, which is hard to solve directly.

We observe that the objective function in problem (8), which is provided in (3), is a non-increasing function of α. Meanwhile, according to (8b), we can obtain

$$\alpha \geq \frac{\frac{1}{2}\sum_{n=1}^{N} p_n^R}{\frac{1}{2}\sum_{n=1}^{N} p_n^R + G} \tag{9}$$

Thus, the optimal α satisfies the equation

$$\alpha = \frac{\frac{1}{2}\sum_{n=1}^{N} p_n^{\mathrm{R}}}{\frac{1}{2}\sum_{n=1}^{N} p_n^{\mathrm{R}} + G} \tag{10}$$

Submitting (10) into (3), we have

$$R = \frac{GB}{N\left(\sum_{n=1}^{N} p_n^{\mathrm{R}} + 2G\right)} \sum_{n=1}^{N} \log_2\left(1 + \frac{p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}} p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}}}{p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}} + p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}} + 1}\right) \tag{11}$$

At high-SNR region, we can rewrite (11) to obtain the upper-bound approximation of the end-to-end achievable rate such as

$$R_{\mathrm{up}} = \frac{GB}{N\left(\sum_{n=1}^{N} p_n^{\mathrm{R}} + 2G\right)} \sum_{n=1}^{N} \log_2\left(1 + \frac{p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}} p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}}}{p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}} + p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}}}\right) \tag{12}$$

This upper bound becomes tight as the channel gains of the subcarriers increase [15]. Therefore, the IPA solution obtained using (12) is asymptotically optimal.

Then, we can simplify problem (8) such as

$$\max_{\mathbf{p}_{\mathrm{I}}} \; R_{\mathrm{up}} \quad \text{s.t.} \; \sum_{n=1}^{N} p_n^{\mathrm{S,I}} \le \mathcal{P}_{\mathrm{S}} \tag{13}$$

However, the objective function of problem (13), which is provided in (12), is still non-convex. Fortunately, we observe that this function is quasi-concave, which is just as stated in the proposition as follows

Proposition 3 *The rate function in (12) is a quasi-concave function.*

Proof: The rate function in (12) is quasi-concave if all superlevel sets of the objective function are convex. For any $\lambda \ge 0$, the superlevel set of the objective function is expressed as follows

$$\mathcal{S}_\lambda = \left\{ \mathbf{p}_{\mathrm{I}} : GB\sum_{n=1}^{N} \log_2\left(1 + \frac{p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}} p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}}}{p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}} + p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}}}\right) - \lambda N\left(\sum_{n=1}^{N} p_n^{\mathrm{R}} + 2G\right) \ge 0 \right\}.$$

It can be proved that $\mathcal{S}_\lambda$ is a convex set. ■

In addition, the constraints in problem (13) are affine. Therefore, we propose to solve the problem with the quasi-convex optimization method [18] on the basis of proposition 3. Firstly, problem (13) can be equivalently rewritten as

$$\begin{aligned} &\min: \; \mu \\ &\text{s.t.} \\ &\left[\begin{array}{l} \max\limits_{\mathbf{p}_{\mathrm{I}}} \; \frac{GB}{N\left(\sum_{n=1}^{N} p_n^{\mathrm{R}} + 2G\right)} \sum_{n=1}^{N} \log_2\left(1 + \frac{p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}} p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}}}{p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}} + p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}}}\right) \le \mu \\ \text{s.t.} \; \sum_{n=1}^{N} p_n^{\mathrm{S,I}} \le \mathcal{P}_{\mathrm{S}} \end{array}\right] \end{aligned} \tag{14}$$

by introducing a variable μ as an upper bound on problem (13). This step holds since minimizing μ is the same as finding the least upper bound of the objective function in problem (13). This is equal to the maximum value of the objective function in problem (13), which exists, as seen by straightforward continuity argument.

Furthermore, problem (14) can be equivalently rewritten as follows

$$\begin{aligned} &\min: \ \mu \\ &\text{s.t.} \\ &\left[\begin{array}{l} \min\limits_{\mathbf{p}_\mathrm{I}} \mu N\left(\sum_{n=1}^{N} p_n^{\mathrm{R}} + 2G\right) - GB\sum_{n=1}^{N}\log_2\left(1+\frac{p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}}p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}}}{p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}}+p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}}}\right) \geq 0 \\ \text{s.t. } \sum_{n=1}^{N} p_n^{\mathrm{S,I}} \leq \mathcal{P}_\mathrm{S} \end{array}\right] \end{aligned} \tag{15}$$

Then, μ is a true upper bound if the problem

$$\begin{aligned} &\min_{\mathbf{p}_\mathrm{I}} \ \mu N\left(\sum_{n=1}^{N} p_n^{\mathrm{R}} + 2G\right) - GB\sum_{n=1}^{N}\log_2\left(1+\frac{p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}}p_n^{R}\gamma_n^{\mathrm{RD}}}{p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}}+p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}}}\right) \\ &\quad \text{s.t. } \sum_{n=1}^{N} p_n^{\mathrm{S,I}} \leq \mathcal{P}_\mathrm{S} \end{aligned} \tag{16}$$

has a non-negative optimal value. It can be easily proved that the objective function in (16) is convex [18]. Thus, problem (16) is a convex problem, which has a globally unique optimal solution. By using the Karush-Kuhn-Tucker (KKT) conditions [18] (details in Appendix 1), we can obtain the optimal solution of problem (16) as

$$\begin{cases} \overline{p}_n^{\mathrm{S,I}} = \dfrac{W}{\sqrt{\upsilon\gamma_n^{\mathrm{RD}}}}\left[Q\dfrac{\gamma_n^{\mathrm{RD}}}{W^2} - \dfrac{1}{\gamma_n^{\mathrm{SR}}}\right]^+, \forall n & (17) \\ \overline{p}_n^{\mathrm{R}} = \dfrac{W}{\sqrt{\mu N\gamma_n^{\mathrm{SR}}}}\left[Q\dfrac{\gamma_n^{\mathrm{SR}}}{W^2} - \dfrac{1}{\gamma_n^{\mathrm{RD}}}\right]^+, \forall n & (18) \end{cases}$$

where $W \triangleq \sqrt{\mu N\gamma_n^{\mathrm{SR}}} + \sqrt{\upsilon\gamma_n^{\mathrm{RD}}}$, $x^+ \triangleq \max(x,0)$, $Q = \frac{GB}{\log(2)}$ and υ is the Lagrange multiplier determined by the maximum available power constraint in (16). It is noted that the optimal power allocation in (17) and (18) is a multi-level water-filling policy, since the water surface is different for different subcarriers.

As the optimal solution for problem (16) is obtained, we can solve problem (13) using a standard bisection procedure as summarized in Algorithm 1, where ϵ denotes the predefined accuracy of bisection search over μ and U is the upper bound of the optimal value of problem (13),

$$U = BN\log_2\left(1+\mathcal{P}_\mathrm{S}\max_n \gamma_n^{\mathrm{SR}}\right)$$

which is obtained by considering the maximum achievable rate from source to relay.

3.4 Complexity analysis

We have derived the optimal resource allocation scheme for TSR-based OFDM AF relay systems in previous subsections. Meanwhile, as mentioned in section 1, the optimal resource allocation for TSR-based OFDM AF relay systems can also be obtained with the BFS scheme by optimizing the EH time with brute-force search, which is just as those in [7] and [13] for TSR-based wireless relay systems. As for the BFS scheme, we have to exhaustively search for the optimal α for problem (8) and solve problem

Algorithm 1: Joint IPA and EH time optimization for OFDM AF relay systems with SWIET

1: **Initialization:** μ_l = 0, μ_u = U and $t = 0$;

2: **Repeat:**

$\mu(t) = \frac{1}{2}(\mu_l + \mu_u)$;

Obtain $\overline{p}_n^{\mathrm{S,I}}$ and $\overline{p}_n^{\mathrm{R}}$, $n \in \overline{\mathcal{N}}$, by multi-level waterfilling;

$p_n^{\mathrm{S,I}}(t) = \overline{p}_n^{\mathrm{S,I}}$, $p_n^{\mathrm{R}}(t) = \overline{p}_n^{\mathrm{R}}$;

If $\mu N \left(\sum\limits_{n=1}^{N} p_n^{\mathrm{R}}(t) + 2G \right) - GB \sum_{n=1}^{N} \log_2 \left(1 + \frac{p_n^{\mathbf{S,I}} \gamma_n^{\mathrm{SR}} p_n^{\mathrm{R}} \gamma_n^{\mathrm{RD}}}{p_n^{S,I} \gamma_n^{\mathrm{SR}} + p_n^{\mathrm{R}} \gamma_n^{\mathrm{RD}}} \right) \geq 0$

$\mu_u = \mu(t)$;

else $\mu_l = \mu(t)$;

$t = t + 1$;

3: **Until:** $\mu_u - \mu_l < \epsilon$

4: **Obtain the optimal IPA and EH time:**

$\overline{p}_n^{\mathrm{S,I}} = p_n^{\mathrm{S,I}}(t)$, $\overline{p}_n^{\mathrm{R}} = p_n^{\mathrm{R}}(t)$;

Calculate the optimal EH time according to (10).

(8) at each search point of α. As a result, the BFS scheme has much higher computational complexity than our proposed resource allocation scheme, which can be validated as follows.

Firstly, for our proposed resource allocation scheme, the computational complexity of the bisection search used in Algorithm 1 is $\mathcal{O}\left(\log_2\left(\epsilon^{-1}\right)\right)$. To calculate the optimal IPA, one optimal Lagrange multiplier υ is to be determined (refer to (17) and (18)). Let δ be the predefined accuracy of calculating the optimal Lagrange multiplier by the gradient method [18]. Then, the computational complexity is $\mathcal{O}\left(\log_2\left(\delta^{-1}\right)\right)$ as the bisection search method is employed to calculate the optimal Lagrange multiplier. As the optimal Lagrange multiplier υ is determined, the complexity of the calculation IPA by (17) and (18) is N. Thus, the general computational complexity of our proposed resource allocation scheme is $\mathcal{O}\left(\log_2\left(\epsilon^{-1}\right)\left(\log_2\left(\delta^{-1}\right) + N\right)\right)$.

Secondly, for the BFS scheme, the computational complexity of the brute-force search can be evaluated as $\mathcal{O}\left(\frac{1}{\varDelta\alpha}\right)$, where $\varDelta\alpha$ is an update step for the exhaustive search on a continuous interval $[0, 1]$. Furthermore, the calculation of IPA in a traditional AF OFDM relay system [15] requires calculating two optimal Lagrange multipliers, whose computational complexity is $\mathcal{O}\left(1/\delta^2\right)$ as the gradient method is used to calculate the two optimal Lagrange multipliers. Thus, the general computational complexity of the BFS scheme is $\mathcal{O}\left(\frac{1}{\varDelta\alpha}\left(\frac{1}{\delta^2} + N\right)\right)$.

Without loss of generality, let $\epsilon = \delta = \varDelta\alpha = \varsigma = 0.001$ and $N = 2048$. Then, for our proposed resource allocation scheme, the general computational complexity is about 2×10^4. Meanwhile, for the BFS scheme, the general computational complexity is about 10^9. That is, the computational complexity of our scheme is about 5×10^4 times lower than that of the BFS scheme.

4 Simulation results

We verify the performance of our proposed resource allocation scheme by computer simulations in this section. In the simulations, the total number of subcarriers in our OFDM AF relay system is set as $N = 32$. The channel responses over each subcarrier are independent and identically distributed complex Gaussian random variables with unit variances, which means that the amplitudes of channel responses are Rayleigh distributed. The large-scale path loss is modeled as d^{-2}, where d is the distance between two nodes. The distance between the source and the destination is set as d_{SD} =100m. The total system bandwidth is set as $B =$ 5MHz. The maximum allowable transmit power at the source is set as $\mathcal{P}_{\text{S}} =$ 10dBm. The variance of the receiving noise at the relay and destination is set as $\sigma_R^2 = \sigma_D^2 = -70$dBm. The energy conversion efficiency is $\tau = 0.9$. All configuration parameters mentioned above will not change in the following simulations unless specified otherwise.

In Fig.2, we compare the end-to-end achievable rates achieved by different resource allocation schemes as the location of the relay varies. The compared schemes are our proposed optimal resource allocation scheme (denoted as 'Proposed optimal scheme' in the legend), the BFS scheme and the scheme with fixed α (denoted as "Fixed EH time"). The BFS scheme is obtained by solving problem (13) as α is given and optimizing α with brute-force search. The Fixed EH time is obtained by solving problem (8) with fixed α. For our proposed optimal resource allocation scheme, we illustrate the upper bound (denoted as "UB" in the legend) and lower bound (denoted as "LB" in the legend) of the achieved rates. For other schemes, we only illustrate the upper bound of the achieved rates for convenience of observation. The upper bound is obtained by optimizing the resource allocation with the end-to-end achievable rate at high SNR approximation. The lower bound is obtained with the actual rate calculated with the optimal solutions at high SNR approximation. From Fig.2, it is observed that both the proposed optimal resource allocation scheme and the BFS scheme achieve the maximum rates. Also, it is observed that the proposed optimal resource allocation scheme outperforms the schemes with fixed EH time. Furthermore, for our proposed optimal scheme, we magnify the upper and lower bounds of the achieved rates as κ =0.7, 0.8 and 0.9. From the magnified results, it is observed that the lower bound and upper bound of the proposed optimal resource allocation scheme are almost the same for all relay locations, except for the case that as $\kappa = 0.9$, where the upper bound is a little larger than the lower bound. This indicates that the actual rates obtained with the lower bound are nearly optimal. Moreover, it is interesting to note that the optimal rates are larger as the relay locates near by the source or the destination, compared to those as the relay locates halfway between the source or the destination. This is contrast with that for traditional wireless relay systems with fixed power supply at the relay, where the largest rate is achieved just as $\kappa = 0.5$ [15, 16].

In Fig.3, we illustrate the optimal α and the consumed power at the source and the relay as the relay location varies. From Fig.3, it is observed that the the optimal α is the largest as $\kappa = 0.5$. This indicates that the relay harvests most energy as the relay locates halfway between the source and the destination. Moreover, it is observed that the power consumed at the source is equal to its allowable transmit power wherever the relay locates, while the power consumed at the relay decreases as the relay moves from the source towards the destination. Furthermore, according to Fig.2 and Fig.3, it is observed at as κ varies

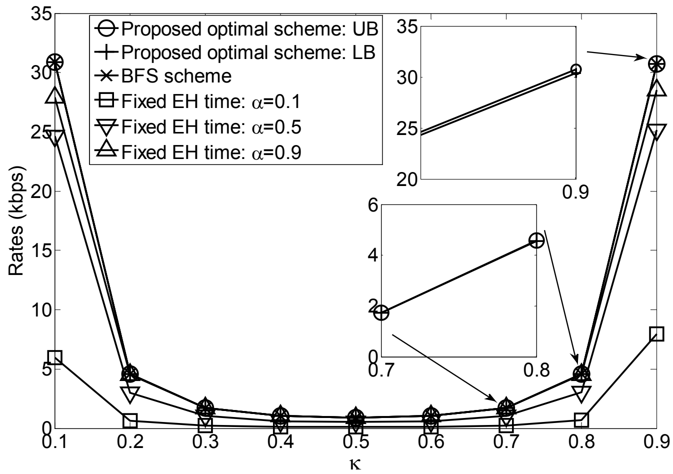

Fig. 2 Average rate as the relay location varies; comparison of the proposed optimal resource allocation scheme with the schemes with fixed EH time.

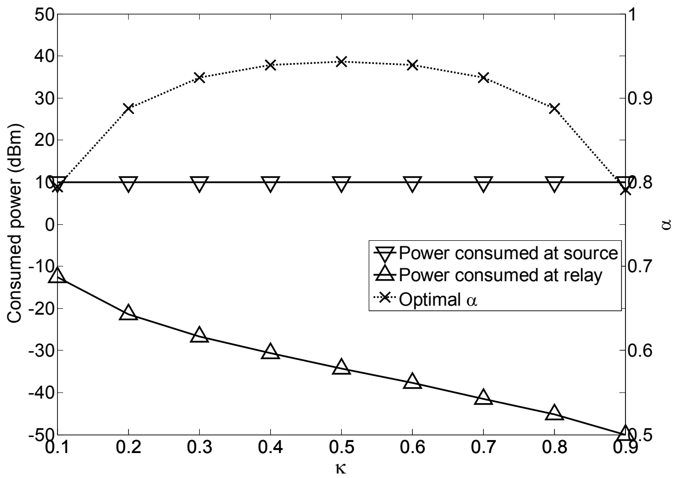

Fig. 3 Optimal EH time and consumed power at the source and the relay as the relay location varies

from 0.2 to 0.8, the optima α is approximately equal to 0.9, while the rate achieved by the the scheme with fixed EH time $\alpha = 0.9$ approaches to that achieved by the proposed optimal scheme.

In Fig.4, we compare the rates achieved by different resource allocation schemes as $\mathcal{P}_{\rm S}$ varies, where the relay location indicator is set as $\kappa = 0.8$. The compared schemes are the same as Fig.2. In addition, we also illustrate the optimal α in Fig.4. From Fig.4, it is observed that both the proposed optimal resource allocation scheme and the BFS scheme achieve the maximum end-to-end rates for all $\mathcal{P}_{\rm S}$. Also, it is observed that the proposed optimal resource allocation scheme outperforms the schemes with fixed EH time for all $\mathcal{P}_{\rm S}$. Moreover, it is also observed that the EH time decreases as the $\mathcal{P}_{\rm S}$ increases, which means that more time can be used for information transmission and thus the achieved rate can be increased. Especially, it is noted that as $\mathcal{P}_{\rm S}$ decreases to 0dBm, the optimal α is approaches to 0.9 and the rate achieved by the scheme with fixed EH time $\alpha = 0.9$ approaches to that achieved by the optimal scheme. Meanwhile, as $\mathcal{P}_{\rm S} = 40$dBm, the optimal α is approximately equal to 0.5 and the rate achieved by the scheme with fixed EH time $\alpha = 0.5$ approaches to that achieved by the optimal scheme.

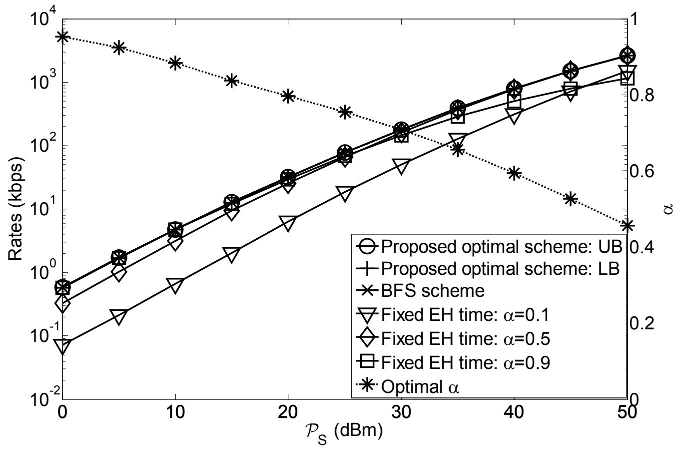

Fig. 4 Average rate and optimal α as $\mathcal{P}_{\rm S}$ varies and $\kappa = 0.8$; comparison of the optimal resource allocation scheme with the schemes with fixed EH time.

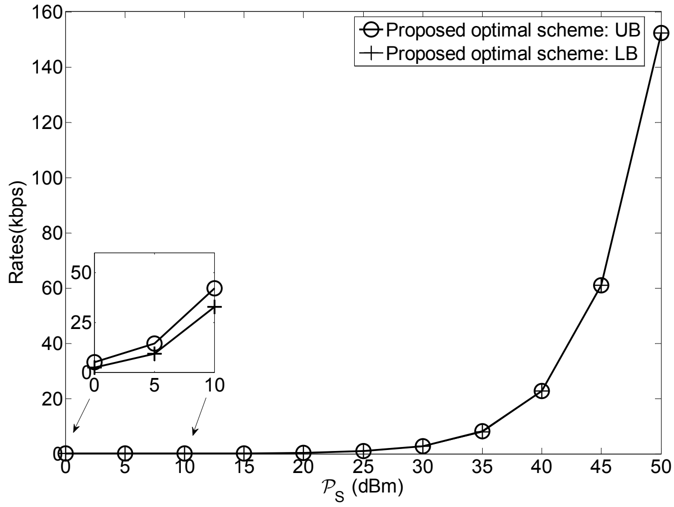

Fig. 5 Performance loss evaluation of our proposed scheme at low SNR regime; comparison of the upper bound and lower bound of optimal rate achieved by our proposed scheme.

Since our proposed scheme is obtained by high SNR approximation, there will be performance loss at low SNR regime, which is just illustrated in Fig.5. For observation convenience, we set the noise power as -50dBm to obtain the results in Fig.5, which is larger than that in the previous simulations. By Fig.5, it is observe that as SNR is low, i.e. $\mathcal{P}_{\rm S}$ varies from 0dBm to 10dBm, the achieved rate by our proposed scheme with the lower bound is smaller than that depicted as the upper bound, though the gap can be negligible at high SNR regime.

In Fig. 6, to evaluate the impact of SP, we compare the actual end-to-end achievable rates achieved by the ordered-SNR and no ordered-SNR SP schemes as the number of subcarriers N varies, where the no ordered-SNR SP scheme means that the relay pairs the nth subcarrier at the SR link with the nth subcarrier at the RD link without sorting these subcarriers in advance. For both SP schemes, the EH time and PA are optimized according to the statement in previous section. The relay location indicator κ is set as 0.1, 0.5 and 0.9, respectively. From Fig.6, it is observed that the ordered-SNR SP achieves

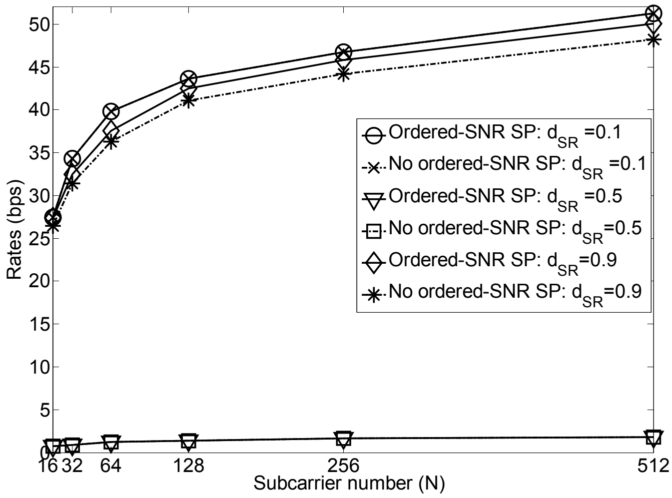

Fig. 6 Average end-to-end achievable rate versus N; comparison of ordered-SNR SP scheme with no ordered-SNR SP scheme.

the maximum end-to-end rate. Especially, for the case that the relay locates near by the destination, i.e. $\kappa = 0.9$, ordered-SNR SP can achieve a significant performance gain over no ordered-SNR SP.

5 Conclusion

In this paper, we investigated the problem of optimizing the EH time, SP, EPA and IPA in order to maximize end-to-end achievable rates for OFDM nonregenerative AF relay systems with SWIET. An optimal EPA policy was first proposed to transfer energy from the source to the relay. Then, we validated that ordered-SNR SP is globally optimal. After that, the EH time and IPA was jointly optimized by solving a quasi-convex programming problem with bisection search. Our theoretical analyses showed that the proposed optimal resource allocation has much lower computational complexity than the peer studies do in the literature. Finally, the simulation results demonstrated the optimality of our proposed resource allocation scheme.

Acknowledgements The research work was supported by the Guangdong natural science foundation (No. 2014A030310349), Guangdong science and technology project (No. 2016A010101032, 2013B010402018, 2013B020200016, 2012A010800009), Guangzhou science and technology project (No. 2014J4100142, 2014J4100233), Guangzhou education bureau science and technology project (No. 2012A082) and Guangzhou college and university science and technology project (No. 1201421329). The work of Dr. Tu in this paper was supported by the U.K. EPSRC under Grant EP/J017159/1.

APPENDIX 1 : DERIVATION FOR THE OPTIMAL SOLUTIONS IN (17) and (18)

Since (16) is a convex optimization problem, the solution can be obtained with KKT conditions. Using standard optimization technique, construct the Lagrange function as follows:

$$\begin{aligned} L &= \mu N\left(\sum_{n=1}^{N} p_n^{\mathrm{R}} + 2G\right) - GB\sum_{n=1}^{N}\log_2\left(1 + \frac{p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}}p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}}}{p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}} + p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}}}\right) + \upsilon\left(\sum_{n=1}^{N} p_n^{\mathrm{S,I}} - \mathcal{P}_{\mathrm{S}}\right) \\ &= \mu N\left(\sum_{n=1}^{N} p_n^{\mathrm{R}} + 2G\right) - GB\sum_{n=1}^{N}\log_2\left(p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}} + p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}} + p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}}p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}}\right) \\ &\quad + GB\sum_{n=1}^{N}\log_2\left(p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}} + p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}}\right) + \upsilon\left(\sum_{n=1}^{N} p_n^{\mathrm{S,I}} - \mathcal{P}_{\mathrm{S}}\right) \end{aligned} \tag{19}$$

where $\upsilon \geq 0$ is the Lagrange multiplier. Then the optimal solutions $\overline{p}_n^{\mathrm{S,I}}$ and $\overline{p}_n^{\mathrm{R}}$ must satisfy the following equations:

$$\begin{cases} \dfrac{\partial L}{\partial p_n^{\mathrm{S,I}}} = \dfrac{GB}{\log 2}\left[\dfrac{\gamma_n^{\mathrm{SR}}}{p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}} + p_n^{p_n^{\mathrm{R}}}\gamma_n^{\mathrm{RD}}} - \dfrac{\gamma_n^{\mathrm{SR}} + p_n^{R}\gamma_n^{\mathrm{SR}}\gamma_n^{\mathrm{RD}}}{p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}} + p_n^{R}\gamma_n^{\mathrm{RD}} + p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}}p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}}}\right] + \upsilon = 0 & (20) \\ \dfrac{\partial L}{\partial p_n^{\mathrm{R}}} = \dfrac{GB}{\log 2}\left[\dfrac{\gamma_n^{\mathrm{RD}}}{p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}} + p_n^{R}\gamma_n^{\mathrm{RD}}} - \dfrac{\gamma_n^{\mathrm{RD}} + p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}}\gamma_n^{\mathrm{RD}}}{p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}} + p_n^{R}\gamma_n^{\mathrm{RD}} + p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}}p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}}}\right] + \mu N = 0 & (21) \end{cases}$$

By (20) and (21), we have

$$\begin{cases} \gamma_n^{\mathrm{SR}}\left(p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}}\right)^2 = \dfrac{\upsilon\log 2}{GB}\left(p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}} + p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}}\right)\left(p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}} + p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}} + p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}}p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}}\right) & (22) \\ \gamma_n^{\mathrm{RD}}\left(p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}}\right)^2 = \dfrac{\mu N\log 2}{GB}\left(p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}} + p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}}\right)\left(p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}} + p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}} + p_n^{\mathrm{S,I}}\gamma_n^{\mathrm{SR}}p_n^{\mathrm{R}}\gamma_n^{\mathrm{RD}}\right) & (23) \end{cases}$$

Then, by (22) and (23), we have

$$p_n^{\mathrm{R}} = p_n^{\mathrm{S,I}}\sqrt{\frac{\upsilon\gamma_n^{\mathrm{SR}}}{\mu N\gamma_n^{\mathrm{RD}}}} \tag{24}$$

Substituting (24) into (22), we obtain the optimal solution $\overline{p}_n^{\mathrm{S,I}}$ as in (17). After obtaining $\overline{p}_n^{\mathrm{S,I}}$, we can obtain $\overline{p}_n^{\mathrm{R}}$ as in (18) by (24).

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