Linearized DQ Averaged Model of Modular Multilevel Converter

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***Abstract*—This paper proposes a linearized model of Modular Multilevel Converter (MMC) in DQ frame. The proposed MMC model has a modular structure and can be linked with other power elements such as AC and DC subsystems. The main challenge of developing DQ model of MMC is to deal with the multiplication terms in dynamic equations of MMC. In this paper, the multiplication terms are done in ABC frame and results are transferred to DQ frame after ignoring the higher harmonics. The detailed model is implemented in PSCAD/EMTDC and the proposed linearized model is implemented in Matlab in a modular form. The results of the two models show very good matching which in turn confirm the accuracy of the proposed model. Therefore, this model permits modern control design techniques to be employed on MMC, including eigenvalues studies and frequency domain analysis.**

***Index Terms*-- Modular Multilevel converter, modelling, averaged model, small signal model.**

I. INTRODUCTION

The recent years have witnessed a vast application of modular multilevel converter (MMC) in both high-power transmission/distribution systems and in the industrial applications [1]-[6]. The MMC have stood out amongst the available two-level voltage source converter (VSC), multilevel converter topologies because it permits advanced power-handling ability with lower switching power losses and harmonic distortion [5].The MMC can operate at both fundamental switching frequency and high switching frequency Pulse Width Modulation (PWM). It should be prominent that lower switching frequency generally means lower switching loss and higher efficiency. As a result of this, the MMC has been an active research topic in recent years.

As shown in Fig.1 the basic component of an MMC is called a submodule (SM) and it can be increased or decreased with a corresponding increases or decreases in the number of levels to get the preferred output voltage with minimum harmonic distortion. The cascade configuration of the SMs allows the choice to link required amount of modules without increasing the complication of the circuit. To get the preferred output voltage the converter arm characterizes a controllable voltage source controlled with modulation techniques.

Reference [7] presents the model of VSC based transmission that can be used for wide range of stability studies and controller design was presented in MATLAB environment. It is strategic to use a model with a modular structure as every subsystem individually can show discrete analysis before they are coupled together and are easily extended for multi-terminal systems. This analytical model engages every constraint and variable with physical significance to study the variations in the structures of AC and DC systems.

The same level of experience does not exist for MMC. Most of the literature can be found only on modelling in static ABC frame, diverse modulation techniques, with different level of detail, and different assumptions[8]-[10]. The unavailability of a linearized MMC HVDC models in power system modelling and design software has to be a limitation to its application. The dynamic analysis of MMC is very complex and usually carried out by using software like PSCAD/EMTDC[11]. Nevertheless, these simulations offer only trial-and-error type analysis in the time domain. The essential design task and analysis takes a longer period of time since a large search has to be performed to find the best solution [12].

On the other hand, much faster design methods are provided by convenient and accurate analytical system models. These models provide a facility to apply dynamic system analysis techniques such as frequency-domain, eigenvalue analysis and modern control design theories, resulting in less consumption of time for design and advanced controller configuration outcomes [7].

In contrast to conventional two-level VSCs, the analytical modelling of MMC significantly becomes a challenging task. This is because of technical issues such as higher order system, the discontinuous and non-linear nature of signal transfer through converters, the complexity of the interaction equations between the AC and DC variables, and harmonic frequency conversion through AC side and DC side of the converter.

This paper presents a linearized DQ average model of an MMC that is suitable for small signal stability analysis and control design. A modular modelling methodology is adopted to characterize the complication of the system with the assistance that each separate subsystem can be analysed individually. However AC, DC and Phase Locked Loop (PLL) subsystem models can be found from [7],[12] and only MMC subsystem model is presented in this paper. This analytical MMC model consists of all variables and parameters with physical importance to analyse variations of structures in AC and DC systems.

II. OPERATIONAL PRINCIPLE AND EQUIVALENT STRUCTURE OF MMC

Fig.1 shows the electrical equivalent schematic diagram of the MMC. The IGBT elements which perform the switching acts as VSC valve and distributes DC capacitors in SMs. The valve operates as a controllable voltage source, which is connected between the AC side of the corresponding phase unit, and one of the DC terminals.

The model consists of two arms (Upper and Lower) of the converter, each one formed by a series connection of SMs and an inductor whose function is to limit the arm fault current. For *n*-level converter we need 2(*n*-1) SMs where '*n*' is a number of levels, for example, we need 16 SMs for the 9-level converter.

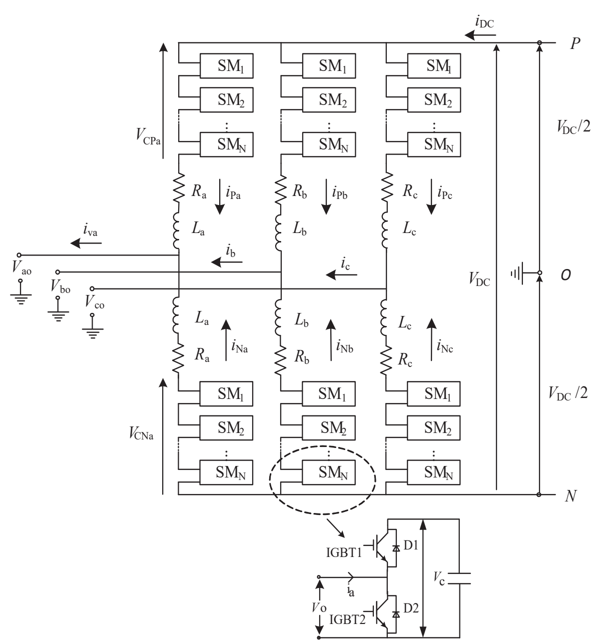

Fig 1. Circuit configuration of MMC

Fig. 1 also represents the structure of one SM of an MMC. When the switching device IGBT1 is on and IGBT2 is off (always complementary to each other), output voltage $V_O = V_C$. When the switching device IGBT1 is off and IGBT2 is on, voltage $V_O = 0$.

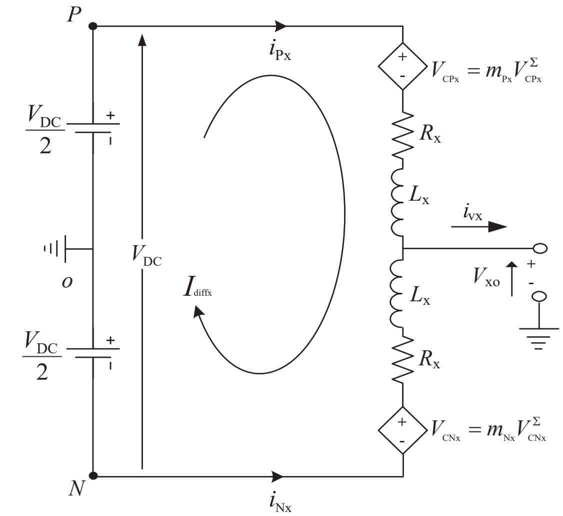

Fig 2. Equivalent electrical circuit for one phase leg of MMC [8]

Fig. 2 is the equivalent circuit of MMC. R_x and L_x are inductance and equivalent arm resistance of respective phase x where(x=a,b,c). V_{xo} and I_{vx} are the AC side converter voltage and line current .V_{DC} and I_{DC} are the DC bus voltage and current. V_{CPx} and V_{CNx} are upper and lower arm voltages. V_{CPx}^{Σ} and V_{CNx}^{Σ} are voltage stored in upper and lower arm submodules. I_{diffx} is differential current. m_{Px} and m_{Nx} are modulation indexes of upper and lower arm.

According to [8] and [9], taking $I_{diffx}, V_{CPx}^{\Sigma}, V_{CNx}^{\Sigma}$ as the state variables, V_{DC}, i_{vx} as the inputs, I_{DC}, internal voltage e_x as outputs, the state-space model of an MMC converter is stated as

$$\frac{d}{dt}\begin{bmatrix} I_{diffx} \\ V_{CPx}^{\Sigma} \\ V_{CNx}^{\Sigma} \end{bmatrix} = \begin{bmatrix} -\frac{R_x}{L_x} & \frac{-m_{Px}}{2L_x} & \frac{-m_{Nx}}{2L_x} \\ \frac{m_{Px}}{C^{arm}} & 0 & 0 \\ \frac{m_{Nx}}{C^{arm}} & 0 & 0 \end{bmatrix} \begin{bmatrix} I_{diffx} \\ V_{CPx}^{\Sigma} \\ V_{CNx}^{\Sigma} \end{bmatrix} + \begin{bmatrix} \frac{V_{DC}}{2L_x} \\ \frac{m_{Px} i_{vx}}{2C^{arm}} \\ \frac{-m_{Nx} i_{vx}}{2C^{arm}} \end{bmatrix} \quad (1)$$

Output equation is

$$\begin{bmatrix} I_{DC} \\ e_x \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -\frac{m_{Px}}{2} & \frac{m_{Nx}}{2} \end{bmatrix} \begin{bmatrix} I_{diffx} \\ V_{CPx}^{\Sigma} \\ V_{CNx}^{\Sigma} \end{bmatrix} \quad (2)$$

III. NONLINEAR MMC MODEL IN DQ FRAME

Equations (1)and (2) cannot be directly transferred to DQ frame because of multiplication terms. In this section, a solution to overcome this problem is given. The proposed solution is based on some definitions and assumptions to simplify the complexity of the system model. These assumptions are considering 1) the AC currents as ideal sinusoidal without DC offsets. 2) the differential currents as DC offset plus second harmonic sinusoidal components and 3)The upper and lower modulation indexes as ideal sinusoidal plus DC offset components.

The alternating current for phase a is assumed as

$$i_{\mathrm{va}} = I_{\mathrm{vm}} \sin(\omega_0 t + \theta_{\mathrm{i}}) \tag{3}$$

where I_{vm} is a magnitude, ω_0 is angular frequency and θ_{i} is the phase angle

Based on reference [9], the differential current comprises two parts (i.e., a DC part representing one-third of the overall DC current and an AC part resultant to the circulating current with double fundamental frequency $2\omega_0$). The expressions of the differential current for phase a is therefore assumed as

$$I_{\mathrm{diffa}} = \frac{I_{\mathrm{DC}}}{3} + I_{\mathrm{cirm}} \sin(2\omega_0 t + \theta_{\mathrm{cir}}) \tag{4}$$

where I_{cirm} is the magnitude and θ_{cir} is phase angle of circulating current.

The modulation index of upper arm (for phase a) is considered as

$$m_{\mathrm{Pa}} = \frac{1 - M \sin(\omega_0 t + \theta_{\mathrm{m}})}{2} \tag{5}$$

And for the Lower arm as

$$m_{\mathrm{Na}} = \frac{1 + M \sin(\omega_0 t + \theta_{\mathrm{m}})}{2} \tag{6}$$

where M is modulation index and θ_m is phase angle.

The above definitions for I_{vx}, I_{diffx} , m_{Px} and m_{Nx} should be placed in the equation (1) and (2).To prevent multiple mathematical calculations, we first do the multiplication for a general case.

Consider the following two variables X(t) and Y(t)

$$X(t) = X_{\mathrm{m}} \cos(\omega_0 t + X_{\mathrm{a}}) + X_0 \tag{7}$$

$$Y(t) = Y_{\mathrm{m}} \cos(\omega_0 t + Y_{\mathrm{a}}) + Y_0 \tag{8}$$

where X_{m} and Y_{m} are magnitudes, X_{a} and Y_{a} are phase angles, X_0 and Y_0 are DC offset values and ω_0 is the angular frequency in a static frame.

By direct product of (7) and (8), we get

$$\begin{aligned} X(t)Y(t) &= \left(X_0 Y_0\right) + \frac{1}{2} X_{\mathrm{m}} \cos\left(X_{\mathrm{a}}\right) Y_{\mathrm{m}} \cos\left(Y_{\mathrm{a}}\right) + \frac{1}{2} X_{\mathrm{m}} \sin\left(X_{\mathrm{a}}\right) Y_{\mathrm{m}} \sin\left(Y_{\mathrm{a}}\right) \\ &+ \left(X_0 Y_{\mathrm{m}} \cos Y_{\mathrm{a}} + Y_0 X_{\mathrm{m}} \cos X_{\mathrm{a}}\right) \cos \omega t - (X_0 Y_{\mathrm{m}} \sin Y_{\mathrm{a}} + Y_0 X_{\mathrm{m}} \sin X_{\mathrm{a}}) \sin \omega t \\ &+ \frac{X_{\mathrm{m}} Y_{\mathrm{m}}}{2} \sin(2\omega t)\left(\sin(X_{\mathrm{a}}) \cos(Y_{\mathrm{a}}) + \cos(X_{\mathrm{a}}) \sin(Y_{\mathrm{a}})\right) \\ &+ \frac{X_{\mathrm{m}} Y_{\mathrm{m}}}{2} \cos(2\omega t)\left(\cos(X_{\mathrm{a}}) \cos(Y_{\mathrm{a}}) - \sin(X_{\mathrm{a}}) \sin(Y_{\mathrm{a}})\right) \end{aligned} \tag{9}$$

Equation (9) can be transferred to DQ coordinate frame using parks transformation matrix P as

$$\begin{bmatrix} \left(X(t)Y(t)\right)_{\mathrm{d}} \\ \left(X(t)Y(t)\right)_{\mathrm{q}} \\ \left(X(t)Y(t)\right)_{0} \end{bmatrix} = \left[P\right] \begin{bmatrix} X_{\mathrm{a}}(t)\, Y_{\mathrm{a}}(t) \\ X_{\mathrm{b}}(t)\, Y_{\mathrm{b}}(t) \\ X_{\mathrm{c}}(t)\, Y_{\mathrm{c}}(t) \end{bmatrix} \tag{10}$$

where P is the transformation matrix given by

$$P = \frac{2}{3} \begin{bmatrix} \cos\left(\omega_0 t\right) & \cos\left(\omega_0 t - 2\pi/3\right) & \cos\left(\omega_0 t + 2\pi/3\right) \\ -\sin\left(\omega_0 t\right) & -\sin\left(\omega_0 t - 2\pi/3\right) & -\sin\left(\omega_0 t + 2\pi/3\right) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \tag{11}$$

By considering

$$\begin{aligned} X_{\mathrm{d}} &= X_{\mathrm{m}} \cos(X_{\mathrm{a}}) \\ X_{\mathrm{q}} &= X_{\mathrm{m}} \sin(X_{\mathrm{a}}) \\ X_0 &= \frac{1}{2} X_{\mathrm{m}} \cos(X_{\mathrm{a}}) + \frac{1}{2} X_{\mathrm{m}} \sin(X_{\mathrm{a}}) + X_0 \end{aligned} \tag{12}$$

and neglecting the higher harmonics (considering only fundamental and DC terms in (9)), we have

$$\begin{aligned} \left(X(t)Y(t)\right)_{\mathrm{d}} &= X_0 Y_{\mathrm{d}} + Y_0 X_{\mathrm{d}} \\ \left(X(t)Y(t)\right)_{\mathrm{q}} &= -X_0 Y_{\mathrm{q}} - Y_0 X_{\mathrm{q}} \\ \left(X(t)Y(t)\right)_{0} &= \frac{1}{2} X_{\mathrm{d}} Y_{\mathrm{d}} + \frac{1}{2} X_{\mathrm{q}} Y_{\mathrm{q}} + X_0 Y_0 \end{aligned} \tag{13}$$

Using definitions (4)-(6) and (13), the I_{diffx} of dynamic equation (1) can be transferred to DQ frame as

$$\begin{aligned} \frac{d}{dt} \begin{bmatrix} I_{\mathrm{diffd}} \\ I_{\mathrm{diffq}} \\ I_{\mathrm{diff0}} \end{bmatrix} &= \begin{bmatrix} 0 & \omega_0 & 0 \\ -\omega_0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_{\mathrm{diffd}} \\ I_{\mathrm{diffq}} \\ I_{\mathrm{diff0}} \end{bmatrix} \\ &- \frac{1}{4L_x} \begin{bmatrix} 4R_x I_{\mathrm{diffd}} + 2m_{\mathrm{P0}} V^{\Sigma}_{\mathrm{CPd}} + 2V^{\Sigma}_{\mathrm{CP0}} m_{\mathrm{Pd}} + 2m_{\mathrm{N0}} V^{\Sigma}_{\mathrm{CNd}} + 2V^{\Sigma}_{\mathrm{CN0}} m_{\mathrm{Nd}} \\ 4R_x I_{\mathrm{diffq}} + 2m_{\mathrm{P0}} V^{\Sigma}_{\mathrm{CPq}} + 2V^{\Sigma}_{\mathrm{CP0}} m_{\mathrm{Pq}} + 2m_{\mathrm{N0}} V^{\Sigma}_{\mathrm{CNq}} + 2V^{\Sigma}_{\mathrm{CN0}} m_{\mathrm{Nq}} \\ 4R_x I_{\mathrm{diff0}} + 2V^{\Sigma}{}_{\mathrm{CP0}} m_{\mathrm{P0}} + V^{\Sigma}_{\mathrm{CPq}} m_{\mathrm{Pq}} + V^{\Sigma}_{\mathrm{CPd}} m_{\mathrm{Pd}} + 2V^{\Sigma}{}_{\mathrm{CN0}} m_{\mathrm{N0}} + V^{\Sigma}_{\mathrm{CNq}} m_{\mathrm{Nq}} + V^{\Sigma}_{\mathrm{CNd}} m_{\mathrm{Nd}} - 2V_{\mathrm{DC}} \end{bmatrix} \end{aligned} \tag{14}$$

And the upper and lower arm capacitor voltage as

$$\frac{d}{dt}\begin{bmatrix} V^{\Sigma}_{CPd} \\ V^{\Sigma}_{CNd} \\ V^{\Sigma}_{CPq} \\ V^{\Sigma}_{CNq} \\ V^{\Sigma}_{CP0} \\ V^{\Sigma}_{CN0} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \omega_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_0 & 0 & 0 \\ -\omega_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\omega_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} V^{\Sigma}_{CPd} \\ V^{\Sigma}_{CNd} \\ V^{\Sigma}_{CPq} \\ V^{\Sigma}_{CNq} \\ V^{\Sigma}_{CP0} \\ V^{\Sigma}_{CN0} \end{bmatrix} + \frac{1}{2C^{arm}}\begin{bmatrix} 2m_{P0}I_{diffd} + 2m_{Pd}I_{diff0} + m_{Pd}I_{v0} + m_{P0}I_{vd} \\ 2m_{N0}I_{diffd} + 2m_{Nd}I_{diff0} - m_{Nd}I_{v0} - m_{N0}I_{vd} \\ 2m_{P0}I_{diffq} + 2m_{Pq}I_{diff0} + m_{Pq}I_{v0} + m_{P0}I_{vq} \\ 2m_{N0}I_{diffq} + 2m_{Nq}I_{diff0} - m_{Nq}I_{v0} - m_{N0}I_{vq} \\ 2m_{P0}I_{diff0} + m_{Pd}I_{diffd} + m_{Pq}I_{diffq} + m_{P0}I_{v0} + \frac{m_{Pd}I_{vd}}{2} + \frac{m_{Pq}I_{vq}}{2} \\ 2m_{N0}I_{diff0} + m_{Nd}I_{diffd} + m_{Nq}I_{diffq} - m_{N0}I_{v0} - \frac{m_{Nd}I_{vd}}{2} - \frac{m_{Nq}I_{vq}}{2} \end{bmatrix} \quad (15)$$

In steady state the differential current DQ components will be zero and only DC current remains (zero sequence component). This DC current transfers the active power transfer between both sides of the converter. Based on this further assumption the (14) will be

$$\frac{d}{dt}\left[I_{diff0}\right] = -\frac{R_x I_{diff0}}{L_x} - \frac{V^{\Sigma}_{CP0}m_{P0}}{2L_x} - \frac{V^{\Sigma}_{CPd}m_{Pd}}{4L_x} - \frac{V^{\Sigma}_{CNd}m_{Nd}}{4L_x} - \frac{V^{\Sigma}_{CN0}m_{N0}}{2L_x} - \frac{V^{\Sigma}_{CPq}m_{Pq}}{4L_x} - \frac{V^{\Sigma}_{CNq}m_{Nq}}{4L_x} + \frac{V_{DC}}{2L_x} \quad (16)$$

Considering the above assumption ($I_{diffd} = I_{diffq} = 0$), and also the upper and lower arm zero sequence modulation indexes are just DC offset value ($m_{P0}, m_{N0} = 1/2$), and zero sequence components of AC side output current is zero ($I_{v0} = 0$), equation. (15) can be written as

$$\frac{d}{dt}\begin{bmatrix} V^{\Sigma}_{CPd} \\ V^{\Sigma}_{CNd} \\ V^{\Sigma}_{CPq} \\ V^{\Sigma}_{CNq} \\ V^{\Sigma}_{CP0} \\ V^{\Sigma}_{CN0} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \omega_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_0 & 0 & 0 \\ -\omega_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\omega_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} V^{\Sigma}_{CPd} \\ V^{\Sigma}_{CNd} \\ V^{\Sigma}_{CPq} \\ V^{\Sigma}_{CNq} \\ V^{\Sigma}_{CP0} \\ V^{\Sigma}_{CN0} \end{bmatrix} + \frac{1}{2C^{arm}}\begin{bmatrix} 2m_{Pd}I_{diff0} + \frac{I_{vd}}{2} \\ 2m_{Nd}I_{diff0} - \frac{I_{vd}}{2} \\ 2m_{Pq}I_{diff0} + \frac{I_{vq}}{2} \\ 2m_{Nq}I_{diff0} - \frac{I_{vq}}{2} \\ 2m_{P0}I_{diff0} + m_{Pd}I_{vd}/2 + m_{Pq}I_{vq}/2 \\ 2m_{N0}I_{diff0} - m_{Nd}I_{vd}/2 - m_{Nq}I_{vq}/2 \end{bmatrix} \quad (17)$$

And the output signals (2) in do frame will be

$$e_d = \frac{m_{N0}V^{\Sigma}_{CNd} + V^{\Sigma}_{CN0}m_{Nd} - m_{P0}V^{\Sigma}_{CPd} - V^{\Sigma}_{CP0}m_{Pd}}{2}$$
$$e_q = \frac{m_{N0}V^{\Sigma}_{CNq} + V^{\Sigma}_{CN0}m_{Nq} - m_{P0}V^{\Sigma}_{CPq} - V^{\Sigma}_{CP0}m_{Pq}}{2} \quad (18)$$
$$I_{DC} = 3I_{diff0}$$

IV. LINEARISED MODEL IN DQ FRAME

To perform small signal analysis equation(16),(17) and (18) should be linearized at their operating point. The small signal analysis using linear techniques provides important information about the inherent dynamic characteristics of the modelled system. The aim of this approach is to create the subsystems (AC model, DC Model, PLL and the controller) individually, then link them together according to their interaction[7]. The AC and DC controllers are therefore developed in a matrix form within their subsystems in order to incorporate all the states that may have an influence and/or interaction. This model, however, allows modern control design techniques to be employed, including eigenvalues studies and frequency domain analysis. The AC, DC and PLL subsystems' models can be found in [7] and only MMC subsystem model is presented in this paper. The linearization principle can be found in [13].

Equations (16)-(18) are linearized and are given in standard state-space form as

$$\frac{d}{dt}\begin{bmatrix} V^{\Sigma}_{CPd} \\ V^{\Sigma}_{CNd} \\ V^{\Sigma}_{CPq} \\ V^{\Sigma}_{CNq} \\ V^{\Sigma}_{CP0} \\ V^{\Sigma}_{CN0} \\ I_{diff0} \end{bmatrix} = A\begin{bmatrix} V^{\Sigma}_{CPd} \\ V^{\Sigma}_{CNd} \\ V^{\Sigma}_{CPq} \\ V^{\Sigma}_{CNq} \\ V^{\Sigma}_{CP0} \\ V^{\Sigma}_{CN0} \\ I_{diff0} \end{bmatrix} + B\begin{bmatrix} m_{Pd} \\ m_{Nd} \\ I_{vd} \\ m_{Pq} \\ m_{Nq} \\ I_{vq} \\ V_{DC} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{\dot{V}^{\Sigma}_{CP0}}{4L_x} - \frac{\dot{V}^{\Sigma}_{CN0}}{4L_x} \end{bmatrix} \quad (19)$$

The output signals can be stated is

$$\begin{bmatrix} e_d \\ e_q \\ I_{DC} \end{bmatrix} = C\begin{bmatrix} V^{\Sigma}_{CPd} \\ V^{\Sigma}_{CNd} \\ V^{\Sigma}_{CPq} \\ V^{\Sigma}_{CNq} \\ V^{\Sigma}_{CP0} \\ V^{\Sigma}_{CN0} \\ I_{diff0} \end{bmatrix} + D\begin{bmatrix} m_{Pd} \\ m_{Nd} \\ I_{vd} \\ m_{Pq} \\ m_{Nq} \\ I_{vq} \\ V_{DC} \end{bmatrix} \quad (20)$$

where

$$A=\begin{bmatrix} 0 & 0 & \omega_0 & 0 & 0 & 0 & \frac{\dot{m}_{\mathrm{Pd}}}{C^{arm}} \\ 0 & 0 & 0 & \omega_0 & 0 & 0 & \frac{\dot{m}_{\mathrm{Nd}}}{C^{arm}} \\ -\omega_0 & 0 & 0 & 0 & 0 & 0 & \frac{-\dot{m}_{\mathrm{Pq}}}{C^{arm}} \\ 0 & -\omega_0 & 0 & 0 & 0 & 0 & \frac{-\dot{m}_{\mathrm{Nq}}}{C^{arm}} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ -1/4L_x & -1/4L_x & -1/4L_x & -1/4L_x & -1/2L_x & -1/2L_x & -\frac{R_x}{L_x} \end{bmatrix}$$

$$B=\begin{bmatrix} \frac{\dot{I}_{\mathrm{diff0}}}{C^{arm}} & 0 & \frac{1}{4C^{arm}} & 0 & 0 & 0 & 0 \\ 0 & \frac{\dot{I}_{\mathrm{diff0}}}{C^{arm}} & \frac{1}{4C^{arm}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-\dot{I}_{\mathrm{diff0}}}{C^{arm}} & 0 & \frac{-1}{4C^{arm}} & 0 \\ 0 & 0 & 0 & 0 & \frac{-\dot{I}_{\mathrm{diff0}}}{C^{arm}} & \frac{-1}{4C^{arm}} & 0 \\ \frac{\dot{I}_{\mathrm{vd}}}{2} & 0 & \frac{\dot{m}_{\mathrm{Pd}}}{2} & \frac{\dot{I}_{\mathrm{vq}}}{2} & 0 & \frac{\dot{m}_{\mathrm{Pq}}}{2} & 0 \\ 0 & \frac{\dot{I}_{\mathrm{vd}}}{2} & \frac{\dot{m}_{\mathrm{Nd}}}{2} & 0 & \frac{\dot{I}_{\mathrm{vq}}}{2} & \frac{\dot{m}_{\mathrm{Nq}}}{2} & 0 \\ -\frac{\dot{V}^{\Sigma}_{\mathrm{CPd}}}{4L_x} & -\frac{\dot{V}^{\Sigma}_{\mathrm{CNd}}}{4L_x} & 0 & -\frac{\dot{V}^{\Sigma}_{\mathrm{CPq}}}{4L_x} & -\frac{\dot{V}^{\Sigma}_{\mathrm{CNq}}}{4L_x} & 0 & \frac{1}{2L_x} \end{bmatrix}$$

$$C=\begin{bmatrix} -\frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{-\dot{m}_{\mathrm{Pd}}}{2} & \frac{\dot{m}_{\mathrm{Nd}}}{2} & 0 \\ 0 & 0 & -\frac{1}{4} & \frac{1}{4} & -\frac{\dot{m}_{\mathrm{Pq}}}{2} & \frac{\dot{m}_{\mathrm{Nq}}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}, D=\begin{bmatrix} -\dot{V}^{\Sigma}_{\mathrm{CP0}} & \dot{V}^{\Sigma}_{\mathrm{CN0}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\dot{V}^{\Sigma}_{\mathrm{CP0}} & \dot{V}^{\Sigma}_{\mathrm{CN0}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (21)$$

where superscript $\dot{x}$ denotes steady state value before applying the step.

V. SIMULATION RESULTS

The test system is an open loop MMC system including MMC, DC and AC system with PLL as shown in Fig 3. The dynamic equations of DC, AC and PLL subsystems are given in [12] and V_{Sdq} is the DQ components of the AC voltage at the point of common coupling. The MMC parameters of the detailed model using selective harmonic elimination modulation are given in Table 1.

The detailed benchmark model of the test system is implemented in PSCAD/EMTDC and the linearized DQ model is developed in Matlab and the results are compared together. The modulation index M_{d} is stepped from 0.945 to 0.845(10%) at 1.5 sec while the M_{q} is fixed to 0.05. Fig 4 - Fig 8 shows the variable comparisons for the two models. It is seen that the linearized model gives good matching with the original detailed model.

Table 1 Parameters for MMC using Selective harmonic elimination

Power Rating	1000 MVA
AC Voltage Rating	230 kV
DC voltage	400 kV
System Frequency	50 Hz
Number of SM per arm	8
Number of levels	9
Module Capacitance C	5000 µF
Arm Inductance L	1.26 mH
Total switching resistor per arm	0.08 ohm

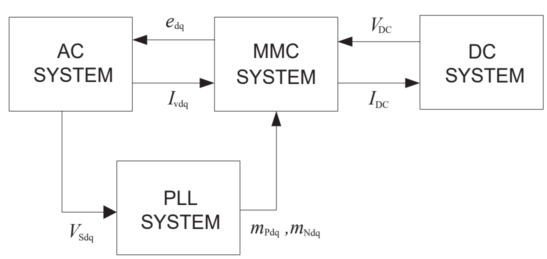

Fig 3. Block diagram of MMC transmission model

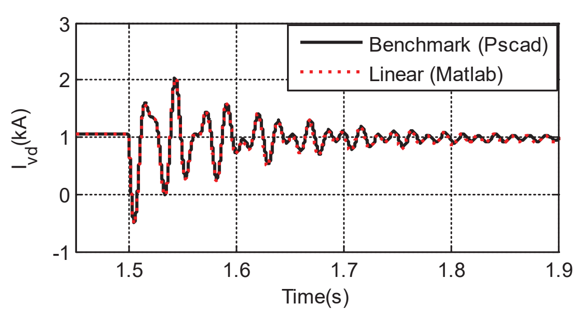

Fig 4. Comparison of input current I_{vd}

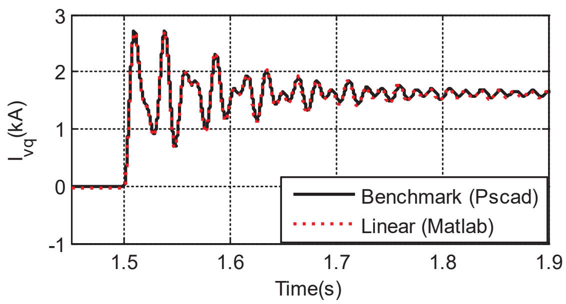

Fig 5. Comparison of input current I_{vq}

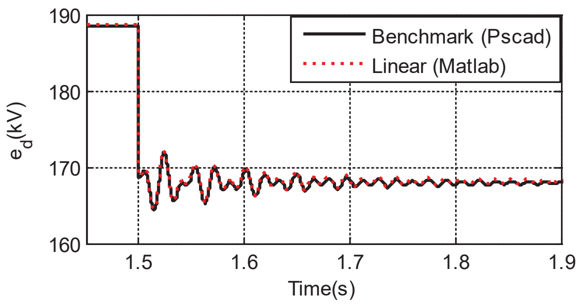

Fig 6. Comparison of output voltage e_d

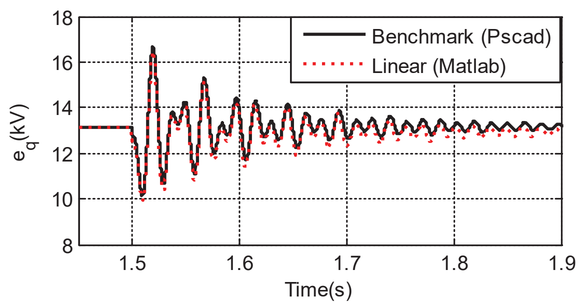

Fig 7. Comparison of output voltage e_q

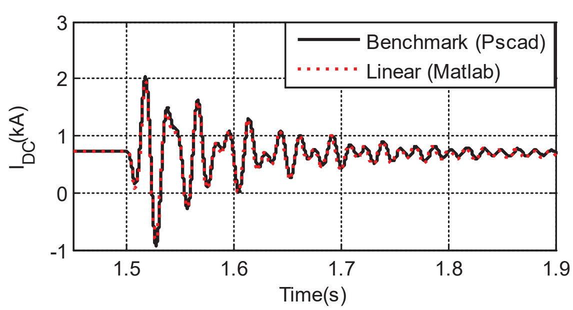

Fig 8. Comparison of output current I_{DC}

VI. Conclusions

The analytical modelling of an MMC in DQ frame is a challenging task due to non-linear nature and configuration of the actual MMC system. In this paper, the dynamic equations of MMC were transferred to DQ frame using the time multiplication of the variables and ignoring the higher harmonics. Then, the achieved nonlinear DQ MMC model was linearized around the nominal operating point.

A benchmark test system is built in PSCAD and a linearized Matlab model, which it is developed in a modular form comprising of MMC, AC, DC and PLL subsystems. The results of the benchmark detailed test system were compared against the linearized model. The results show very good matching and confirm the accuracy of the proposed linearized model. This model allows modern control design techniques to be employed, including eigenvalues studies and frequency domain analysis.

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