Averaged Model of Modular Multilevel Converter in Rotating DQ frame

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***Abstract*—This paper proposes an average model of Modular Multilevel Converter (MMC) in rotating DQ frame. The proposed MMC model has a modular structure and can be linked with other power elements such as AC and DC subsystems. Modelling in DQ frame has numerous advantages over traditional ABC frame in terms of simulation speed and convenience for linearization. The main challenge of developing DQ model of MMC is to deal with the multiplication terms of dynamic equations of MMC. To overcome this complexity, a generic form is first introduced for each product variable mathematical equations of the average MMC model in ABC frame and then the multiplication results are transferred to DQ frame after ignoring the higher harmonics. The detailed model and the proposed DQ average model are implemented in PSCAD/EMTDC. The simulation results of the two models show very good matching which in turn confirms the accuracy of the proposed model. Also, the DQ average model is considerably faster than the detailed and even ABC average models.**

***Index Terms*-- Modular Multilevel converter, modelling, averaged model, DQ frame.**

I. INTRODUCTION

In recent years, Modular Multilevel Converter (MMC) has become progressively popular converter for High Voltage Direct Current (HVDC) and industrial drives applications [1]-[4]. In contrast to a conventional two-level voltage source converter (VSC), MMC use a number of (low voltage) series-connected capacitors to generate high AC voltage. This allows higher power-handling capability with reduced switching power losses and harmonic distortion [5]. The MMC can operate at both fundamental switching frequency and high switching frequency Pulse Width Modulation (PWM). It should be noted that lower switching frequency usually means lower switching loss and higher efficiency. As a result of this, the MMC has been an active research topic in recent years.

The non-linear detailed dynamic models of MMC with different modulation techniques are presented [2],[6],[7]. These models are developed in PSCAD/EMTDC [8], which denotes detailed characteristics of all switches. Therefore, these detailed models represent the real system accurately.

These detailed models are discrete in nature and involves a considerable amount of simulation time. The model complexity and computation burden is going to increase when the system levels increase. At the same time, the detailed models are discontinuous and therefore it does not provide an analytical approach for stability analysis and controller design purposes. This complicates the study of transients especially when these systems are integrated into a large network.

The average model is introduced to overcome the technical complications of detailed models[7],[10].The average value models and their purpose is to replicate the average response of switching devices, converters and controls by using mathematical equations and controlled voltage or current sources. However, the proposed average models for MMC are in ABC frame. These ABC frame average MMC models are not suitable for developing linearized small signal models which are needed for analytical studies and eigenvalue analysis [9],[11].

Transferring the ABC frame average model of MMC to DQ frame is not a straightforward task. It is a challenge mainly because of multiplication terms and higher harmonics in the dynamic equations of the ABC frame average model.

This paper aims to give a solution to this challenge and develop an MMC model in DQ frame. By using a simple elementary sinusoidal signal derivation, the ABC frame MMC model equations are transferred to the DQ frame. The developed model can be easily linearized and used for eigenvalue analysis and controller design. It also runs faster than the detailed and ABC frame average models thanks to its less mathematical equations and elimination of oscillatory variables.

II. OPERATIONAL PRINCIPLE AND EQUIVALENT STRUCTURE OF MMC

A. Electrical circuit structure

Fig.1 shows the electrical structure of MMC [7]. It consists of three phase legs and two arms (upper and lower) per each phase leg. Each arm comprises of N sub-modules (SMs), one equivalent resistor Ra, and one inductor La in order to smooth the voltage difference and phase current. Each SM is a simple half-bridge with a capacitor bank where its output voltage is either equal to its capacitor voltage or zero depending on the switching states. The switching states are determined by modulation and voltage balancing strategy [6].The number of output levels depends on the number of SMs in each arm.

A detailed model based on Fig.1 can be developed in PSCAD to study the MMC behaviour. This model is very accurate. However, it needs very long simulation time to run. This simulation time will be further increased if the number of either MMC in a grid or SMs in the MMC is increased.

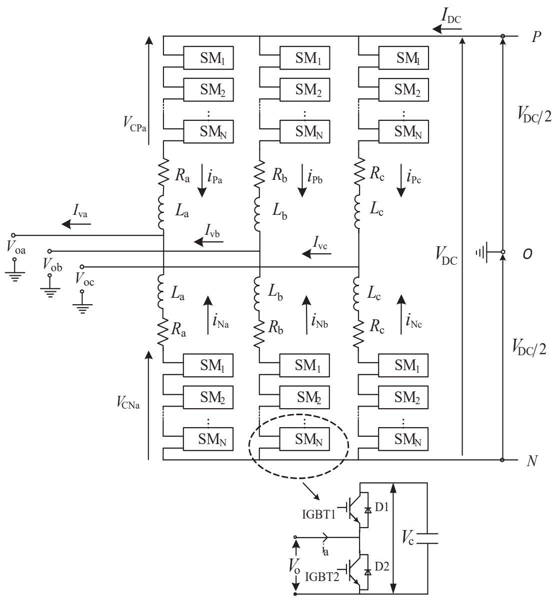

Fig 1. Circuit configuration of MMC

B. *Average model in ABC frame*

Fig. 2 shows the equivalent circuit for one phase leg of MMC. It is based on substituting the SMs of each arm with an equivalent variable voltage supply. The value of the equivalent voltage supply is calculated continuously using the following mathematical equations [10]

$$\frac{d}{dt}\begin{bmatrix} I_{\mathrm{diffx}} \\ V_{\mathrm{CPx}}^{\Sigma} \\ V_{\mathrm{CNx}}^{\Sigma} \end{bmatrix} = \begin{bmatrix} -\frac{R_x}{L_x} & \frac{-m_{\mathrm{Px}}}{2L_x} & \frac{-m_{\mathrm{Nx}}}{2L_x} \\ \frac{m_{\mathrm{Px}}}{C^{arm}} & 0 & 0 \\ \frac{m_{\mathrm{Nx}}}{C^{arm}} & 0 & 0 \end{bmatrix} \begin{bmatrix} I_{\mathrm{diffx}} \\ V_{\mathrm{CPx}}^{\Sigma} \\ V_{\mathrm{CNx}}^{\Sigma} \end{bmatrix} + \begin{bmatrix} \frac{V_{\mathrm{DC}}}{2L_x} \\ \frac{m_{\mathrm{Px}} I_{\mathrm{vx}}}{2C^{\mathrm{arm}}} \\ \frac{-m_{\mathrm{Nx}} I_{\mathrm{vx}}}{2C^{\mathrm{arm}}} \end{bmatrix} \tag{1}$$

where C^{arm}=C/N, C is the capacitance of one SM, N is the number of SMs in an arm, R_x and L_x are inductance and equivalent arm resistance of respective phase x (x=a,b,c), I_{diffx} is the differential current, V_{CPx} and V_{CNx} are the upper and lower arm voltage, $V_{\mathrm{CPx}}^{\Sigma}$ and $V_{\mathrm{CNx}}^{\Sigma}$ are voltage stored in upper and lower arm capacitor, V_{ox} and I_{vx} are the AC side converter voltage and line current, V_{DC} and I_{DC} are the DC bus voltage and current, and m_{Px} and m_{Nx} are modulation indexes of upper and lower arm.

The output of MMC is defined

$$I_{\mathrm{DC}} = \sum_{x=a,b,c} I_{\mathrm{diffx}}$$
$$e_x = \frac{V_{\mathrm{CNx}}}{2} - \frac{V_{\mathrm{CPx}}}{2} = \frac{m_{\mathrm{Nx}} V_{\mathrm{CNx}}^{\Sigma}}{2} - \frac{m_{\mathrm{Px}} V_{\mathrm{CPx}}^{\Sigma}}{2} \tag{2}$$

where e_x is the internal voltage of MMC.

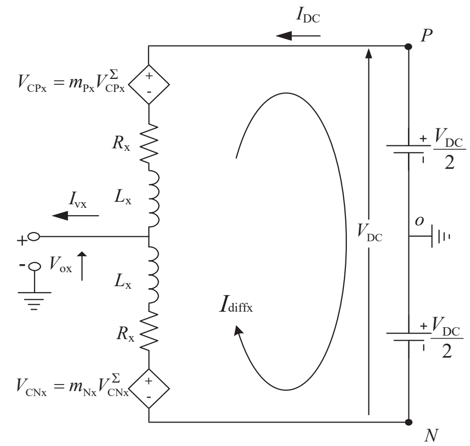

Fig 2. Equivalent circuit for one phase leg

The above ABC frame average model shows very good matching with the detailed model and runs significantly faster.

III. AVERAGE MMC MODEL IN DQ FRAME

Analyzing DC grids and designing controllers in time-domain softwares such as PSCAD is a very time-consuming task especially when we are dealing with large grids. One common way to do analytical study and controller design for such a system is eigenvalue study which needs a linear small-signal model. This linear model can be easily developed from the DQ frame model of the power system.

Developing a DQ average model for the system with VSCs is straightforward. However, it is a challenging task for MMC because of the multiplication terms in equations (1) and (2).

The proposed solution is based on considering a generic representation for each product variable and multiplying them in ABC frame and then transferring the results to DQ frame after ignoring the higher harmonics. In the rest of this section, an initial model in DQ frame is given first and then by manipulating the product terms, the final model will be developed.

A. *Initial Model in DQ-frame*

The average model in ABC frame can be divided into two sections; 1) the capacitor cells voltages and 2) the differential current.

The capacitor cell voltages equations can be rewritten as

$$\frac{d}{dt}\begin{bmatrix} V^{\Sigma}_{\mathrm{CPx}} \\ V^{\Sigma}_{\mathrm{CNx}} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} V^{\Sigma}_{\mathrm{CPx}} \\ V^{\Sigma}_{\mathrm{CNx}} \end{bmatrix} + \begin{bmatrix} \frac{1}{C^{\mathrm{arm}}} & 0 & \frac{1}{2C^{\mathrm{arm}}} & 0 \\ 0 & \frac{1}{C^{\mathrm{arm}}} & 0 & \frac{-1}{2C^{\mathrm{arm}}} \end{bmatrix}\begin{bmatrix} m_{\mathrm{Px}} I_{\mathrm{diffx}} \\ m_{\mathrm{Nx}} I_{\mathrm{diffx}} \\ m_{\mathrm{Px}} I_{\mathrm{vx}} \\ m_{\mathrm{Nx}} I_{\mathrm{vx}} \end{bmatrix} \tag{3}$$

and the differential current equation as

$$\frac{d}{dt}\left[I_{\mathrm{diffx}} \right] = \left[-\frac{R_x}{L_x} \right] I_{\mathrm{diffx}} + \begin{bmatrix} \frac{1}{2L_x} & \frac{-1}{2L_x} & \frac{-1}{2L_x} \end{bmatrix}\begin{bmatrix} V_{\mathrm{DC}} \\ m_{\mathrm{Px}} V^{\Sigma}_{\mathrm{CPx}} \\ m_{\mathrm{Nx}} V^{\Sigma}_{\mathrm{CNx}} \end{bmatrix} \tag{4}$$

The output equations are

$$\begin{aligned} e_{\mathrm{x}} &= \frac{V_{\mathrm{CN\,x}}}{2} - \frac{V_{\mathrm{CP\,x}}}{2} = \frac{m_{\mathrm{Nx}} V^{\Sigma}_{\mathrm{CNx}}}{2} - \frac{m_{\mathrm{Px}} V^{\Sigma}_{\mathrm{CPx}}}{2} \\ I_{\mathrm{DC}} &= \sum_{x=a,b,c} I_{\mathrm{diffx}} = I_{\mathrm{diff\,a}} + I_{\mathrm{diff\,b}} + I_{\mathrm{diff\,c}} \end{aligned} \tag{5}$$

Equations (3),(4) and (5) can be transferred to DQ frame as

$$\frac{d}{dt}\begin{bmatrix} V^{\Sigma}_{\mathrm{CPD}} \\ V^{\Sigma}_{\mathrm{CND}} \\ V^{\Sigma}_{\mathrm{CPQ}} \\ V^{\Sigma}_{\mathrm{CNQ}} \\ V^{\Sigma}_{\mathrm{CP0}} \\ V^{\Sigma}_{\mathrm{CN0}} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \omega_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_0 & 0 & 0 \\ -\omega_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\omega_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} V^{\Sigma}_{\mathrm{CPD}} \\ V^{\Sigma}_{\mathrm{CND}} \\ V^{\Sigma}_{\mathrm{CPQ}} \\ V^{\Sigma}_{\mathrm{CNQ}} \\ V^{\Sigma}_{\mathrm{CP0}} \\ V^{\Sigma}_{\mathrm{CN0}} \end{bmatrix} + \frac{1}{C^{arm}} \begin{bmatrix} 1 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{-1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{-1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{-1}{2} \end{bmatrix}\begin{bmatrix} \left(m_{\mathrm{P}} I_{\mathrm{diff}}\right)_{\mathrm{D}} \\ \left(m_{\mathrm{N}} I_{\mathrm{diff}}\right)_{\mathrm{D}} \\ \left(m_{\mathrm{P}} I_{\mathrm{v}}\right)_{\mathrm{D}} \\ \left(m_{\mathrm{N}} I_{\mathrm{v}}\right)_{\mathrm{D}} \\ \left(m_{\mathrm{P}} I_{\mathrm{diff}}\right)_{\mathrm{Q}} \\ \left(m_{\mathrm{N}} I_{\mathrm{diff}}\right)_{\mathrm{Q}} \\ \left(m_{\mathrm{P}} I_{\mathrm{v}}\right)_{\mathrm{Q}} \\ \left(m_{\mathrm{N}} I_{\mathrm{v}}\right)_{\mathrm{Q}} \\ \left(m_{\mathrm{P}} I_{\mathrm{diff}}\right)_{0} \\ \left(m_{\mathrm{N}} I_{\mathrm{diff}}\right)_{0} \\ \left(m_{\mathrm{P}} I_{\mathrm{v}}\right)_{0} \\ \left(m_{\mathrm{N}} I_{\mathrm{v}}\right)_{0} \end{bmatrix} \tag{6}$$

$$\frac{d}{dt}\begin{bmatrix} I_{\mathrm{diffD}} \\ I_{\mathrm{diffQ}} \\ I_{\mathrm{diff0}} \end{bmatrix} = \begin{bmatrix} -\frac{R_x}{L_x} & \omega_0 & 0 \\ -\omega_0 & -\frac{R_x}{L_x} & 0 \\ 0 & 0 & -\frac{R_x}{L_x} \end{bmatrix}\begin{bmatrix} I_{\mathrm{diffD}} \\ I_{\mathrm{diffQ}} \\ I_{\mathrm{diff0}} \end{bmatrix} + \frac{1}{2L_x} \begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} \left(V_{\mathrm{DC}}\right)_{\mathrm{D}} \\ \left(m_{\mathrm{P}} V^{\Sigma}_{\mathrm{CP}}\right)_{\mathrm{D}} \\ \left(m_{\mathrm{N}} V^{\Sigma}_{\mathrm{CN}}\right)_{\mathrm{D}} \\ \left(V_{\mathrm{DC}}\right)_{\mathrm{Q}} \\ \left(m_{\mathrm{P}} V^{\Sigma}_{\mathrm{CP}}\right)_{\mathrm{Q}} \\ \left(m_{\mathrm{N}} V^{\Sigma}_{\mathrm{CN}}\right)_{\mathrm{Q}} \\ \left(V_{\mathrm{DC}}\right)_{0} \\ \left(m_{\mathrm{P}} V^{\Sigma}_{\mathrm{CP}}\right)_{0} \\ \left(m_{\mathrm{N}} V^{\Sigma}_{\mathrm{CN}}\right)_{0} \end{bmatrix} \tag{7}$$

The output equations in DQ frame are

$$\begin{aligned} e_{\mathrm{D}} &= \frac{V_{\mathrm{CND}} - V_{\mathrm{CPD}}}{2} = \frac{\left(m_{\mathrm{N}} V^{\Sigma}_{\mathrm{CN}}\right)_{\mathrm{D}}}{2} - \frac{\left(m_{\mathrm{P}} V^{\Sigma}_{\mathrm{CP}}\right)_{\mathrm{D}}}{2} \\ e_{\mathrm{Q}} &= \frac{V_{\mathrm{CNQ}} - V_{\mathrm{CPQ}}}{2} = \frac{\left(m_{\mathrm{N}} V^{\Sigma}_{\mathrm{CN}}\right)_{\mathrm{Q}}}{2} - \frac{\left(m_{\mathrm{P}} V^{\Sigma}_{\mathrm{CP}}\right)_{\mathrm{Q}}}{2} \\ e_{0} &= \frac{V_{\mathrm{CN0}} - V_{\mathrm{CP0}}}{2} = \frac{\left(m_{\mathrm{N}} V^{\Sigma}_{\mathrm{CN}}\right)_{0}}{2} - \frac{\left(m_{\mathrm{P}} V^{\Sigma}_{\mathrm{CP}}\right)_{0}}{2} \\ I_{\mathrm{DC}} &= 3 I_{\mathrm{diff0}} \end{aligned} \tag{8}$$

B. *Multiplication of two oscillating signals*

The multiplication terms of equations (6)-(8) cannot be transferred into DQ frame in a straightforward way. To do this, we considered the generic form of two general signals *X(t)* and *Y(t)* as

$$\begin{aligned} X(t) &= X_{\mathrm{m}} \cos(\omega t + \theta_{\mathrm{X}}) + X_{\mathrm{DC}} \\ Y(t) &= Y_{\mathrm{m}} \cos(\omega t + \theta_{\mathrm{Y}}) + Y_{\mathrm{DC}} \end{aligned} \tag{9}$$

The product signal *Z(t)* is

$$\begin{aligned} &Z(t) = X(t)Y(t) = \\ &\left(X_{\mathrm{m}} \cos(\omega t + \theta_{\mathrm{X}}) + X_{\mathrm{DC}}\right)\left(Y_{\mathrm{m}} \cos(\omega t + \theta_{\mathrm{Y}}) + Y_{\mathrm{DC}}\right) \end{aligned} \tag{10}$$

By considering

$$\begin{aligned} X_{\mathrm{D}} &= X_{\mathrm{m}} \cos(\theta_{\mathrm{X}}), X_{\mathrm{Q}} = X_{\mathrm{m}} \sin(\theta_{\mathrm{X}}), X_0 = X_{\mathrm{DC}} \\ Y_{\mathrm{D}} &= Y_{\mathrm{m}} \cos(\theta_{\mathrm{Y}}), Y_{\mathrm{Q}} = Y_{\mathrm{m}} \sin(\theta_{\mathrm{Y}}), Y_0 = Y_{\mathrm{DC}} \end{aligned} \tag{11}$$

The equation (10) can be written as

$$Z(t)=\left(X_{\mathrm{D}}\cos\omega t-X_{\mathrm{Q}}\sin\omega t+X_0\right)\left(Y_{\mathrm{D}}\cos\omega t-Y_{\mathrm{Q}}\sin\omega t+Y_0\right) \quad (12)$$

After multiplication and rearranging,

$$\begin{aligned}Z(t)&=\frac{1}{2}X_{\mathrm{D}}Y_{\mathrm{D}}+\frac{1}{2}X_{\mathrm{Q}}Y_{\mathrm{Q}}+X_0Y_0\\&+\left(X_{\mathrm{D}}Y_0+X_0Y_{\mathrm{Q}}\right)\cos\omega t-\left(X_{\mathrm{Q}}Y_0+X_0Y_{\mathrm{Q}}\right)\sin\omega t\\&+\frac{1}{2}\left(X_{\mathrm{D}}Y_{\mathrm{D}}-X_{\mathrm{Q}}Y_{\mathrm{Q}}\right)\cos(2\omega t)-\frac{1}{2}\left(X_{\mathrm{D}}Y_{\mathrm{Q}}+X_{\mathrm{Q}}Y_{\mathrm{D}}\right)\sin(2\omega t)\end{aligned} \quad (13)$$

By considering only DC and fundamental frequency components and ignoring the higher harmonic terms,

$$\begin{aligned}Z_{\mathrm{D}}&=\left(X(t)Y(t)\right)_{\mathrm{D}}=X_0Y_{\mathrm{D}}+Y_0X_{\mathrm{D}}\\Z_{\mathrm{Q}}&=\left(X(t)Y(t)\right)_{\mathrm{Q}}=-X_0Y_{\mathrm{Q}}-Y_0X_{\mathrm{Q}}\\Z_0&=\left(X(t)Y(t)\right)_0=\frac{1}{2}X_{\mathrm{D}}Y_{\mathrm{D}}+\frac{1}{2}X_{\mathrm{Q}}Y_{\mathrm{Q}}+X_0Y_0\end{aligned} \quad (14)$$

C. Input signals

The input signals of MMC are the AC current I_{vx}, The DC voltage V_{DC} and the upper and lower modulation index m_{Px} and m_{Nx}.

The AC current is assumed to be an ideal sinusoidal wave as

$$I_{\mathrm{vx}}=I_{\mathrm{vm}}\cos(\omega_0 t+\theta_{\mathrm{i}}) \quad (15)$$

where I_{vm} is the magnitude and θ_{i} is the phase angle. The AC current has no DC offset, therefore $I_{\mathrm{v0}}=0$.

The modulation index of upper and lower arms are considered as

$$\begin{aligned}m_{\mathrm{Pa}}&=\frac{1-M\cos(\omega_0 t+\theta_{\mathrm{m}})}{2}\\m_{\mathrm{Na}}&=\frac{1+M\cos(\omega_0 t+\theta_{\mathrm{m}})}{2}\end{aligned} \quad (16)$$

where M is the magnitude and θ_{m} is the phase angle. It is easily concluded that $m_{\mathrm{P0}}=m_{\mathrm{N0}}=1/2$.

The DC voltage has no oscillatory part, therefore $(V_{\mathrm{DC}})_0=V_{\mathrm{DC}}$ and $(V_{\mathrm{DC}})_{\mathrm{D}}=0$, $(V_{\mathrm{DC}})_{\mathrm{Q}}=0$.

D. Final Model in DQ-frame

Using circulating current suppress control [6], the equation (7) can be verified that $I_{\mathrm{diffD}}\approx I_{\mathrm{diffQ}}\approx 0$. Therefore, the differential current equation can be simplified as

$$\frac{d}{dt}\left[I_{\mathrm{diff0}}\right]=\left[-\frac{R_x}{L_x}\right]I_{\mathrm{diff0}}+\frac{1}{2L_x}\begin{bmatrix}1 & -1 & -1\end{bmatrix}\begin{bmatrix}\left(V_{\mathrm{DC}}\right)_0\\\left(m_{\mathrm{P}}V_{\mathrm{CP}}^{\Sigma}\right)_0\\\left(m_{\mathrm{N}}V_{\mathrm{CN}}^{\Sigma}\right)_0\end{bmatrix} \quad (17)$$

By substituting equation (14) in (6) and also considering the simplification results of the input signals of the previous subsection, we have

$$\frac{d}{dt}\begin{bmatrix}V_{\mathrm{CPD}}^{\Sigma}\\V_{\mathrm{CND}}^{\Sigma}\\V_{\mathrm{CPQ}}^{\Sigma}\\V_{\mathrm{CNQ}}^{\Sigma}\\V_{\mathrm{CP0}}^{\Sigma}\\V_{\mathrm{CN0}}^{\Sigma}\end{bmatrix}=\begin{bmatrix}0&0&\omega_0&0&0&0\\0&0&0&\omega_0&0&0\\-\omega_0&0&0&0&0&0\\0&-\omega_0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{bmatrix}\begin{bmatrix}V_{\mathrm{CPD}}^{\Sigma}\\V_{\mathrm{CND}}^{\Sigma}\\V_{\mathrm{CPQ}}^{\Sigma}\\V_{\mathrm{CNQ}}^{\Sigma}\\V_{\mathrm{CP0}}^{\Sigma}\\V_{\mathrm{CN0}}^{\Sigma}\end{bmatrix}+\frac{1}{2C^{\mathrm{arm}}}\begin{bmatrix}2&0&1&0&0&0&0&0&0&0&0\\0&2&0&-1&0&0&0&0&0&0&0\\0&0&0&0&2&0&1&0&0&0&0\\0&0&0&0&0&2&0&-1&0&0&0\\0&0&0&0&0&0&0&2&0&1&0\\0&0&0&0&0&0&0&0&2&0&-1\end{bmatrix}\begin{bmatrix}m_{\mathrm{PD}}I_{\mathrm{diff0}}\\m_{\mathrm{ND}}I_{\mathrm{diff0}}\\\frac{1}{2}I_{\mathrm{vD}}\\\frac{1}{2}I_{\mathrm{vD}}\\-m_{\mathrm{PQ}}I_{\mathrm{diff0}}\\-m_{\mathrm{NQ}}I_{\mathrm{diff0}}\\-\frac{1}{2}I_{\mathrm{vQ}}\\-\frac{1}{2}I_{\mathrm{vQ}}\\\frac{1}{2}I_{\mathrm{diff0}}\\\frac{1}{2}I_{\mathrm{diff0}}\\\frac{1}{2}m_{\mathrm{PD}}I_{\mathrm{vD}}+\frac{1}{2}m_{\mathrm{PQ}}I_{\mathrm{vQ}}\\\frac{1}{2}m_{\mathrm{ND}}I_{\mathrm{vD}}+\frac{1}{2}m_{\mathrm{NQ}}I_{\mathrm{vQ}}\end{bmatrix} \quad (18)$$

$$\frac{d}{dt}\left[I_{\mathrm{diff0}}\right]=\left[-\frac{R_x}{L_x}\right]I_{\mathrm{diff0}}+\frac{1}{2L_x}\begin{bmatrix}1 & -1 & -1\end{bmatrix}\begin{bmatrix}V_{\mathrm{DC}}\\\frac{1}{2}V_{\mathrm{CP0}}^{\Sigma}+\frac{1}{2}V_{\mathrm{CPD}}^{\Sigma}m_{\mathrm{PD}}+\frac{1}{2}V_{\mathrm{CPQ}}^{\Sigma}m_{\mathrm{PQ}}\\\frac{1}{2}V_{\mathrm{CN0}}^{\Sigma}+\frac{1}{2}V_{\mathrm{CND}}^{\Sigma}m_{\mathrm{ND}}+\frac{1}{2}V_{\mathrm{CNQ}}^{\Sigma}m_{\mathrm{NQ}}\end{bmatrix} \quad (19)$$

And the output equations will be

$$\begin{bmatrix}e_{\mathrm{D}}\\e_{\mathrm{Q}}\\e_0\end{bmatrix}=\frac{1}{2}\begin{bmatrix}\frac{1}{2}V_{\mathrm{CND}}^{\Sigma}+m_{\mathrm{ND}}V_{\mathrm{CN0}}^{\Sigma}-\frac{1}{2}V_{\mathrm{CPD}}^{\Sigma}-m_{\mathrm{PD}}V_{\mathrm{CP0}}^{\Sigma}\\\frac{1}{2}V_{\mathrm{CNQ}}^{\Sigma}+m_{\mathrm{NQ}}V_{\mathrm{CN0}}^{\Sigma}-\frac{1}{2}V_{\mathrm{CPQ}}^{\Sigma}-m_{\mathrm{PQ}}V_{\mathrm{CP0}}^{\Sigma}\\0\end{bmatrix} \quad (20)$$

$$I_{\mathrm{DC}}=3I_{\mathrm{diff0}}$$

The achieved nonlinear model of MMC can be easily linearized and developed in MATLAB for eigenvalue analysis and controller design purposes [11].

IV. Computer Timings of Detailed and Average Models

The main objective of proposing the DQ frame average model of MMC is to develop a small-signal model for analytical studies which can be implemented now easily. However, the proposed model runs even faster than the ABC average model. It is mainly because of the following two reasons:

1) The DQ variables are almost constant (especially in steady-state) and therefore the computer simulation of DQ average model is execute faster compared to the ABC average model with oscillatory variables.

2) The number of equations in the DQ average model is approximately 2/3 of the ABC average model.

Table 1 summarizes the computer times for the three models; the detailed model using selective harmonic elimination modulation (SHE), ABC average and DQ average models. The simulations were performed on a computer with a 3.10-GHz Inter Core i5-2400 Processor and 4 GB of RAM.

Table 1 Computer Timings for the detailed, ABC and DQ Average models

Model	**Time step(µs)**	**Computer time(s)**
SHE Detailed Model	10	70.39
ABC-Average Model	10	9.59
DQ-Average Model	10	2.85

It is seen that the proposed DQ average model performs significantly better in terms of computer speed. A simulation of 4 sec. with 10 µs time step for the DQ average model can be performed 24.7 times faster than the detailed model and 3.3 times faster than the ABC average model with acceptable matching between their steady-state and transient responses. In addition, it is possible to increase the simulation time steps for the average models to reduce further the simulation time.

V. Simulation Results

The DQ average model of MMC was applied in an open loop system including MMC, DC and AC subsystems with PLL shown in Fig 3. The dynamic equations of DC, AC and PLL subsystems are given in [11] and V_{acDQ} is the DQ components of the AC voltage at the point of common coupling. The MMC parameters for the detailed model are given in Table 2.

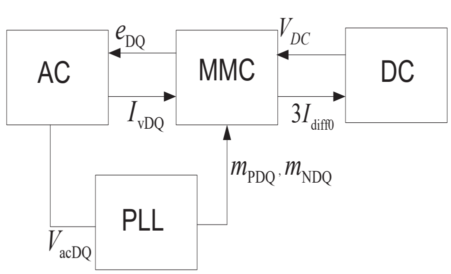

Fig 3. Block diagram of MMC model

Table 2 Parameters for MMC using selective harmonic elimination

Power Rating	1000 MVA
AC Voltage Rating	230 kV
DC voltage	400 kV
Switching Frequency	50 Hz
Number of SM per arm	8
Number of levels	9
Module Capacitance C	5000 µF
Arm Inductance L_X	1.26 mH
Arm equivalent resistance R_X	0.32 ohm

The detailed and the DQ average model of the test system were verified in PSCAD/EMTDC and the results are compared. The modulation index M_D is fixed at 0.945 and M_Q is stepped up from 0.05 to 0.1 at 1.5 sec. Fig 4 to Fig 7 shows the variable comparisons for the two models. It is seen that the DQ average model gives good matching with the benchmark PSCAD detailed model.

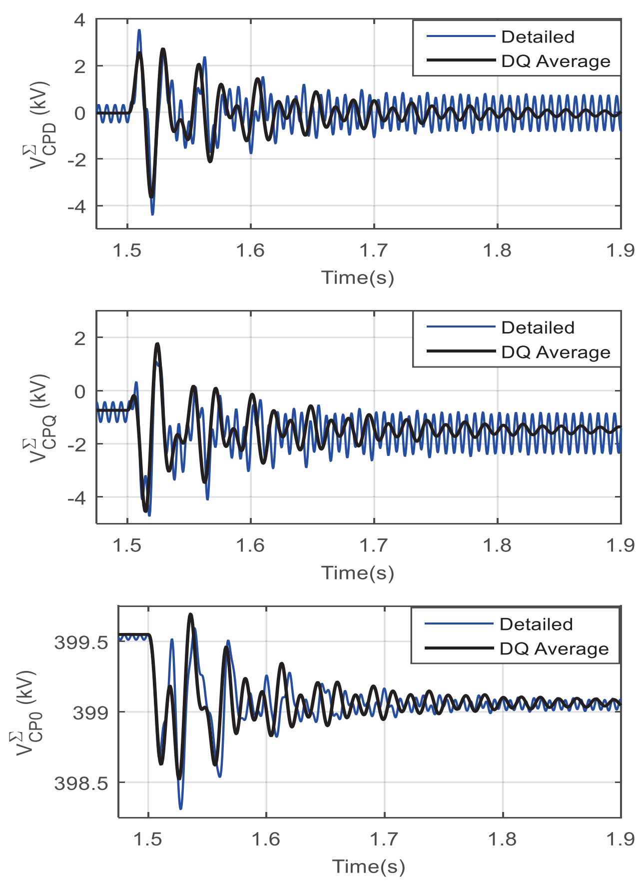

Fig 4. Comparison of positive pole capacitor voltages of DQ MMC average model against the benchmark detailed PSCAD model

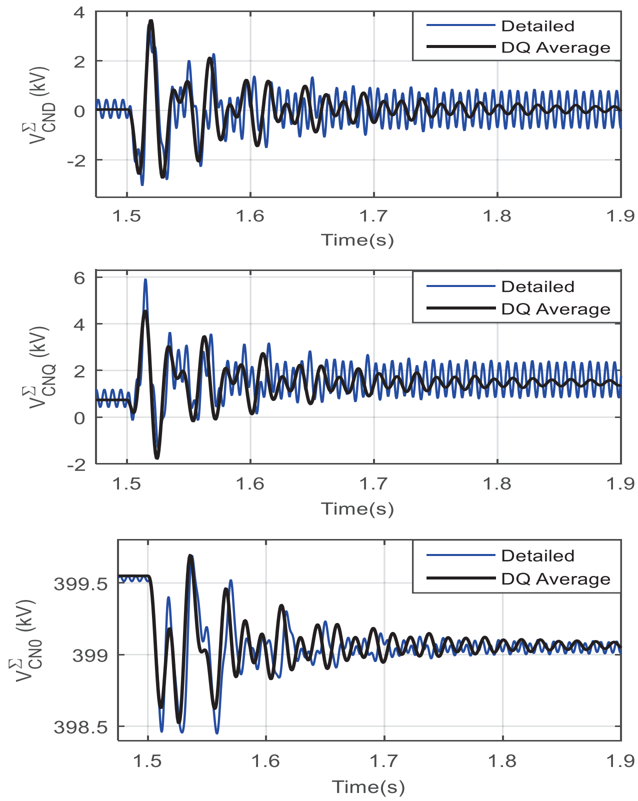

Fig 5. Comparison of negative pole capacitor voltages of DQ MMC average model against the benchmark detailed PSCAD model

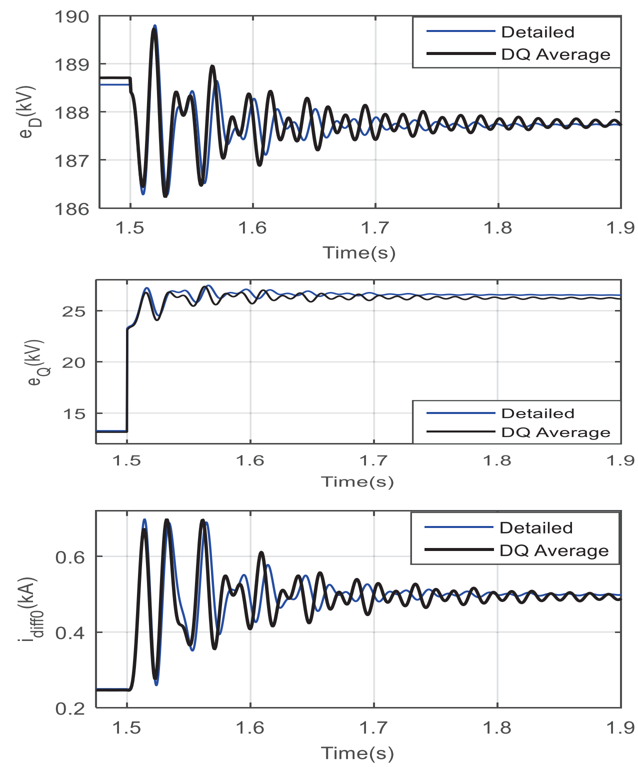

Fig 6. Comparison of output voltages & zero sequence differential current of DQ MMC average model against the benchmark detailed PSCAD model

VI. Conclusions

In this paper, an average model of MMC in rotating DQ frame has been proposed. Developing the MMC average model in DQ frame was a challenging task because of the multiplication terms in the MMC average model in ABC frame. The proposed approach to overcome this challenge is considering generic form for the product variables and multiplying them in ABC frame and then transferring only the DC and fundamental frequency components of the results to DQ frame.

This nonlinear DQ average model can be easily linearized and used as a small-signal model for eigenvalue analysis and controller design purposes. It also runs even faster than the average ABC model due to its fewer dynamic equations and neglecting oscillatory variables.

A test system using the proposed DQ average model was developed in PSCAD and its results were compared with the detailed model. The results show good matching and confirm the accuracy of the proposed DQ average model.

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