Numerical investigation of the effect of bed height and coefficient of restitution on the minimum fluidization velocity of a cylindrical fluidized bed.

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Abstract

Numerical simulation for 4 different ratios of initial bed heights (H) to base diameter (D), were performed; viz. 0.5, 1, 2 and 3. Glass beads of density 2600kg/m\textsuperscript{3} and with an average diameter of 550µm were used for all the simulations. Simulations were performed using the commercial CFD software, STAR-CCM+. The minimum fluidization velocity was identified by measuring pressure drop across the entire domain and found to remain same for all the above mentioned ratios. The present CFD results show excellent agreement with the experimental findings of Escudero & Heindel (2010).

Introduction

Fluidized beds have a wide range of application in the chemical, pharmaceutical, mineral and oil-gas industries. The reason for their widespread usage is the better mixing properties and the high contact surface area it provides between the continuous and dispersed phases.

Several complexities are involved in numerical modeling of fluidized beds, the presence of gas-solid intermixing media - with a continuously changing interface, the highly transient nature and the interaction between the phases. This compounded nature of fluidized beds has been a hindrance in completely understanding the physics involved. With the advent of CFD, considerable progress has been made in conducting investigative studies relating to bed hydrodynamics.

Minimum fluidization velocity, $U_{mf}$, is one of the most important parameters to characterize a bed (Caicedo et al. 2002). It is the velocity at which the weight of the bed is just balanced by the inertial force carried by the air coming into the bed. At velocities just equal to or above minimum fluidization velocity the bed attains a suspended state. This velocity is a characteristic property because it depends on the particle property/geometry, bed geometry and fluid properties. Gunn & Hilal (1997) and Cranfield & Geldart (1974) have both showed that $U_{mf}$ is independent of bed height for a certain types of beds like spouting beds and pseudo 2D beds.

In the present work, the Eulerian multiphase modeling approach has been employed using CFD tool, STAR-CCM+. The Eulerian model assumes that the phases are continuous and inter-penetrating. The inter-phase interaction is accounted for by choosing an appropriate drag model. A cylindrical fluidized bed, same as that used by Escudero & Heindel (2010) has been modeled. The details about the experimental setup are explained by Escudero & Heindel (2010). Owing to the rotational symmetry of the problem, only a 2D slice of the bed has been modeled.

Numerical Scheme

A 2D Cartesian simulation is performed as opposed to a 3D cylindrical geometry, to save simulation time. 2D simulations must be used with caution and should be used only for sensitivity analysis, they predict the bed height and pressure drop with good accuracy but, for predicting the spatial position of particles it is preferable to use 3D simulations.

The transport equations for momentum and continuity are solved for both the gas and the solid phase. The equations for the phases are linked together through the drag law. The solid phase has additional equations solved for the kinetic, collisional and frictional regime fundamentally based on the kinetic theory of granular flow (Gidaspow 1994). Assuming local dissipation of the granular energy, the granular temperature ($\Theta_s$) is evaluated using an algebraic equation which account for the collisions between particles.

$$\Theta_s = \left[ -K_1 \epsilon_s \text{tr}(\bar{\tau}_s) + \frac{K_2 \epsilon_s^2 (\text{tr}(\bar{\tau}_s)^2 + 4K_4 \epsilon_s \text{tr}(\bar{\tau}_s^2) + 2K_3 \epsilon_s \text{tr}(\bar{\tau}_s^3))}{2\epsilon_s K_4} \right]^2$$

Where $\bar{\tau}_s$ is the solid stress tensor and,

$$K_1 = 2(1 + \epsilon)\rho_g g_0$$

$$K_2 = \frac{4}{3\sqrt{\pi}} d_p \rho_s (1 + \epsilon) \rho_g g_0 - \frac{2}{3} K_3$$
The solid bulk viscosity is given by

\[
K_3 = \frac{d_0 \rho_s}{2} \left( \frac{\sqrt{\pi}}{3(3-e)} \right) \left[ 1 + \frac{2}{5}(1+e)(3e - 1)\varepsilon_s g_0 \right] 
+ \frac{8 \varepsilon_s}{5\pi} g_0 (1+e) 
\]

\[
K_4 = \frac{12(1-e^2)\rho_s g_6}{d_s \sqrt{\pi}} 
\]

The solid pressures and viscosities in the kinetic and collisional regimes are given by

\[
P_k = \rho_s \dot{e}_s \theta_s 
\]

\[
P_c = 2g_0 \rho_s \dot{e}_s^2 \theta_s (1+e) 
\]

\[
\mu_k = \frac{2 \mu_{di}}{g_0 (1+e)} \left[ 1 + \frac{4}{5} (1+e)\varepsilon_s g_0 \right] \] 

\[
\mu_c = \frac{4}{5} \varepsilon_s \rho_s d_s (1+e) \frac{\theta_s}{\sqrt{\pi}} 
\]

The solid bulk viscosity is given by

\[
\mu_b = \frac{5 \sqrt{\pi}}{96} (\varepsilon_s \rho_s) \left( \frac{d_s}{\dot{e}_s} \right) \sqrt{\theta_s} 
\]

In regions where the contact between the particles is not instantaneous but continuous the friction between particles has to be considered. The model equations, originally described by Schaeffer (1987), describe the plastic flow of a granular material and relate the shear stress to the normal stress. The Schaeffer model is only activated when the volume fraction of the particle exceeds a certain maximum packing limit (which is set as 0.65 in our case). The frictional pressure and viscosity are modeled as follows

\[
P_f = \begin{cases} 
10^{25} (\varepsilon_s - \varepsilon_s^{max})^{10}, & \varepsilon_s > \varepsilon_s^{max} \\
0, & 0 \leq \varepsilon_s \leq \varepsilon_s^{max} 
\end{cases} 
\]

\[
\mu_f = \begin{cases} 
\mu_{max} \left( \frac{P_f \sin(\Phi)}{4I_{2D}}, & \varepsilon_s > \varepsilon_s^{max} \\
0, & 0 \leq \varepsilon_s \leq \varepsilon_s^{max} 
\end{cases} 
\]

\[
\mu_{max} = 1000 \eta 
\]

\[
I_{2D} = \frac{1}{6} [(D_{s11} - D_{s22})^2 + (D_{s22} - D_{s33})^2 
+ (D_{s33} - D_{s11})^2] + D_{s12}^2 + D_{s23}^2 + D_{s31}^2 
\]

\[
D_{sij} = \frac{1}{2} \left( \frac{\partial u_{si}}{\partial x_j} + \frac{\partial u_{sj}}{\partial x_i} \right) 
\]

The total solid pressure and viscosity are given by

\[
P_s = P_f + P_k + P_c 
\]

\[
\mu_s = \mu_f + \mu_k + \mu_c 
\]

Drag force is the most important force in fluidized beds as it is the only source of inter-phase interaction in fluidized beds. Some drag laws are obtained by experimental pressure drop data of packed beds. Ergun equation is one such mathematical model obtained for a packed bed. The Gidaspow (1989) drag model is used in the present study. The inter-phase drag coefficient for the Gidaspow model is given by

\[
\beta_{gs} = \frac{3}{4} C_D \frac{e_s \rho_g [u_g - u_s]}{d_s} \varepsilon_s^{-2.65} \quad \text{for} \quad \varepsilon_s > 0.8 
\]

\[
C_D = \begin{cases} 
24 \frac{1}{\varepsilon_s Re_s} [1 + 0.15 (\varepsilon_s Re_s)^{0.687}], & Re_s < 1000 \\
0.44, & Re_s > 1000 
\end{cases} 
\]

\[
\beta_{gs} = \frac{150 \varepsilon_s^2 \mu_g}{\varepsilon_s d_s} + 1.75 \frac{\varepsilon_s \rho_g [u_g - u_s]}{d_s} \quad \text{for} \quad \varepsilon_s < 0.2 
\]

**Simulation setup**

The geometry and the mesh employed in the present work is shown in Figure 1. The mesh comprises of 8484 cells.

**Table 1:** Bulk density values reported in experiment of Escudero & Heindel (2010) and the initial particle volume fraction in the bed in present simulations.

<table>
<thead>
<tr>
<th>H/D</th>
<th>Bed mass (g)</th>
<th>Bulk density (kg/m³)</th>
<th>Particle Volume fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>670</td>
<td>1610±70</td>
<td>0.62</td>
</tr>
<tr>
<td>1</td>
<td>1320</td>
<td>1590±70</td>
<td>0.61</td>
</tr>
<tr>
<td>2</td>
<td>2560</td>
<td>1540±70</td>
<td>0.59</td>
</tr>
<tr>
<td>3</td>
<td>3610</td>
<td>1440±70</td>
<td>0.55</td>
</tr>
</tbody>
</table>

**Figure 1:** The mesh comprising of 8484 cells employed in all the simulations.

The initial packing fraction of the bed was chosen based on the bulk density reported in the experiment of Escudero & Heindel (2010). The density of the particle is the same in all cases, and the ratio of mean bulk density-to-particle density gives the average volume fraction of particle in the bed, before starting the air flow. The bulk density values are reported in Table 1. An adiabatic (zero-gradient) boundary condition is applied for the granular temperature at the walls. Particles are allowed to slip along the walls, while their wall-normal velocity component is set to zero. The other simulation parameters are listed in Table 2.
Results and Discussion

The pressure drop across the bed is estimated by measuring the difference between the surface averaged pressure across the bottom (inlet) and the top boundary (outlet). Data for the initial 5s is neglected since it takes time for the bed to reach a statistically stationary state. The averaging is carried out between 5 – 15s. Figure 2 shows the time history of the pressure drop for different superficial gas velocities for the case where H/D = 1.0. It is observed that the mean pressure drop is nearly the same for superficial gas velocities above U = 0.18 m/s.

Figure 3 shows the pressure drop across the bed as a function of superficial gas velocity for different H/D ratios. It is observed that the pressure drop attains a constant value after a certain superficial gas velocity. This value of the superficial gas velocity is typically referred to as the minimum fluidisation velocity, \( U_{mf} \). It is observed that \( U_{mf} \) is nearly same for the different H/D ratios. Hence, it can be concluded that there is no correlation between minimum fluidization velocity and bed height for cylindrical fluidized beds. The value of minimum fluidization velocity is approximately obtained to be at around 0.18m/s as shows in Figure 3. It is noted in Figure 1 that the pressure drop oscillates for velocities above minimum fluidization. Similar behavior was reported by Goldschmidt et al. (2001). The amplitude of the oscillations was found to be increasing with increasing superficial velocities at superficial velocities greater than the minimum fluidization velocity; below minimum fluidization velocity the oscillations are negligible.

Figure 4 shows the comparison between the pressure drop obtained from present simulations with the experimental results of Escudero & Heindel (2010) for H/D = 1.0 case. Similar comparison was observed for the other H/D cases. Below minimum fluidization velocity, the bed dynamics are dominated by particle-particle contacts. As such, the frictional model used may play a dominant role. In the present work, the model by Schaeffer (1987) is used. However there is scope for exploring the frictional model proposed by Johnson & Jackson (1987) to improve agreement with experiment below \( U_{mf} \).

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle density</td>
<td>2600kg/m³</td>
</tr>
<tr>
<td>Gas density</td>
<td>1.2kg/m³</td>
</tr>
<tr>
<td>Particle diameter(d)</td>
<td>550 µm</td>
</tr>
<tr>
<td>Coefficient of restitution(e)</td>
<td>0.9</td>
</tr>
<tr>
<td>Superficial gas velocity(U)</td>
<td>0.1m/s-0.3m/s</td>
</tr>
<tr>
<td>Bed width(D)</td>
<td>0.102m</td>
</tr>
<tr>
<td>Total bed height</td>
<td>0.91m</td>
</tr>
<tr>
<td>Static bed height(H)</td>
<td>0.051m-0.306m</td>
</tr>
<tr>
<td>Time step</td>
<td>0.0001-0.0005s</td>
</tr>
<tr>
<td>Maximum physical time</td>
<td>16s</td>
</tr>
</tbody>
</table>

Table 2: Simulation parameters in the present work.

Figure 2: Pressure drop across the bed for different superficial gas velocities. H/D = 1.0 for all the simulations in this plot.

Figure 3: Pressure drop across the bed as a function of superficial gas velocity for different H/D ratios.

Figure 4: Pressure drop across the bed from present simulations compared with the experimental results of Escudero & Heindel (2010) for H/D = 1.0 case.
Another possible improvement to get better agreement with experiment could be use of partial slip boundary condition for particles at the wall as employed by Patil et al. (2005).

A force balance between the pressure drop and the weight of the bed is plotted as shown in Figure 5. The value of the knee along the y-axis is approximately 1 showing that beyond minimum fluidization the inertial force of the incoming air exactly balances the weight of the bed.

Conclusions

Simulations of the dynamics of a cylindrical fluidized bed were performed using commercial CFD code, STAR-CCM+. Simulations were performed for different static bed heights and it was found that the minimum fluidization velocity for the different bed heights is same. This corroborates the experimental observations that the minimum fluidization velocity is independent of bed height for certain types of beds. Moreover it is clearly demonstrated that at the minimum fluidization velocity, the bed weight is exactly balanced by the inertial force of the incoming air.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{v}_s$</td>
<td>Velocity of solid (vector)</td>
<td></td>
</tr>
<tr>
<td>$\bar{v}_g$</td>
<td>Velocity of gas (vector)</td>
<td></td>
</tr>
<tr>
<td>$P_s$</td>
<td>Solid pressure</td>
<td></td>
</tr>
<tr>
<td>$P_f$</td>
<td>Frictional pressure</td>
<td></td>
</tr>
<tr>
<td>$P_k$</td>
<td>Kinetic pressure</td>
<td></td>
</tr>
<tr>
<td>$P_c$</td>
<td>Collisional pressure</td>
<td></td>
</tr>
<tr>
<td>$g_o$</td>
<td>Radial distribution function</td>
<td></td>
</tr>
<tr>
<td>$e$</td>
<td>Coefficient of restitution</td>
<td></td>
</tr>
<tr>
<td>$d_s$</td>
<td>Diameter of particle</td>
<td></td>
</tr>
<tr>
<td>$C_D$</td>
<td>Standard drag coefficient</td>
<td></td>
</tr>
<tr>
<td>$Re_p$</td>
<td>Particle Reynolds number</td>
<td></td>
</tr>
<tr>
<td>$d$</td>
<td>Mean particle diameter</td>
<td></td>
</tr>
</tbody>
</table>

Greek letters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_s$</td>
<td>Volume fraction of particle/solid</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_g$</td>
<td>Volume fraction of air</td>
<td></td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>Density of particle</td>
<td></td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>Density of air</td>
<td></td>
</tr>
<tr>
<td>$\tau_s$</td>
<td>Stress tensor for solid</td>
<td></td>
</tr>
<tr>
<td>$\tau_g$</td>
<td>Stress tensor for gas</td>
<td></td>
</tr>
<tr>
<td>$\lambda_g$</td>
<td>Bulk viscosity of gas</td>
<td></td>
</tr>
<tr>
<td>$\beta_{gs}$</td>
<td>Drag coefficient</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon^\text{max}_s$</td>
<td>Maximum volume fraction of particle</td>
<td></td>
</tr>
<tr>
<td>$\mu^\text{max}_m$</td>
<td>Maximum viscosity of particle</td>
<td></td>
</tr>
<tr>
<td>$\mu_s$</td>
<td>Solid phase viscosity</td>
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<td>$\mu_f$</td>
<td>Frictional viscosity</td>
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<td>$\mu_k$</td>
<td>Kinetic viscosity</td>
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<td>$\mu_c$</td>
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</tr>
<tr>
<td>$\mu_b$</td>
<td>Bulk viscosity</td>
<td></td>
</tr>
</tbody>
</table>

References

Escudero, D. & Heindel, T.J., Bed height and material density effects on minimum fluidization velocity in a cylindrical fluidized bed, 7th International Conference on Multiphase Flow, Tampa, FL, USA (2010)


Johnson, P.C., and Jackson, R., Frictional-collisional