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Analysis of futures and spot electricity markets under risk aversion.

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Analysis of futures and spot electricity markets under risk aversion

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A B S T R A C T

We analyze the procurement problem in the electricity supply chain, focusing on the interaction between futures and spot prices. The supply chain network analyzed in our study includes risk-averse generators and retailers, both with the ability to use conditional value at risk (CV@R) in their decision processes. In this supply chain, the futures price is computed to clear the futures market, without imposing the constraint that the expected spot price equals the futures price. As major methodological contributions: we compute the Nash equilibrium of the problem using CV@R and considering conjectural variations; we derive analytical relationships between the futures and the spot market outcomes and study the implications of demand and marginal cost uncertainty, as well as the level of the players' risk aversion, on market equilibrium; we introduce the concept of risk-adjusted expectation to derive the futures market price as a function of the players' expected losses or profits in the spot market; and we use consistent spot and wholesale price derivatives to calculate the players' reaction functions. Finally, we illustrate our model with several numerical examples in the context of the Spanish electricity market, studying how the shape of the forward curve and the relationship between spot and futures prices depend on seasonality, risk aversion, generators' market power, and hydrological resources. Surprisingly we observed that risk aversion increases the profit and reduces firms' risk, and that the consumer utility is higher in the scenarios in which retailers behave *a la Cournot* in the wholesale market.

Keywords:
Electricity market; Futures prices; Non-cooperative games; Risk aversion; Supply chain management

1. Introduction

The electricity supply chain is rather complex due to the technical constraints associated with the production and distribution of electricity, which is very difficult to store and needs to be produced on demand, and due to the wide variety of production technologies that co-exist in the same market: coal, gas, nuclear, hydro power, wind and solar generation plants. These varied ways of generating electricity exhibit uncertain production costs as they are affected by the price of gas, oil, coal, uranium and by weather conditions. Within this supply chain network, generators sell electricity to retailers using bilateral contracts, and they participate in the spot and futures markets.

In this context, it is important to explain how uncertainty management is translated into contracts that enable the firms to act upon the received information (Conejo, Carrión, & Morales, 2010a).

In order to achieve this goal we propose a stochastic programming model of a complex supply network that accounts for the two major sources of uncertainty, i.e., generating costs and demand behavior. Moreover, we analyze the impact of these uncertainties on the electricity retail, spot and futures markets.

Although complex, the electricity supply network has a very well defined structure. The consumers (private households and large industrial companies) decide how much they want to consume. Some of these consumers are on a fixed tariff, i.e., they pay the same price per kWh consumed independently of the quantity bought (this is the traditional scheme for small households). Larger consumers and, increasingly in the more sophisticated market, also small households, have real-time meters that are used to measure electricity consumption and price in real time. As illustrated in Section 4, in the Spanish electricity market, demand response is an important part of total consumption.

The retailers, the intermediaries between generation and final consumers, buy the electricity required to meet demand using bilateral trading and organized wholesale markets (spot and futures).

Given the expected demand and respective response to price, the retailers determine their bidding curves for buying forward using bilateral contracts (or equivalently in the futures market), their real-time bidding curves to buy in the spot markets, and the retail price charged to the responsive consumers. Similarly, the generators decide their offering curves to sell their energy in the different markets. At equilibrium, the total quantity sold by the generators equals the quantity bought by the retailers in each market, and the total electricity traded in both markets equals consumer demand. This equilibrium is attained due to the change in the electricity prices in the three markets. Generalizing from Allaz (1992), the structure of the electricity market includes a supply chain with the possibility of risk-aversion. The closest article to ours is Oliveira, Ruiz, and Conejo (2013), which we now extend by including risk aversion.

As CV@R can be solved as a Linear Programming (LP) problem or in the form of a Mixed-Complementarity Problem (MCP) for games, it is very flexible due to the easy parameterization and, moreover, it is distribution independent. These are its major advantages when compared to other risk measures.

Table 1
Forward Curve (€ /MWh) for Nov. 30, 2017. Source: OMIP (2020).

Price	Base	Peak	Price	Base	Peak	Price	Base	Peak
Spot	69.56	76.41	Q1-18	56.85	65.47	M Dec-17	62.30	68.98
YR-18	52.35	58.84	Q2-18	48.25	53.35	M Jan-18	63.20	72.78
YR-19	49.25	55.48	Q3-18	51.65	58.05	M Feb-18	57.60	66.28
YR-20	47.79	53.79	Q4-18	52.71	58.50	M Mar-18	49.81	57.09
YR-21	47.31	53.29	Q1-19	53.48	60.24	M Apr-18	44.04	48.70
			Q2-19	45.39	51.13	M May-18	48.92	54.09
			Q3-19	48.59	54.73			

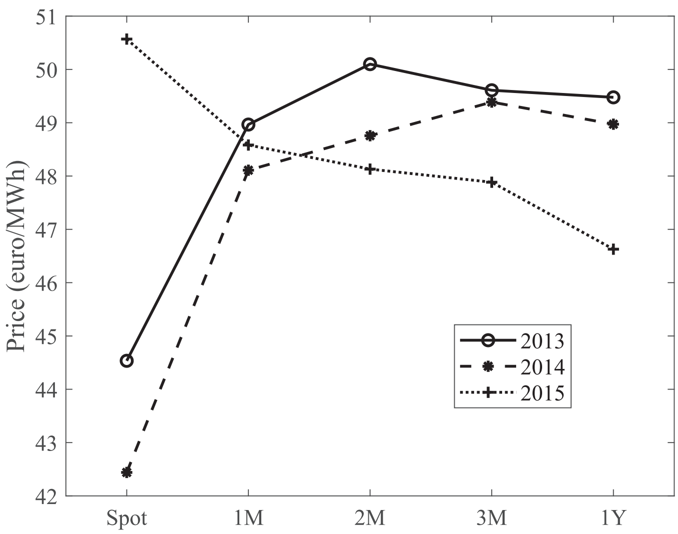


Fig. 1. Normal vs. inverted forward curve in the Spanish electricity market. Source: OMIP (2020).

The main motivation for this article is understanding the behavior of the Spanish electricity market. One of the major features of this market is the low proportion of trade in the futures market, which made up 30% of total trading in 2015/16 (e.g., OMIE, 2020). The second focus is to better understand the relationship between futures and spot electricity markets, as illustrated by the shape of the forward curve. This curve represents the relationship between the spot price and the futures prices over different durations. As an example, Table 1 presents the forward curve resulting from the futures markets trading on Nov. 30, 2017 in the Spanish market (OMIP, 2020). Fig. 1 depicts three forward curves (for the years 2013, 2014, and 2015, respectively) in the Spanish electricity market: the prices presented are for the spot and futures markets with durations of one, two, three months, and one year.

Before proceeding with the description of the market, we need to define the concepts of a normal versus an inverted forward curve. A futures market is in a normal state when the spot price is lower than the one-month futures price, which is less than the two-month futures price, and so on. In this situation supply is currently abundant when compared to demand. A futures market is inverted if the spot price is higher than the one-month futures, price, which is higher than the two-month futures prices, and so on. This is an indication that currently supply is scarce when compared to demand.

The years 2013 and 2014 were normal futures markets: the spot prices were lower than the futures prices and there was, therefore, an expectation that the spot prices were due to increase in the horizon of 1 year. On the other hand, the year 2015 represents an inverted market: the electricity prices were expected to decrease, which together with a larger average spot price in 2015 clearly illustrates how tight the electricity market throughout the year, due to a heat wave that increased consumption and decreased hydro generation.

The major contributions of this article are: (a) to propose a supply chain model of the electricity industry based on a concentrated market structure with a few producers and retailers, considering risk aversion using the CV@R, and solving the equilibrium of the game for different degrees of market power; (b) to derive analytical relationships between the futures and spot market outcomes, and to study the implications of demand and marginal cost uncertainties and the level of the players' risk aversion for market equilibrium, without imposing the constraint that the future price equals the expected spot price, which has been previously used in articles considering uncertainty (e.g., Allaz, 1992, Gulpinar & Oliveira, 2012, Popescu & Seshadri, 2013); (c) to introduce the concept of risk-adjusted expectation which allows the futures market price to be derived as a function of the players' expected losses or profits in the spot market; (d) to compute consistent derivatives of the spot and wholesale prices in respect to the quantities traded by generators and retailers, both of which are scenario-independent; (e) to present several numerical examples in the context of the Spanish electricity market to illustrate the qualitative impact the players' risk aversion and market power may have on the relationship between the futures and spot markets.

We believe both retail and generation companies in liberalized electricity markets can benefit from the proposed methodology for several reasons. The ability to explicitly compute a futures price as a result of market equilibrium, but one that is possibly different from the expected spot price, gives a signal to the purchasing teams for when to go long and short depending on the state of the futures market. We have discovered that, crucially, the state of the slope of the forward curve depends on both market-power and risk aversion (in the presence of risk aversion and Cournot generators we have normal backwardation; when all players are risk-neutral and behave *a la Cournot*, we observe contango). This is a new and important insight, and, in the case of the Spanish market, it is also based on the availability of hydro resources. For these reasons, firms need to take into account market structure and behavioral considerations when deciding, possibly a few months ahead, how to sell and procure the required electricity. It is also evident that in the case of countries where

hydrological resources are a significant part of the production mix, forecasting weather (i.e., accumulated rain fall during the time horizon) a few months ahead of the delivery date is a crucial component of an effective policy.
Overall, these results are a major departure from the previous literature reporting that the introduction of futures markets reduces producers' profitability. Our results show that, under risk aversion, the inverse is correct: producers are able to increase spot prices and trade mostly in this market, thus increasing profitability, at the expense of retailers and consumers in the presence of futures markets.

The article is organized as follows. We start in Section 2 with a literature review. In Section 3 we present the model of the electricity supply chain, deriving the analytical properties of the model. In Section 4 we apply our model to a realistic electricity market and Section 5 concludes the article.

2. Literature review

In this section we review the literature on supply chain management, focusing on risk-averse stochastic models, and analyzing the relationship between spot and forward markets. The interaction between oligopolistic market power and futures trading can be traced back to Greenstone (1981), who argued that coffee producing countries used the futures markets to increase spot prices. This theory was, however, challenged by Allaz (1992), and Allaz and Vila (1993), who concluded, through the analysis of a duopoly model, that producers are worse off with a futures market. It is therefore important for procurement departments to better understand how and when, to use futures markets (or bilateral trading) instead of spot markets. For this reason the topic has been addressed both in the management and economics literatures. Powell (1993) presents an analysis of the relationship between spot and long-term contracts trading with risk-averse retailers, concluding that, in this case, the contract price is larger than the expected spot price. Newbery (1998) studies the interaction between long- term contracts and new entry. Dong and Liu (2007) have also used a model based on Allaz and Vila (1993) and applied it to study a one-to-one supply chain considering risk aversion. Anderson and Hu (2008) analyze the use of forward contracts when the buyer (retailer) has market power and offers forward contracts to two different generators, concluding that, in this case, there is an increase in the social optimum. An equilibrium model is used by Aïd, Chemla, Porchet, and Touzi (2011) to study the relationship between forward, spot and retail markets when firms can be vertically integrated. The authors concluded that, under demand uncertainty, both vertical integration and forward hedging decrease retail prices.

Moreover, in the context of electricity markets, the relationship between spot and forward contracts is analyzed by Bessembinder and Lemmon (2002) considering risk aversion, profit variance, and homogeneous generators, but without considering market power.
Carrión, Conejo, and Arroyo (2007) presents a stochastic programming model to identify optimal forward contracts for risk-averse (and price-taking) retailers. From the perspective of a risk-averse producer, Conejo, García-Bertrand, Carrión, Caballero, and de Andres (2008) develop optimal trading strategies in both spot and futures markets showing that futures trading may help to hedge the profit volatility risk. Also in this context (Murphy & Smeers, 2010) have analyzed the relationship between spot and forward contracts in an investment model with oligopolistic players and capacity constraints, and Oliveira and Costa (2018) studied the relationship between investment and futures prices, in an oligopoly, using indirect reinforcement-learning. Further, Kazempour, Conejo, and Ruiz (2012) study how the introduction of a futures market may alter the investment decisions of generators with market power in both futures and spot markets. Finally, Gulpinar and Oliveira (2012) propose a worst-case analysis of the relationship between futures and spot markets, Oliveira et al. (2013) develop an integrated model of the electricity supply chain analyzing the interaction between generators and retailers under different market structures and contractual arrangements, and Oliveira (2017) analyzes the impact of capacity constraints and price caps on the relationship between futures and spot markets.

As exemplified in Powell (1993), Newbery (1998), Bessembinder and Lemmon (2002), Anderson and Hu (2008), Oliveira et al. (2013), Popescu and Seshadri (2013), and Oliveira (2017), among other studies, the equilibrium model presented in this article assumes that prices are determined by industry participants as we do not consider financial speculators. There are two reasons for choosing this option: first, this is a realistic assumption as these markets tend to have low liquidity and do not attract many outside traders; second, we want to analyze the nature of the relationship between spot and future prices in an oligopolistic market.

Crucially, in this article we emphasize the interaction between risk aversion and market power in determining optimal trading strategies in the electricity market supply chain. For this reason, we develop a model of the supply chain using conditional value at risk (CV@R). The CV@R enables the computation of the optimal policy to minimize the expected loss under extreme conditions in low probability scenarios. In previous research, Chen, Shum, and Simchi-Levi (2014) study a decentralized supply chain with demand uncertainty and risk-averse firms (a single retailers and multiple suppliers) by applying a CV@R objective formulation. In another study, Downward, Young, and Zakeri (2016) use the CV@R to model risk-averse retailers and analyze the impact of risk aversion on spot and retail prices; Gersema and Wozabal (2018) select optimal portfolios of electricity generation technologies using risk-optimized pooling of solar and wind plants to consider market and volume risks and the impact of feed-in tariffs. Falbo and Ruiz (2019) analyze the optimal sales-mix of a generation plant using CV@R; Wozabal and Rameseder (2020) study the optimal bidding of a virtual power producer in the Spanish day-ahead and intraday electricity markets using CV@R; and Moret, Pinson, and Papakonstantinou (2020) propose the use of financial contracts, in decentralized electricity markets, considering heterogeneous risk preferences, modeled using CV@R.

In the literature, the stochastic factor studied is almost always demand, but Balakrishnan, Geunes, and Pangburn (2004), Mendelson and Tunca (2007) and Lucheroni and Mari (2019) have also considered cost uncertainty. Depending on the specific objectives of the study, some models have included endogenous prices whereas some others assume that firms are price takers. Game theory problems have also been studied, including Stackelberg models, e.g., Hsieh and Lu (2010), Mendelson and Tunca (2007), Adida and DeMiguel (2011) and Shi, Zhang, and Ru (2013), as well as supply chain Nash equilibrium (e.g., Lin, Cai, & Xu, 2010, and Caliskan-Demirag, Chen, & Li, 2011).

Mean-variance models, with different variants are the most common focus of studies (e.g., Balakrishnan et al., 2004; Chen, Sim, Simchi-Levi, & Sun, 2007; Miyaoka & Hausman, 2008; Lin et al., 2010; Adida & DeMiguel, 2011; Aïd et al., 2011), while utility based models using loss aversion (e.g., Wang & Webster, 2009; Ma, Zhao, Xue, Cheng, & Yan, 2012; Deng, Xie, & Xiong, 2013) and exponential utility functions (e.g., Choi & Ruszczynski, 2011) are also used, but are less popular. CV@R (e.g., Artzner, Delbaen, Eber, & Heath, 1999; Chen, Sim, Sun, & Teo, 2010; Rockafellar & Uryasev, 2000; Shapiro, 2009; Shapiro, 2011) is applied by Hsieh and Lu (2010) and Caliskan-Demirag et al. (2011): in both these articles the manufacturer is considered risk neutral whereas retailers are risk averse. Ehrenmann and Smeers (2011) propose a stylized model using CV@R to analyze production capacity expansion, in which the price-taking decision makers represent the different technologies. Their analysis is very different from ours. We provide a detailed model of the the supply chain interactions between retailers and generators, these are firms managing their own businesses, and we account for market power when analyzing the re lationship between spot and futures prices.

3. Modeling the electricity supply chain

We consider an electricity market in which G generators and R retailers interact in different trading floors. In particular, we analyze a two-stage setting composed of a futures market and a subsequent spot market (e.g., Conejo, García-Bertrand, Carrión, & Pineda, 2010b). In the first stage, generators and retailers participate in a futures market. As a result, generator g sells a quantity q_g^F and retailer r buys q_r^F at a price W. W is the equilibrium price that clears the futures market, such that supply equals demand.

In the second stage, generators and retailers participate in the spot market where they sell and buy their electricity, $q_{g\omega}^S$ and $q_{r\omega}^S$ respectively, at the spot price S_ω, which depends on the specific scenario ω for demand and costs. Finally, retailers sell the energy purchased in the futures and spot markets to two market segments: (a) elastic consumers at the single retail price P_ω and (b) consumers who have signed with the retailer at a fixed tariff, i.e., a quantity $\overline{q}_r$ at a price $\overline{P}_r$, where $\overline{q}_r$ and $\overline{P}_r$ are considered as exogenous parameters in the current model. These market segments also represent the degree to which a retailer can differentiate the service offered to the final consumers without them changing supplier, even when paying a higher price. This product differentiation may reflect the reputation of the firm, the quality of its services (e.g., dealing better with consumers), or may just reflect different types of demand (e.g., consumers with a high degree of risk aversion will prefer to stay with the same retailer and pay a fixed tariff rather than changing to a flexible price contract).

Under this market setting, generators and retailers have to make the first stage decisions (futures market) in the face of uncertainty associated with consumer behavior and generation costs. This uncertainty is thus characterized by scenarios $\omega = 1, \dots, \Omega$ for the possible realizations of the inverse demand curve representing the behavior of final consumers, and of the electricity generation costs.

The profit function of generator g in scenario ω, i.e., $\Pi_{g\omega}$, is described by (1) where the first term stands for the generator's income from selling its electricity in the futures market, the second term is the income from the spot market, and the last term accounts for the generation costs. Generator $g's$ total cost function $C_{g\omega} = C_{g\omega}\left(q_g^F, q_{g\omega}^S\right)$ varies with the scenario and is a function of total production, which equals the sum of the electricity traded in the futures and in the spot market, i.e., $q_g^F + q_{g\omega}^S$.

$$\Pi_{g\omega} = q_g^F W + q_{g\omega}^S S_\omega - C_{g\omega} \qquad g = 1, \ldots, G \quad \omega = 1, \ldots, \Omega. \tag{1}$$

Moreover, the futures market variables q_g^F and W do not depend on the scenario ω, as the futures market is settled before the uncertain parameters (inverse demand curve and generation costs) are observed. Additionally, the expected profit function of generator g can be computed as $\mathbb{E}[\Pi_g] = \sum_{\omega=1}^{\Omega} \sigma_{g\omega}\Pi_{g\omega}$, where $\sigma_{g\omega}$ is the probability assigned by generator g to scenario ω.

The profit function of retailer r in scenario ω, i.e., $\Pi_{r\omega}$, is formulated as (2) where the first term represents the retailer's income from selling the electricity to consumers under a fixed tariff, and the second term is the income from selling the remaining purchased electricity, i.e., $q_{r\omega}^S + q_r^F - \overline{q}_r$, to the final elastic consumers at a price P_ω. The third and fourth terms are the costs from buying in the futures and spot markets, respectively. Again, the futures decision variables (q_r^F) do not depend on the scenario ω. Similarly, the expected profit of retailer r can be computed as $\mathbb{E}[\Pi_r] = \sum_{\omega=1}^{\Omega} \sigma_{r\omega}\Pi_{r\omega}$, where $\sigma_{r\omega}$ is the probability assigned by retailer r to scenario ω.

$$\Pi_{r\omega} = \overline{P}_r\overline{q}_r + \left(q_{r\omega}^S + q_r^F - \overline{q}_r\right)P_\omega - q_r^F W - q_{r\omega}^S S_\omega$$
$$r = 1, \ldots, R \quad \omega = 1, \ldots, \Omega. \tag{2}$$

Finally, the elastic consumers' preferences in scenario ω are represented by the inverse linear demand curve (3) where $\nu_\omega > 0$ and $\beta > 0$. The uncertainty associated with the consumer's behavior is represented by the parameter ν_ω, which is scenario dependent.

$$P_\omega = \nu_\omega - \beta \sum_{r=1}^{R} \left(q_{r\omega}^S + q_r^F - \overline{q}_r\right). \tag{3}$$

In order to characterize the global equilibrium of the electricity supply chain we first need to compute the equilibrium in the spot market (stage two) assuming that the futures market has already been settled. Therefore, the resulting spot market equilibrium price and quantities, one per scenario ω, are a function of the futures decision variables q_g^F and q_r^F , and the futures price W. This allows the generators' and retailers' profits, (1) and (2), respectively, to be rewritten as a function of q_g^F, q_r^F and W. We then move one step backwards to stage one and obtain the futures market equilibrium (and thus the supply chain equilibrium) by simultaneously maximizing the profits of generators and retailers, under different levels of risk aversion, as analyzed in the following subsections.

3.1. Characterizing the equilibrium in the spot market (Stage Two)

In this section we describe the equilibrium conditions for the spot market. We start by defining a key model assumption and then derive the equilibrium purchases and sales by retailers and generators in the spot market, together with their respective aggregate inverse demand and offer curves.

Assumption 1. All the retailers (generators) are identical with respect to their conjectural variations, i.e., all the retailers (generators) assume the same impact of their purchases (sales) in the spot and futures prices, as for their rival retailers' purchases (generators' sales).

Assumption 1 is equivalent to considering the same level of competition (Cournot, Bertrand, perfect competition, etc.) among generators and retailers. However, generators and retailers can be asymmetric in terms of costs, share of fixed tariff consumers and levels of risk aversion.

Proposition 1. *Under Assumption 1, the equilibrium purchases of re- tailer r in scenario ω in the spot market are described by Eq. (4).*

$$q_{r\omega}^S = \frac{S_\omega - P_\omega - \left(q_r^F - \overline{q}_r\right)\frac{\partial P_\omega}{\partial q_{r\omega}^S}}{\frac{\partial P_\omega}{\partial q_{r\omega}^S} - \frac{\partial S_\omega}{\partial q_{r\omega}^S}} \tag{4}$$

Proof. Assuming monotonicity of P_ω and S_ω with respect to $q_{r\omega}^S$, we consider the maximization of the retailers profit (2) as: $\frac{\partial \Pi_{r\omega}}{\partial q_{r\omega}^S} = (q_r^F - \bar{q}_r)\frac{\partial P_\omega}{\partial q_{r\omega}^S} + P_\omega - S_\omega - q_{r\omega}^S \frac{\partial S_\omega}{\partial q_{r\omega}^S} = 0$, which can be easily rearranged to obtain (4). □

Eq. (4) shows that, although retailers in almost all scenarios behave as net buyers, we may find some scenarios where $q_{r\omega}^S < 0$. This would indicate that the retailer sells back in the spot market part of the electricity bought through futures, probably caused by a relatively low demand realization in that scenario.

Proposition 2. *Under Assumption 1, the aggregate demand function for the retailers in the spot market, and its inverse, are* *represented by* Eqs. (5) and (6), *respectively:*

$$\sum_{r=1}^{R} q^S_{r\omega} = \frac{R(S_\omega - \nu_\omega) + \left(R\beta - \frac{\partial P_\omega}{\partial q^S_{r\omega}}\right)\sum_{r=1}^{R}\left(q^F_r - \bar{q}_r\right)}{\frac{\partial P_\omega}{\partial q^S_{r\omega}} - \frac{\partial S_\omega}{\partial q^S_{r\omega}} - R\beta} \quad (5)$$

$$S_\omega = \nu_\omega + \frac{1}{R}\left(\frac{\partial P_\omega}{\partial q^S_{r\omega}} - \frac{\partial S_\omega}{\partial q^S_{r\omega}} - R\beta\right)\sum_{r=1}^{R} q^S_{r\omega} + \frac{1}{R}\left(\frac{\partial P_\omega}{\partial q^S_{r\omega}} - R\beta\right)\sum_{r=1}^{R}\left(q^F_r - \bar{q}_r\right). \quad (6)$$

Proof. From Assumption 1 lets consider that $\frac{\partial P_\omega}{\partial q^S_{r\omega}} = \frac{\partial P_\omega}{\partial q^S_{r'\omega}}$, $\frac{\partial S_\omega}{\partial q^S_{r\omega}} = \frac{\partial S_\omega}{\partial q^S_{r'\omega}}$ $\forall r$ and $\forall r' \neq r$. At each scenario ω, retailer r selects the optimal quantity to buy in the spot market, $q^S_{r\omega}$, to maximize profit. From (2) we get $\frac{\partial \Pi_{r\omega}}{\partial q^S_{r\omega}} = (q^S_{r\omega} + q^F_r - \bar{q}_r)\frac{\partial P_\omega}{\partial q^S_{r\omega}} + P_\omega - S_\omega - q^S_{r\omega}\frac{\partial S_\omega}{\partial q^S_{r\omega}} = 0$, which, from (3), is equivalent to $\nu_\omega + q^S_{r\omega}(\frac{\partial P_\omega}{\partial q^S_{r\omega}} - \frac{\partial S_\omega}{\partial q^S_{r\omega}}) + (q^F_r - \bar{q}_r)\frac{\partial P_\omega}{\partial q^S_{r\omega}} - \beta\sum_{r'=1}^{R}(q^S_{r'\omega} + q^F_{r'} - \bar{q}_{r'}) - S_\omega = 0$. Then, by summing the quantities bought in the spot market by all retailers, we get $(\frac{\partial P_\omega}{\partial q^S_{r\omega}} - \frac{\partial S_\omega}{\partial q^S_{r\omega}})\sum_{r=1}^{R} q^S_{r\omega} = RS_\omega - R\nu_\omega + R\beta\sum_{r=1}^{R}(q^S_{r\omega} + q^F_r - \bar{q}_r) - \frac{\partial P_\omega}{\partial q^S_{r\omega}}\sum_{r=1}^{R}(q^F_r - \bar{q}_r)$, which, after some term rearrangements, renders $(\frac{\partial P_\omega}{\partial q^S_{r\omega}} - \frac{\partial S_\omega}{\partial q^S_{r\omega}} - R\beta)\sum_{r=1}^{R} q^S_{r\omega} = RS_\omega - R\nu_\omega + R\beta\sum_{r=1}^{R}(q^F_{r'} - \bar{q}_{r'}) - \frac{\partial P_\omega}{\partial q^S_{r\omega}}\sum_{r=1}^{R}(q^F_r - \bar{q}_r)$. From this expression the aggregate demand $\sum_{r=1}^{R} q^S_{r\omega}$, (5), or its inverse, can be easily computed as a function of the spot price S_ω, (6). □

We proceed by deriving the generators' equilibrium sales in the spot market. First, we make the following assumption regarding the structure of the electricity generators' cost function ($C_{g\omega}$).

Assumption 2. The generators' costs functions are linear with respect to the total production $q^S_{g\omega} + q^F_g$ so that $\frac{\partial C_{g\omega}}{\partial q^S_{g\omega}}$ and $\frac{\partial C_{g\omega}}{\partial q^F_g}$ are constant, i.e., let's assume that $C_{g\omega} = (q^S_{g\omega} + q^F_g)c_{g\omega}$ and thus $\frac{\partial C_{g\omega}}{\partial q^S} = \frac{\partial C_{g\omega}}{\partial q^F_{g\omega}} = c_{g\omega}$.

The simplification described by Assumption 2 allows an analytically tractable market model to be derived, while still capturing the basic functioning of real-world electricity markets. Note that in most of these markets, generators are obliged to submit their offers (marginal costs) in the form of stepwise functions, i.e., assuming constant marginal costs per energy block. Hence, for the purposes of our equilibrium analysis we will not explicitly model the whole piecewise-linear costs function, but rather assume it to be in the neighborhood of the equilibrium quantity $q^S_{g\omega} + q^F_g$, where this cost function can be considered strictly linear.

We use another set of assumptions, summarized by Assumption 3, which are standard in this type of model, and relate to the structure of information and uncertainty resolution at the different stages of the game.

Assumption 3. (a) Uncertainty over demand and cost functions decreases monotonically with time, i.e., as the last day of the spot market approaches, firms are able to estimate the parameters with more precision. (b) In the last trading day, there is no uncertainty, i.e., all the firms have common and complete knowledge of the correct parameters, including demand and cost functions.

Proposition 3. *Under Assumption 2, the equilibrium sales of genera- tor g in scenario ω in the spot market are described by:*

$$q^S_{g\omega} = \frac{c_{g\omega} - S_\omega}{\frac{\partial S_\omega}{\partial q^S_{g\omega}}}. \quad (7)$$

Proof. Assuming monotonicity of S_ω with respect to $q^S_{g\omega}$ a generator g selects the optimal quantity $q^S_{g\omega}$ to sell in the spot market so that its profit (1) is maximized at each scenario ω, i.e., $\frac{\partial \Pi_{g\omega}}{\partial q^S_{g\omega}} = \frac{\partial S_\omega}{\partial q^S_{g\omega}} q^S_{g\omega} + S_\omega - c_{g\omega} = 0$, after which it directly follows Eq. (7). Similarly, the concavity of (1) means that (7) are the first-order necessary and sufficient conditions for optimality. This conclusion follows from $\frac{\partial^2 \Pi_{g\omega}}{\partial {q^S_{g\omega}}^2} = 2(\frac{\partial S}{\partial q^S_{g\omega}}) \leq 0$ as $\frac{\partial S}{\partial q^S_{g\omega}} \leq 0$ (Proposition 6). □

We now use a similar process to derive the aggregate supply function (and its inverse) using the generators' equilibrium trades in the spot market.

Proposition 4. *Under Assumptions 1 and 2, the aggregate supply function (and its inverse) for generators in the spot market are represented, respectively, by Eqs. (8) and (9).*

$$\sum_{g=1}^{G} q^S_{g\omega} = \frac{1}{\frac{\partial S_\omega}{\partial q^S_{g\omega}}}\left(\sum_{g=1}^{G} c_{g\omega} - GS_\omega\right) \quad (8)$$

$$S_\omega = \frac{1}{G}\left(\sum_{g=1}^{G} c_{g\omega} - \frac{\partial S_\omega}{\partial q^S_{g\omega}}\sum_{g=1}^{G} q^S_{g\omega}\right). \quad (9)$$

Proof. From Assumption 1 lets consider that $\frac{\partial S_\omega}{\partial q^S_{g\omega}} = \frac{\partial S_\omega}{\partial q^S_{g'\omega}}$ $\forall g$ and $\forall g' \neq g$. Summing the sales of the generators in the spot market (7) to compute $\sum_{g=1}^{G} q^S_{g\omega}$, we obtain the aggregate supply function in the spot market (8), from which the inverse aggregate supply function is derived: $S_\omega = \frac{1}{G}(\sum_{g=1}^{G} c_{g\omega} - \frac{\partial S_\omega}{\partial q^S_{g\omega}}\sum_{g=1}^{G} q^S_{g\omega})$. □

Furthermore, we need to consider the spot market equilibrium condition, which establishes that for each scenario ω, all the energy sold by the generators must be equal to that bought by the retailers, Eq. (10).

$$\sum_{g=1}^{G} q^S_{g\omega} = \sum_{r=1}^{R} q^S_{r\omega}. \quad (10)$$

Then from aggregate demand function (5) and aggregate supply function (8), we compute the price S_w that clears the spot market per scenario ω, i.e., the price that guarantees that supply equals demand as described by Eq. (10). This is derived in Proposition 5.

Proposition 5. *The equilibrium spot price is described by Eq. (11).*

$$S_\omega = \frac{\left(\frac{\partial P_\omega}{\partial q^S_{r\omega}} - \frac{\partial S_\omega}{\partial q^S_{r\omega}} - R\beta\right)\sum_{g=1}^{G} c_{g\omega} + \frac{\partial S_\omega}{\partial q^S_{g\omega}}\left[R\nu_\omega - \left(R\beta - \frac{\partial P_\omega}{\partial q^S_{r\omega}}\right)\sum_{r=1}^{R}\left(q^F_r - \bar{q}_r\right)\right]}{G\left(\frac{\partial P_\omega}{\partial q^S_{r\omega}} - \frac{\partial S_\omega}{\partial q^S_{r\omega}} - R\beta\right) + \frac{\partial S_\omega}{\partial q^S_{g\omega}}} \quad (11)$$

Proof. From (10), (5) and (8) we know that

$$\frac{1}{\frac{\partial S_\omega}{\partial q^S_{g\omega}}}\left(\sum_{g=1}^{G} c_{g\omega} - GS_\omega\right) = \frac{R(S_\omega - \nu_\omega) + \left(R\beta - \frac{\partial P_\omega}{\partial q^S_{r\omega}}\right)\sum_{r=1}^{R}\left(q^F_r - \bar{q}_r\right)}{\frac{\partial P_\omega}{\partial q^S_{r\omega}} - \frac{\partial S_\omega}{\partial q^S_{r\omega}} - R\beta}.$$

By rearranging the terms we obtain

$$S_\omega\left[G\left(\frac{\partial P_\omega}{\partial q^S_{r\omega}} - \frac{\partial S_\omega}{\partial q^S_{r\omega}} - R\beta\right) + \frac{\partial S_\omega}{\partial q^S_{g\omega}}\right] = \left(\frac{\partial P_\omega}{\partial q^S_{r\omega}} - \frac{\partial S_\omega}{\partial q^S_{r\omega}} - R\beta\right)\sum_{g=1}^{G} c_{g\omega} + \frac{\partial S_\omega}{\partial q^S_{g\omega}}\left[R\nu_\omega - \left(R\beta - \frac{\partial P_\omega}{\partial q^S_{r\omega}}\right)\sum_{r=1}^{R}\left(q^F_r - \bar{q}_r\right)\right],$$

and (11). □

3.1.1. Conjectural variations in the spot market

It is evident from the results described in Eq. (11) there are several partial derivatives from generators and retailers ($\frac{\partial P_\omega}{\partial q^S_{r\omega}}$, $\frac{\partial S_\omega}{\partial q^S_{r\omega}}$ and $\frac{\partial S_\omega}{\partial q^S_{g\omega}}$) that influence the spot price and, therefore, the retail price, and the total electricity generated. In this section we analyze their consistent values with respect to predefined conjectural variations. In particular: let $V^S_r = \sum_{r' \neq r}^{R} \frac{\partial q^S_{r'}}{\partial q^S_r}$ and $V^D_r = \sum_{r' \neq r}^{R} \frac{\partial q^S_{r'}}{\partial q^S_r}$ stand for the conjectural variation of r in respect to the total purchases of its competitors r' in the spot and retail markets, respectively; and let $V^S_g = \sum_{g' \neq g}^{G} \frac{\partial q^S_{g'}}{\partial q^S_g}$ represent the conjectural variation of generator g with regard to the total production of their competitors in the spot market. According to Assumption 1, these conjectures are scenario independent and symmetric for all generators or retailers, i.e., $V^S_g = V^S_{g'}$ for $g \neq g'$, $V^S_r = V^S_{r'}$ and $V^D_r = V^D_{r'}$ for $r \neq r'$. Hence, from now on, we adopt the notation: $V^S_{\forall g} \equiv V^S_g$, $V^S_{\forall r} \equiv V^S_r$ and $V^D_{\forall r} \equiv V^D_r$, $\forall r$, $\forall g$.

Proposition 6. *The consistent values for the retail and spot price sensitivities with respect to the quantities procured (sold) by the retailers (generators) are represented by Eqs. (12), (13), (14).*

$$\frac{\partial P_\omega}{\partial q^S_{r\omega}} = -\beta\left(1 + V^D_{\forall r}\right) \qquad \forall r, \forall \omega. \tag{12}$$

$$\frac{\partial S_\omega}{\partial q^S_{r\omega}} = \frac{\beta(1+V^S_{\forall g})(1+V^S_{\forall r})(1+V^D_{\forall r}+R)}{GR - (1+V^S_{\forall r})(1+V^S_{\forall g})} \qquad \forall r, \forall \omega \tag{13}$$

$$\frac{\partial S_\omega}{\partial q^S_{g\omega}} = \frac{-G\beta(1+V^S_{\forall g})(1+V^D_{\forall r}+R)}{GR - (1+V^S_{\forall r})(1+V^S_{\forall g})} \qquad \forall g, \forall \omega \tag{14}$$

Proof. $\frac{\partial P_\omega}{\partial q^S_{r\omega}}$ follows directly from (3) and is equal to $\frac{\partial P_\omega}{\partial q^S_{r\omega}} = -\beta\left(1 + V^D_{\forall r}\right)$.

Generators face the aggregate inverse demand function of the retailers (6) in the spot market. Considering $\sum_{g=1}^{G} q^S_{g\omega} = \sum_{r=1}^{R} q^S_{r\omega}$ and replacing $\frac{\partial P_\omega}{\partial q^S_{r\omega}}$ by (12) renders (15).

$$\frac{\partial S_\omega}{\partial q^S_{g\omega}} = \frac{1}{R}\left(-\beta\left(1+V^D_{\forall r}\right) - \frac{\partial S_\omega}{\partial q^S_{r\omega}} - R\beta\right)(1+V^S_{\forall g}) \tag{15}$$

Similarly, retailers face the aggregate offer curve of the generators (9) in the spot market. Assuming $\sum_{r=1}^{R} q^S_{r\omega} = \sum_{g=1}^{G} q^S_{g\omega}$ renders (16).

$$\frac{\partial S_\omega}{\partial q^S_{r\omega}} = \frac{-1}{G}\frac{\partial S_\omega}{\partial q^S_{g\omega}}(1+V^S_{\forall r}) \tag{16}$$

By solving the linear system (15) and (16) we can derive the explicit expressions for $\frac{\partial S_\omega}{\partial q^S_{r\omega}}$ and $\frac{\partial S_\omega}{\partial q^S_{g\omega}}$ in (13) and (14), respectively. □

Proposition 6 shows that the price sensitivities (12), (13) and (14) are the same within all retailers, generators, and scenarios. Hence, for simplicity, from now on we will use the notation: $\frac{\partial P}{\partial q^S_{\forall r}} \equiv \frac{\partial P_\omega}{\partial q^S_{r\omega}}$, $\frac{\partial S}{\partial q^S_{\forall r}} \equiv \frac{\partial S_\omega}{\partial q^S_{r\omega}}$ and $\frac{\partial S}{\partial q^S_{\forall g}} \equiv \frac{\partial S_\omega}{\partial q^S_{g\omega}}$, $\forall r, \forall g, \forall \omega$.

3.1.2. Equilibrium spot market outcomes

We can use Proposition 5 to replace the explicit formulation of S_ω in Propositions 1–4 and express all the equilibrium market outcomes, i.e., S_ω, P_ω, $q^S_{g\omega}$ and $q^S_{r\omega}$ as a linear function of the futures decision variables $\sum_{r=1}^{R} q^F_r = \sum_{g=1}^{G} q^F_g = q^F$, as represented in Eqs. (17)–(21).

$$S_\omega = \mathcal{A}_\omega + \mathcal{B}q^F \qquad \forall \omega \tag{17}$$

$$\sum_{r=1}^{R} q^S_{r\omega} = \sum_{g=1}^{G} q^S_{g\omega} = \mathcal{F}_\omega + \mathcal{G}q^F \qquad \forall \omega \tag{18}$$

$$P_\omega = \mathcal{J}_\omega + \mathcal{K}q^F \qquad \forall \omega \tag{19}$$

$$q^S_{r\omega} = \mathcal{D}_{r\omega} + \mathcal{E}q^F_r + \mathcal{H}q^F \qquad \forall r, \quad \forall \omega \tag{20}$$

$$q^S_{g\omega} = \mathcal{Q}_{g\omega} + \mathcal{R}q^F \qquad \forall g, \quad \forall \omega \tag{21}$$

The derivation of Eqs. (17) –(21) and the computation of parameters $\mathcal{A}_\omega$, $\mathcal{B}$, $\mathcal{F}_\omega$, $\mathcal{G}$, $\mathcal{J}_\omega$, $\mathcal{K}$, $\mathcal{D}_{r\omega}$, $\mathcal{E}$, $\mathcal{H}$, $\mathcal{Q}_{g\omega}$ and $\mathcal{R}$ can be found in Appendix A.

3.2. Modeling the futures market considering CV@R (Stage One)

We use the equilibrium conditions in the spot market to compute the sales by all the players in this market, as defined by (17)–(21). Then, based on these equations, by backward induction and given a futures price W, we calculate the equilibrium sales and purchases in the futures market (q^F_g, q^F_r and W). With this purpose we move backwards in time to stage one to solve the supply chain equilibrium. This equilibrium is characterized by the joint maximization of all the agents' profits under different levels of risk aversion. Therefore, each generator and retailer sets conjectures about the behavior of its rivals and optimizes its profit by selecting the optimal q^F_g and q^F_r, respectively, while the equilibrium price W comes from the resulting system of optimality conditions, i.e., the equilibrium price W guarantees that all the players' optimality conditions are satisfied, and that supply equals demand. This is an important contribution of this work as the futures price is obtained from the optimality conditions without making any assumption about the relationship between futures and spot prices.

The risk aversion of generators and retailers is considered by using the CV@R. This concept has been applied both in the context of one decision maker facing a static problem (e.g., Artzner et al., 1999; Rockafellar & Uryasev, 2000; Rockafellar & Uryasev, 2002; Andersson, Mausser, Rosen, & Uryasev, 2001; and Chen et al., 2010), and a dynamic problem (e.g., Cheridito, Delbaen, & Kupper, 2006; Boda & Filar, 2006; Klöppel & Schweizer, 2007; Shapiro, 2009; Shapiro, 2011; Philpott, de Matos, & Finardi, 2013, Ansaripoor, Oliveira, & Liret, 2016 and Ansaripoor & Oliveira, 2018).

Shapiro (2011) provides a version of the dynamic CV@R that is time consistent (at each state of the system the optimal decisions cannot depend on the outcome of futures states, which we already know to be unreachable), and Philpott et al. (2013) provide a pro- cedure for computing solutions to multistage stochastic program- ming problems that minimize dynamic coherent risk measures. It is important to be clear that in this article we address the *static* CV@R model only, as the model has two time steps (futures, with uncertainty and spot, without uncertainty).

For a stochastic programming setting where scenarios are used to represent uncertainty, the CV@R associated with a profit distribution can be computed by solving a linear optimization problem (Rockafellar & Uryasev, 2000). In particular, the profit maximization problem solved by the risk-averse generator g is formulated in (9), where $\sigma_{g\omega}$ is the probability assigned by generator g to scenario ω and $1-\alpha_g$ represents the level of significance associated with the CV@R.

$$\underset{q_g^F,\xi_g,\eta_{g\omega}}{\text{Maximize}} \quad \phi_g\left(\sum_{\omega=1}^{\Omega}\sigma_{g\omega}\Pi_{g\omega}(q_g^F,q_{-g}^F,W)\right) + (1-\phi_g)\left(\xi_g - \frac{1}{1-\alpha_g}\sum_{\omega=1}^{\Omega}\sigma_{g\omega}\eta_{g\omega}\right) \tag{22a}$$

subject to:

$$\eta_{g\omega} \geq -\Pi_{g\omega}(q_g^F,q_{-g}^F,W)+\xi_g \qquad \forall\omega \qquad [\lambda_{g\omega}] \tag{22b}$$

$$\eta_{g\omega} \geq 0 \qquad \forall\omega \qquad [\delta_{g\omega}] \tag{22c}$$

The objective function represents a trade-off between the expected value (first term) and the CV@R (second term) for a given confidence level α_g. This trade-off is regulated by the parameter ϕ_g, where $0 \leq \phi_g \leq 1$. Under this setting, $\phi_g = 1$ represents a risk-neutral generator g (maximization of the expected value), while $\phi_g = 0$ corresponds to the maximum level of risk aversion (maximization of the CV@R). Let the value at risk (V@R) be the $1-\alpha_g$ profit left-tail quantile. At the optimal solution, ξ_g equals the V@R and $\eta_{g\ \omega}$ equals (for each scenario ω) the difference between the V@R and the profit $\Pi_{g\ \omega}$ if this difference is positive, or zero otherwise. This condition is imposed by constraints (22b) and (22c). Finally $\lambda_{g\omega}$ and $\delta_{g\omega}$ are the dual variables associated with constraints (22b) and (22c), respectively. The optimization problem solved by retailer r is defined by (23), where $\sigma_{r\ \omega}$ is the probability assigned by retailer r to scenario ω.

$$\underset{q_r^F,\xi_r,\eta_{r\omega}}{\text{Maximize}} \quad \phi_r\left(\sum_{\omega=1}^{\Omega}\sigma_{r\omega}\Pi_{r\omega}(q_r^F,q_{-r}^F,W)\right) + (1-\phi_r)\left(\xi_r - \frac{1}{1-\alpha_r}\sum_{\omega=1}^{\Omega}\sigma_{r\omega}\eta_{r\omega}\right) \tag{23a}$$

subject to:

$$\eta_{r\omega} \geq -\Pi_{r\omega}(q_r^F,q_{-r}^F,W)+\xi_r \qquad \forall\omega \qquad [\lambda_{r\omega}] \tag{23b}$$

$$\eta_{r\omega} \geq 0 \qquad \forall\omega \qquad [\delta_{r\omega}] \tag{23c}$$

Again, the parameter ϕ_r allows a retailer r to balance the tradeoff between the expected profits and the CV@R for a significance level $1-\alpha_r$. Similarly, auxiliary variables ξ_r and $\eta_{r\omega}$ are used in (23b) and (23c) to calculate the positive difference between V@R and profit $\Pi_{r\omega}$ for each scenario ω. The dual variables associated with constraints (23b) and (23c) are $\lambda_{r\omega}$ and $\delta_{r\omega}$, respectively. For a given futures price W, the simultaneous solution of problems (22) and (23) provides the Nash equilibrium quantities for trading in the futures market by generators and retailers.

Moreover, for the specific market configurations studied in this paper (Section 4), (22) and (23) are concave optimization problems (Appendix B) and therefore their solutions are unique. In order to search for specific solutions to the supply chain equilibrium, we replace problems (22) and (23) by their corresponding KKT (Karush–Kuhn–Tucker) system of optimality conditions, which due to the concavity of the profits functions, are first-order necessary and sufficient conditions for optimality (Appendix B). The KKT system associated with problem (22) is:

$$-\phi_g\sum_{\omega=1}^{\Omega}\sigma_{g\omega}\frac{\partial\Pi_{g\omega}}{\partial q_g^F} - \sum_{\omega=1}^{\Omega}\lambda_{g\omega}\frac{\partial\Pi_{g\omega}}{\partial q_g^F} = 0 \qquad \forall g \tag{24a}$$

$$(1-\phi_g)\frac{1}{1-\alpha_g}\sigma_{g\omega} - \lambda_{g\omega} - \delta_{g\omega} = 0 \qquad \forall g,\forall\omega \tag{24b}$$

$$\sum_{\omega=1}^{\Omega}\lambda_{g\omega} = 1-\phi_g \qquad \forall g \tag{24c}$$

$$0 \leq \Pi_{g\omega} - \xi_g + \eta_{g\omega} \perp \lambda_{g\omega} \geq 0 \qquad \forall g,\forall\omega \tag{24d}$$

$$0 \leq \eta_{g\omega} \perp \delta_{g\omega} \geq 0 \qquad \forall g,\forall\omega \tag{24e}$$

Similarly, the KKT system associated with problem (23) is:

$$-\phi_r\sum_{\omega=1}^{\Omega}\sigma_{r\omega}\frac{\partial\Pi_{r\omega}}{\partial q_r^F} - \sum_{\omega=1}^{\Omega}\lambda_{r\omega}\frac{\partial\Pi_{r\omega}}{\partial q_r^F} = 0 \qquad \forall r \tag{25a}$$

$$(1-\phi_r)\frac{1}{1-\alpha_r}\sigma_{r\omega} - \lambda_{r\omega} - \delta_{r\omega} = 0 \qquad \forall r,\forall\omega \tag{25b}$$

$$\sum_{\omega=1}^{\Omega}\lambda_{r\omega} = 1-\phi_r \qquad \forall r \tag{25c}$$

$$0 \leq \Pi_{r\omega} - \xi_r + \eta_{r\omega} \perp \lambda_{r\omega} \geq 0 \qquad \forall r,\forall\omega \tag{25d}$$

$$0 \leq \eta_{r\omega} \perp \delta_{r\omega} \geq 0 \qquad \forall r,\forall\omega \tag{25e}$$

where the complementarity conditions are denoted by $0 \leq x \perp y \geq 0$, which is equivalent to: $x \geq 0$, $y \geq 0$ and $xy = 0$.

For a given futures price W and under the profit concavity assumption, any particular solution q_g^F and q_r^F, $\forall\ g$ and $\forall r$, satisfying the KKT conditions (24) and (25), for both generators and retailers, is also a solution to the Nash equilibrium problem, see Theorem 4.6 in Facchinei and Kanzow (2010).

Among all the possible prices W, we are interested in identifying the value that clears the futures market, i.e., the price that guarantees that supply equals demand, as described by Eq. (26), which was proposed in Oliveira et al. (2013) for the deterministic case.

$$\sum_{g=1}^{G} q_g^F = \sum_{r=1}^{R} q_r^F. \tag{26}$$

Therefore, the supply chain equilibrium is obtained by considering the futures price W as a variable and by solving jointly (24)–(26). There are several methodologies to solve complementarity systems of the type (24)–(26), e.g., Gabriel, Conejo, Fuller, Hobbs, and Ruiz (2013) and Pozo, Sauma, and Contreras (2017). In this article, we have reformulated this system as an optimization problem that minimizes the sum of all the complementarity conditions, subject to the rest of the system's constraints. This formulation is presented in Appendix C where its equivalence is also shown with systems (24)–(26).

3.2.1. *Risk-adjusted expectations*

In this section we introduce the concept of risk-adjusted expectations and demonstrate its usefulness for understanding the price formation mechanism in futures and spot markets. The risk-adjusted probabilities are endogenous to the model and reflect the weight a risk-averse decision maker gives to a scenario. If the dual variables associated with (22b) or (23b) are positive then a given scenario receives a higher weight than if the dual variables are zero.

The major contribution of this analysis is to show the equivalence of computing the optimal policy based on the weighted profit between expected value and CV@R with maximizing the risk-adjusted expected value, which is calculated using the risk-adjusted probabilities. Therefore, in practice, the way the decision maker incorporates risk in the model is reflected in the different weights put on the different scenarios, i.e., the risk-adjusted probabilities take into consideration the influence of the parameterization of the objective function (i.e., ϕ and $1-\alpha$, for both generators and retailers) on the actual probabilities associated with each scenario.

Lemma 1. *(A) Generator g's risk-adjusted probability in scenario ω is equal to $\phi_g\sigma_{g\omega}+\lambda_{g\omega}$. (B) Retailer r's risk-adjusted probability in scenario ω is equal to $\phi_r\sigma_{r\omega}+\lambda_{r\omega}$.*

Proof. By rewriting conditions (24a) and (25a) as $\sum_{\omega=1}^{\Omega}\frac{\partial\Pi_{g\omega}}{\partial q_g^F}\left(\phi_g\sigma_{g\omega}+\lambda_{g\omega}\right)=0$ and $\sum_{\omega=1}^{\Omega}\frac{\partial\Pi_{r\omega}}{\partial q_r^F}\left(\phi_r\sigma_{r\omega}+\lambda_{r\omega}\right)=0$, respectively. Then from (24c) and (25c) it follows that $\sum_{\omega=1}^{\Omega}\left(\phi_g\sigma_{g\omega}+\lambda_{g\omega}\right)=1$ and $\sum_{\omega=1}^{\Omega}\left(\phi_r\sigma_{r\omega}+\lambda_{r\omega}\right)=1$. □

These new probabilities are risk-adjusted as they change the initial weights put on the different scenarios in order to account for risk. The larger the probability associated with a risky scenario (in which the actual profit exceeds the CV@R), the larger the adjustment. This concept of risk-adjusted probability was introduced in Ehrenmann and Smeers (2011). Our concept generalizes theirs by including the trade-off between CV@R and expected value.

Let $\mathbb{E}_g$ and $\mathbb{E}_r$ stand for the risk-adjusted expectation of generator g and retailer r. Eqs. (24a) and (25a) can be interpreted as optimizing the risk-adjusted expected marginal profits for both generators and retailers, i.e., $\mathbb{E}_g\left[\frac{\partial\Pi_{g\omega}}{\partial q_g^F}\right]=0$ and $\mathbb{E}_r\left[\frac{\partial\Pi_{r\omega}}{\partial q_r^F}\right]=0$, where the probability associated with each scenario is $\left(\phi_g\sigma_{g\omega}+\lambda_{g\omega}\right)$ and $\left(\phi_r\sigma_{r\omega}+\lambda_{r\omega}\right)$, respectively.

3.2.2. Conjectural variations in the futures market

In (24) and (25) the partial derivative of the spot profit to the trading in the futures market is computed, respectively, from (2) and (1) as:

$$\frac{\partial\Pi_{r\omega}}{\partial q_r^F}=-W-q_r^F\frac{\partial W}{\partial q_r^F}+\frac{\partial P_\omega}{\partial q_r^F}\left(q_{r\omega}^S+q_r^F-\bar{q}_r\right)+P_\omega\left(\frac{\partial q_{r\omega}^S}{\partial q_r^F}+1\right)-\frac{\partial q_{r\omega}^S}{\partial q_r^F}S_\omega-q_{r\omega}^S\frac{\partial S_\omega}{\partial q_r^F}\qquad\forall r,\forall\omega \tag{27a}$$

$$\frac{\partial\Pi_{g\omega}}{\partial q_g^F}=W+q_g^F\frac{\partial W}{\partial q_g^F}+\frac{\partial q_{g\omega}^S}{\partial q_g^F}S_\omega+q_{g\omega}^S\frac{\partial S_\omega}{\partial q_g^F}-c_{g\omega}\left(1+\frac{\partial q_{g\omega}^S}{\partial q_g^F}\right)\qquad\forall g,\forall\omega \tag{27b}$$

By analyzing the different terms in (27) we can observe there are several partial derivatives from generators and retailers $\frac{\partial W}{\partial q_r^F}$, $\frac{\partial W}{\partial q_g^F}$, $\frac{\partial P_\omega}{\partial q_r^F}$, $\frac{\partial S_\omega}{\partial q_r^F}$, $\frac{\partial S_\omega}{\partial q_g^F}$, $\frac{\partial q_{r\omega}^S}{\partial q_r^F}$ and $\frac{\partial q_{g\omega}^S}{\partial q_g^F}$ that influence the futures market equilibrium. We seek to derive their consistent values with respect to predefined conjectural variations. In particular let $Y_r^F=\sum_{r'\neq r}^{R}\frac{\partial q_{r'}^F}{\partial q_r^F}$ stand for the conjectural variation of retailer r in respect to the total purchases of its competitors in the futures market, r'; and let $Y_g^F=\sum_{g'\neq g}^{G}\frac{\partial q_{g'}^F}{\partial q_g^F}$ represent the conjectural variation of generator g in respect to the total production of their competitors g' in the futures market. Similarly to the spot market conjectures, we assume these to be scenario independent and symmetric within all generators or retailers (Assumption 1), i.e., $Y_g^F=Y_{g'}^F$, for $g\neq g'$ and $Y_r^F=Y_{r'}^F$ for $r\neq r'$. Hence, we will use the notation: $Y_{\forall g}^F\equiv Y_g^F$ and $Y_{\forall r}^F\equiv Y_r^F$ $\forall r,\forall g$.

The partial derivatives of the equilibrium spot market outcomes (P_ω, S_ω, $q_{r\omega}^S$ and $q_{g\omega}^S$) with respect to the retailers' and generators' futures trading (q_r^F and q_g^F) are computed using Eqs. (17) – (21). These partial derivatives are summarized in Eqs. (28) –(32).

$$\frac{\partial P_\omega}{\partial q_r^F}=\mathcal{K}\left(1+Y_{\forall r}^F\right)\qquad\forall r,\forall\omega. \tag{28}$$

$$\frac{\partial S_\omega}{\partial q_r^F}=\mathcal{B}\left(1+Y_{\forall r}^F\right)\qquad\forall r,\forall\omega \tag{29}$$

$$\frac{\partial S_\omega}{\partial q_g^F}=\mathcal{B}\left(1+Y_{\forall g}^F\right)\qquad\forall g,\forall\omega \tag{30}$$

$$\frac{\partial q_{r\omega}^S}{\partial q_r^F}=\mathcal{E}+\mathcal{H}\left(1+Y_{\forall r}^F\right)\qquad\forall r,\forall\omega \tag{31}$$

$$\frac{\partial q_{g\omega}^S}{\partial q_g^F}=\mathcal{R}\left(1+Y_{\forall g}^F\right)\qquad\forall g,\forall\omega \tag{32}$$

Proposition 7 describes the consistent partial derivatives $\frac{\partial W}{\partial q_r}$ and $\frac{\partial W}{\partial q_g^F}$, derived from the equilibrium conditions in the futures market. Moreover, $\frac{\partial W}{\partial q_r^F}$ and $\frac{\partial W}{\partial q_g^F}$ are scenario independent and hence valid for any set of risk-adjusted probabilities.

Proposition 7. *The consistent partial derivatives of W with respect to the quantities procured and sold by retailers and generators in the futures market are represented by Eqs.(33) and (34), respectively.*

$$\frac{\partial W}{\partial q_r^F}=\frac{-\left(1+Y_{\forall r}^F\right)\left[RG\mathcal{X}+\left(1+Y_{\forall g}^F\right)(\mathcal{T}+R\mathcal{U})\right]}{RG-\left(1+Y_{\forall r}^F\right)\left(1+Y_{\forall g}^F\right)}\qquad\forall r \tag{33}$$

$$\frac{\partial W}{\partial q_g^F}=\frac{G\left(1+Y_{\forall g}^F\right)\left[\mathcal{T}+R\mathcal{U}+\mathcal{X}\left(1+Y_{\forall r}^F\right)\right]}{RG-\left(1+Y_{\forall r}^F\right)\left(1+Y_{\forall g}^F\right)}\qquad\forall g \tag{34}$$

where

$$\mathcal{T}=\left(1+Y_{\forall r}^F\right)[\mathcal{K}(\mathcal{E}+1)-\mathcal{B}\mathcal{E}] \tag{35}$$

$$\mathcal{U}=2\left(1+Y_{\forall r}^F\right)\mathcal{H}(\mathcal{K}-\mathcal{B})+\mathcal{K}(\mathcal{E}+1)-\mathcal{B}\mathcal{E} \tag{36}$$

$$\mathcal{X}=2\mathcal{R}\mathcal{B}\left(1+Y_{\forall g}^F\right) \tag{37}$$

The proof for this proposition can be found in Appendix D.

Similar to Proposition 6 for the spot market, Proposition 7 shows that the price sensitivities, (33) and (34), are the same for all retailers, and for all generators, respectively, and are both scenario independent.

Finally, Proposition 8 characterizes the futures market price W. The idea of computing the equilibrium wholesale price without requiring the non-arbitrage condition was originally proposed in Oliveira et al. (2013), but only used in a numerical example. In Proposition 8 we derive the close-form solution for the futures price, taking into account risk aversion. This is a major contribution of this article.

Proposition 8. *The equilibrium futures price is defined as:*

$$W=\frac{-\left(\frac{\partial W}{\partial q_g^F}+G\mathcal{X}\right)\sum_{r=1}^{R}\sum_{\omega=1}^{\Omega}\sigma_{r\omega}^*\mathcal{S}_{r\omega}+\left(-\frac{\partial W}{\partial q_r^F}+\mathcal{T}+R\mathcal{U}\right)\sum_{g=1}^{G}\sum_{\omega=1}^{\Omega}\sigma_{g\omega}^*\mathcal{V}_{g\omega}}{-R\left(\frac{\partial W}{\partial q_g^F}+G\mathcal{X}\right)+G\left(-\frac{\partial W}{\partial q_r^F}+\mathcal{T}+R\mathcal{U}\right)} \tag{38}$$

Proof. In equilibrium (26), price W is such that the retailers' inverse aggregate demand curve (D.5) equals the generators' aggregate supply curve (D.6). □

Both the inverse aggregate demand for retailers (D.5) and the aggregate supply curve for generators (D.6) depend on the risk-adjusted probabilities $\sigma^*_{r\omega} = \phi_r\sigma_{r\omega} + \lambda_{r\omega}$ and $\sigma^*_{g\omega} = \phi_g\sigma_{g\omega} + \lambda_{g\omega}$, which need to be obtained from the joint solution of conditions (24)–(26), rendering also the optimal value for the futures price W. However, Proposition 8 is still meaningful as it shows that for a given set of risk-adjusted probabilities (associated with a particular solution to the Nash equilibrium), the futures price W is unique. Moreover, for the particular case where the market participants are risk neutral, i.e., $\sigma^*_{r\omega} = \sigma_{r\omega}$ and $\sigma^*_{g\omega} = \sigma_{g\omega}$, the equilibrium futures price W is unique and can be directly computed by the closed-form solution provided by Proposition 8. Note that this result is general and allows generators and retailers to differ on their perceived scenario probabilities $\sigma_{r\omega}$ and $\sigma_{g\omega}$, respectively.

4. Numerical analysis

In this section we analyze a numerical example from the Spanish electricity market (OMIE, 2020), liberalized in the late 1990s based on a centralized auction where all the electricity was traded (Crampes & Fabra, 20 05). In the early 20 0 0s it evolved into a bilateral trading mechanism in which a large proportion of the electricity is traded using bilateral contracts between the generators and the retailers (who serve the final consumers), and in which futures contracts are also traded in an exchange (OMIP, 2020). This market is currently composed of three large generators (Endesa, Gas Natural-Fenosa, and Iberdrola), four large retailers (Endesa, Gas Natural-Fenosa, Iberdrola and Hidrocantabrico), and a competitive fringe of small generators. There are also some large consumers participating in the wholesale markets. As their market share is still negligible we do not consider them in our analysis. We have followed the literature in assuming that in equilibrium all retailers charge the same price to the final consumers.
Compared to other deregulated markets, such as the UK or CAISO, the recent deregulations of the Spanish electricity market ("Ley del sector eléctrico 1997 and 2013") do not specifically impose vertical separation between generation and retailing companies, i.e., it is not illegal for a holding company to take an active part in both generation and retailing activities through two affiliated companies. However, the Spanish Energy Regulator ("Comisión Nacional de los Mercados y la Competencia") constantly monitors the behavior of market participants to prevent and penalize any form of unfair competition, including the exercise of vertical market power CNMC, 2014. Hence, in this case study we as- sume independent generators and retailers.

We seek to understand the shape of the forward curves observed at specific times of the year (e.g., Table 1) and the main factors affecting it. In particular, mimicking the functioning of real-world electricity markets (e.g., OMIP, 2020), the generators and retailers participate in several futures markets that take place simultaneously on a given day, for example, Dec. 29, 2016. To illustrate this process, Fig. 3 presents the futures market prices for different products settled on Dec. 29, 2016, together with the resulting monthly spot market prices for 2017. We can observe that the resulting spot market prices during the last quarter of the year 2017 exceeded the quarterly and yearly futures prices settled on Dec. 29, 2016.

We consider only monthly contracts consisting of a single electricity load segment (no distinction between peak or base prices), with a time horizon that extends from 1 to 12 months into the future.

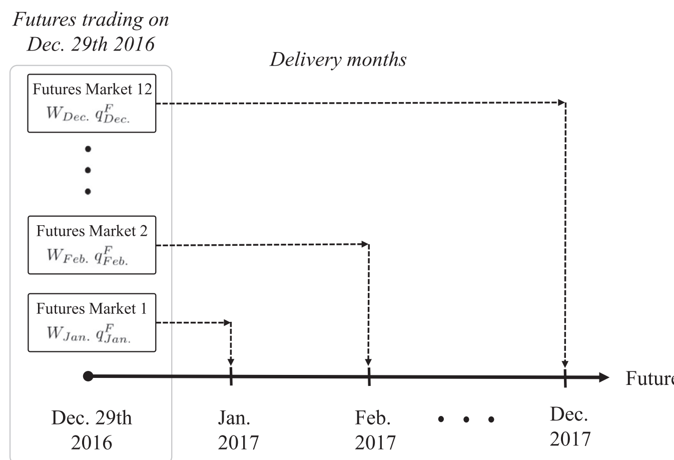


Fig. 2. Futures markets setting for Dec. 29st, 2017.

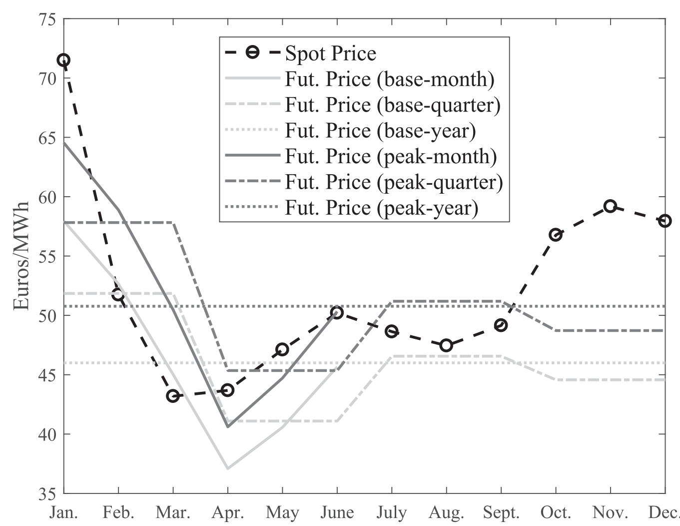


Fig. 3. Prices settled on Dec. 29, 2016, for futures base and peak electricity products for the year 2017 (OMIP, 2020).

Moreover, we model the relationship between the final spot price in the day-ahead market, after all uncertainty has been resolved, and the futures market prices, by using backward induction. Uncertainty evolves as described by Assumption 3: decreases as we get closer to the spot market realization. Given the very large amount of data collected daily, and the very large investment in the data analytics performed by generation and retail firms, this assumption is very realistic, and becomes more so with every passing year.

Fig. 2 illustrates the different futures markets that are traded within the current framework. On the last working day of December the generators and retailers need to plan their futures and spot trading throughout the year to maximize their risk-adjusted profits, given the seasonal effects and the uncertainty associated with each scenario (which depends on the time horizon).
For each of these monthly contracts, we solve our equilibrium model to recreate the negotiation process between generators and retailers. This results in a futures price W_m and in a set of equilibrium energy trades in the futures markets $q^F{}_m$ to be delivered in month $m =$ (Jan., Feb.,..., Dec.). In this process, each market participant decides the optimal level of futures trading by anticipating the scenarios for the demand and supply parameters for month m, and therefore the respective expected prices and production in the spot market.

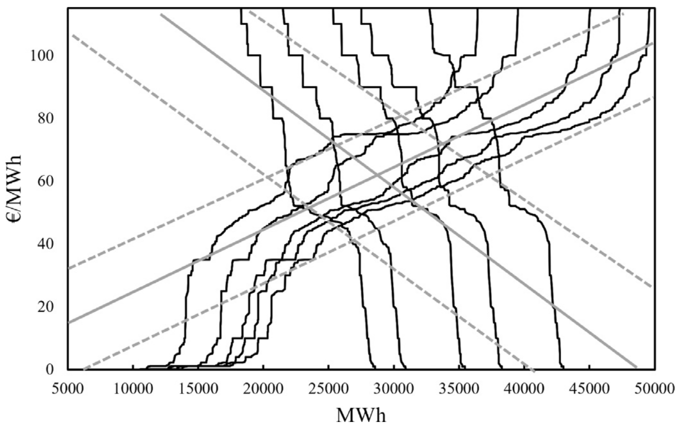


Fig. 4. Inverse demand function and offer curves at a given time period for the Spanish spot electricity market.

In the following sections we first describe how the model parameters are generated for the different simulations, and then test the proposed market settings under different assumptions regarding the level of risk aversion and competitiveness of the market participants. Hence, the main purpose of these simulations is not to reproduce observed outcomes, but rather to show how the relationship between forward and spot markets is highly conditioned by the competitive and risk aversion levels of the market participants.

4.1. Data

The Spanish electricity market has an oligopolistic structure that includes three generators and four retailers. We estimate their cost and demand parameters using market data. As an illustration, Fig. 4 presents the aggregate inverse demand and offer curves for five different hours of a specific day (Nov. 28, 2017) in the Spanish spot market.

By analyzing a larger dataset, demand parameters ν_ω and β are adjusted so that the retailers aggregated demand in the spot market (6) approximates the actual demand observed in the electricity market (decreasing gray lines in Fig. 4). Similarly, generators' costs parameters $c_{g\ \omega}$ are set so that the resulting aggregated supply curves (9) approximate those observed in the market (increasing gray lines in Fig. 4). For this reason, at the present time, when the futures trading for the different planning horizons takes place (Dec. 29, 2016, in our case), neither the demand, market prices, nor the generating costs in any of the months in 2017 are known and need to be estimated. From these estimates, we generate scenarios for the parameters. This scenarios are characterized by an increasing variance for different time spans. Moreover, we assume the companies are perfectly rational and are able to estimate the average value of the parameters actually observed, i.e., based on the electricity market outcomes in each month of 2017. Additionally, we assume that all the participants have common knowledge of the uncertainty factors (i.e., they all use the same scenarios).

As previously indicated, when defining the cost and demand scenarios, we explicitly account for the increasing level of uncertainty associated with the contracts whose delivery months are further in the future, i.e., not only do we take into account the seasonal effects on electricity demand and resource availability, but we also generate a wider range of scenarios for all the uncertain parameters as a function of time.

Moreover, in order to better reflect demand seasonality, we correlate the expected value of its intercept, i.e, $\bar{\nu}$, with the average monthly demand in the spot electricity market during 2017, OMIE (2020). Additionally, we also take into consideration how the production from hydro power affects the generators' cost functions.

Table 2 presents the monthly expected value of the demand intersection ($\bar{\nu}$) and the generating cost for each generator ($\bar{c}_g$) for $g = 1, 2, 3$. Higher generation costs for the last months of the year can be observed as hydro production decreases during 2017 (ESIOS, 2020).

Demand uncertainty is incorporated through parameter ν and cost uncertainty through parameters c_g for $g = 1, 2, 3$. The scenarios for each of these parameters are randomly generated based on normal distributions, centered on their expected values with different coefficients of variation (CV). For example, scenarios ω for the demand parameter ν_ω are generated by Monte Carlo simulation assuming that $\nu \sim N(\bar{\nu}, \mathrm{CV} \times \bar{\nu})$ for $\omega = 1, \ldots, \Omega$. In addition, as the uncertainty increases with the time horizon, this leads to a wider dispersion of the associated scenarios. This is modeled by assuming that the monthly CV increases steadily (from 0 in January to 0.275 in December 2017) the further we are from Dec. 29, 2016, as indicated in Table 2. Note that more realistic models, accounting for stochastic processes and temporal auto-correlations, can be considered to generate costs and demand scenarios with an increasing level of uncertainty. However, this will not have a direct impact on the market equilibrium results reported in the following sections, as we consider a collection of futures markets that are cleared independently.

Finally, the demand slope is set to $\beta = 0.\ 004$ € /MWh^2. For simplicity, we also assume that the four retailers have consumers under a common fixed price of $\bar{P}_1 = \bar{P}_2 = \bar{P}_3 = \bar{P}_4 = 100$ € /MWh, where $\bar{q}_1 = 1000$, $\bar{q}_2 = 1200$, $\bar{q}_3 = 1300$, $\bar{q}_4 = 1500$ MWh, i.e., the total load required by fixed consumers is 5000 MWh. For the CV@R definition, we use an $\alpha_g = \alpha_r = 0.\ 9$ and $\Omega = 500$ equiprobable scenarios, for both generators and retailers.

4.2. Analysis of the effect of generators' and retailers' risk aversion and level of competitiveness

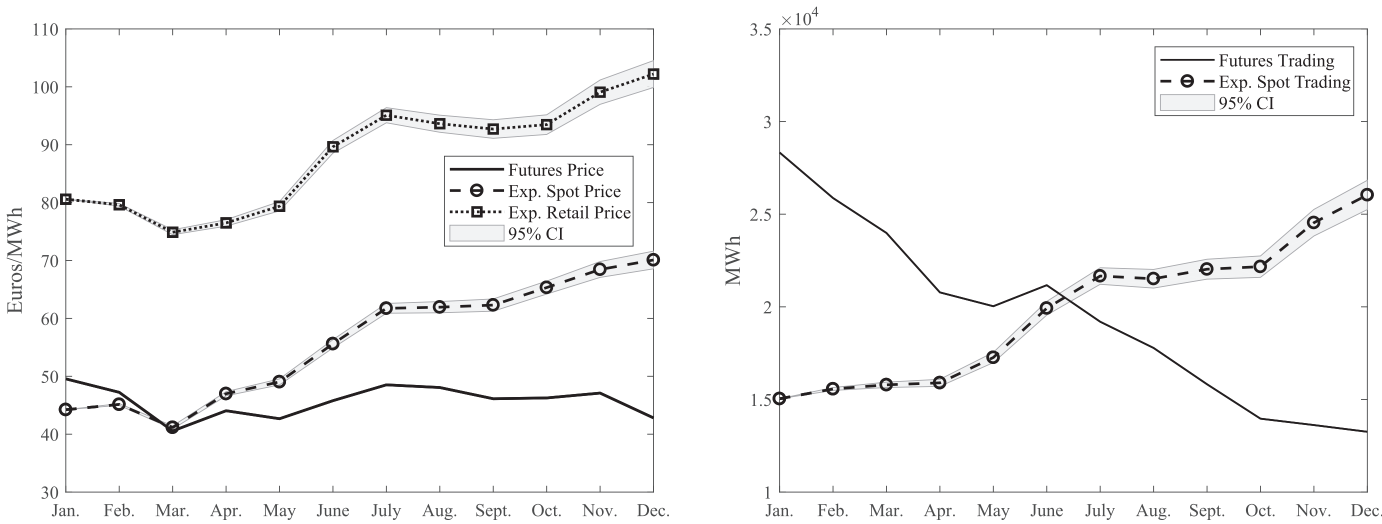
In this section we study the equilibrium properties and how they are influenced by the degree of risk aversion and level of competitiveness of generators and retailers. To this end, four market configurations are analyzed, combining risk-neutral and risk-averse attitudes and Cournot and price-taking behaviors. These configurations are particularly relevant as they reproduce common oligopolistic behaviors in real-world electricity markets. Nevertheless, the proposed model is able to reproduce any other competitive setting, from perfect competition to collusion, for both generators and retailers.

4.2.1. Cournot generators and retailers in both futures and spot markets ($V^S_{\forall g} = V^S_{\forall r} = V^D_{\forall r} = 0$ and $Y^F_{\forall g} = Y^S_{\forall r} = 0$). Risk-averse generators and retailers ($\phi_g = \phi_r = 0$)

Fig. 5a presents the evolution of the monthly futures, and the expected spot and retail prices during 2017, based on Dec. 31, 2016, under this market configuration. To better compare the relationship between these prices, Fig. 5 a also includes the 95% confidence intervals for the expected spot ($\mathbb{E}[S_\omega]$) and retail ($\mathbb{E}[P_\omega]$) prices. The prices paid by consumers are always higher than the spot and futures prices. During January and February the market is in contango, as the futures price is higher than the expected spot price. However, from April to December the expected spot price increases while the futures price remains stationary, so there is normal back-wardation (futures price lower than the expected spot price). This can be explained by noting that, as the cost and demand uncertainty increase with time, both generators and retailers decide to displace the expected trading from the futures to the spot market (the generators motivated by potential high cost and the retailers by potential low demand in the future). This is depicted in Fig. 5b.

Table 2
Expected value for demand and cost parameters (€ /MWh) and coefficient of variation (CV).

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
$\bar{\nu}$	217.61	207.83	198.49	183.44	190.07	211.33	211.58	205.80	197.21	190.56	203.99	209.91
$\bar{c}_1$	15.20	15.14	11.18	16.25	15.93	17.41	20.15	20.87	19.72	22.25	21.46	20.43
$\bar{c}_2$	16.89	16.83	12.42	18.06	17.70	19.34	22.39	23.19	21.91	24.72	23.85	22.70
$\bar{c}_3$	18.58	18.51	13.66	19.86	19.47	21.28	24.63	25.51	24.10	27.20	26.23	24.97
CV	0	0.025	0.050	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250	0.275



(a) Prices. (b) Total Traded Quantities.

Fig. 5. Cournot firms - risk aversion.

Table 3
Monthly profits and CV@R (000 €) for Cournot firms.

(a) Risk-averse generators and retailers.												
Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
E. Profit Generators	1339	1231	1134	1011	1064	1317	1408	1355	1334	1276	1540	1652
E. Profit Retailers	1302	1263	1293	1074	1166	1412	1380	1282	1214	1100	1232	1371
CV@R Generators	1339	1144	974	803	753	855	778	695	625	507	537	457
CV@R Retailers	1302	1116	1031	787	752	807	683	613	557	457	464	456
E. Consumers Utility	2781	2483	2331	1832	1878	2257	2126	1933	1758	1525	1730	1852
(b) Risk-neutral generators and retailers.												
Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
E. Profit Generators	1339	1235	1143	993	1079	1293	1323	1275	1194	1108	1295	1415
E. Profit Retailers	1302	1207	1193	972	1047	1205	1174	1117	1065	946	1098	1197
E. Consumers Utility	2781	2547	2442	1974	2154	2568	2528	2391	2236	1968	2332	2546

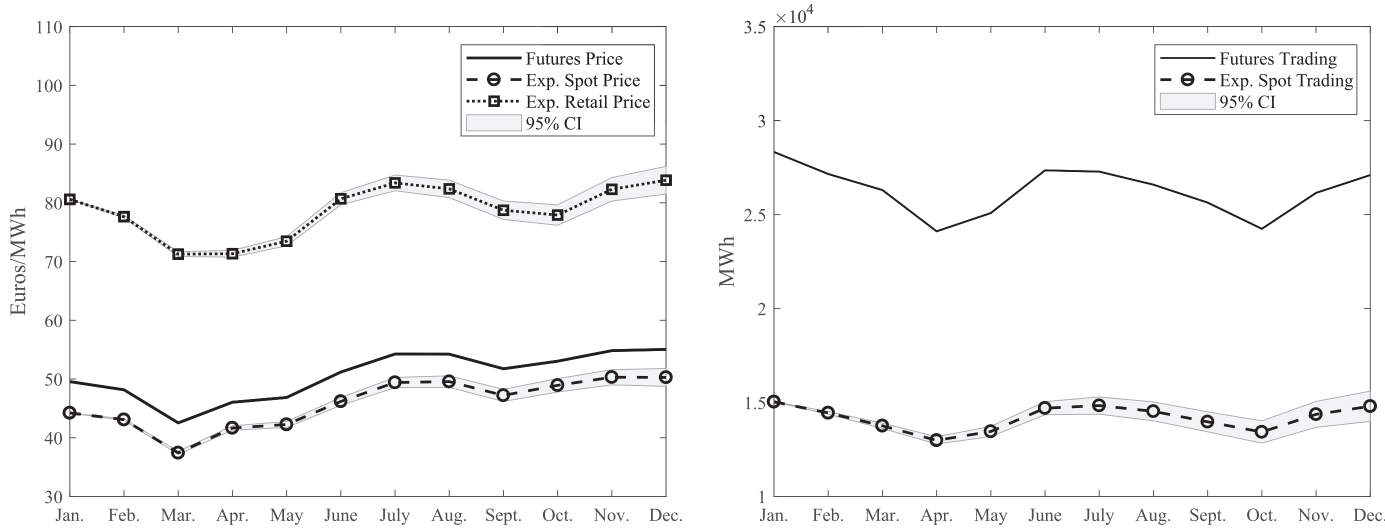
The monthly expected profits and CV@R for generators and retailers are summarized in Table 3 a. Although the expected profit fluctuates following the monthly trend of the market prices, the CV@R for both generators and retailers decreases as we move forward in time. This indicates that generators and retailers face profit distributions with heavier left tails for the last months of the year (with a higher probability of low profits).

4.2.2. Cournot generators and retailers in both futures and spot markets ($V^S_{\forall g} = V^S_{\forall r} = V^D_{\forall r} = 0$ and $Y^F_{\forall g} = Y^S_{\forall r} = 0$). Risk-neutral generators and retailers ($\phi_g = \phi_r = 1$)

We keep the Cournot setting from the previous section but now assume risk-neutral generators and retailers. The equilibrium market prices are presented in Fig. 6a.

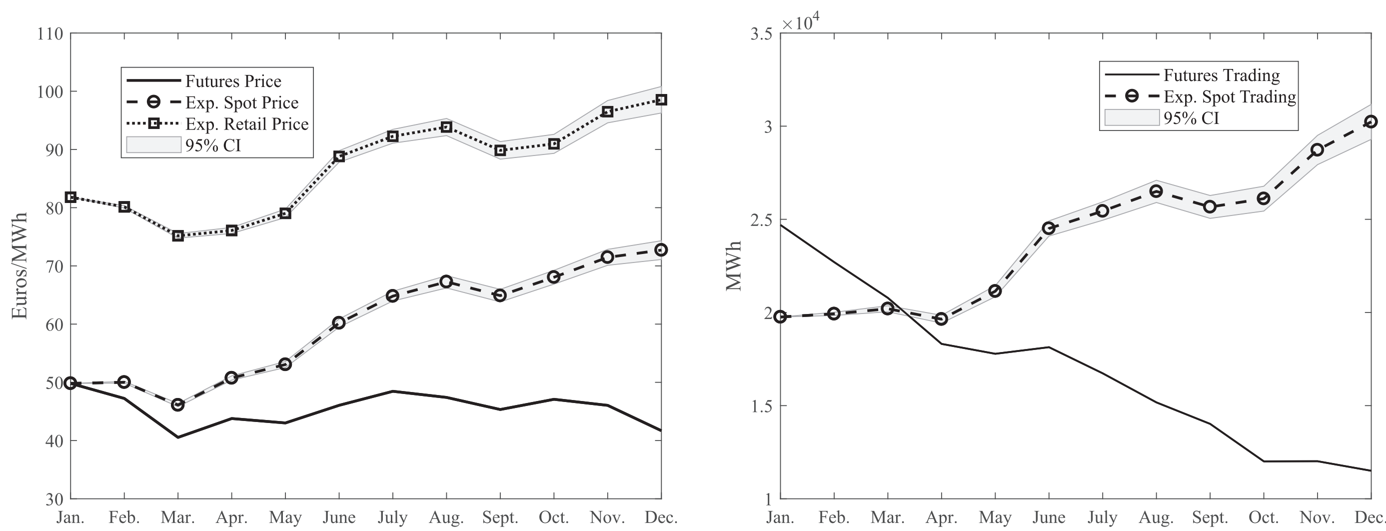
The market is always in contango as the futures prices are higher than the expected spot price. Compared to the risk-averse case, futures trading does not decrease with the time horizon (Fig. 6b), as generators and retailers do not protect themselves from the worst cost or demand scenarios (they focus on maximizing their expected profit). Futures trading maintains high energy volumes, and hence, expected spot and retail prices are lower than in the risk-averse case (Fig. 5 a).

Due to lower market prices and a lower margin between retail and futures prices, the expected profits for generators and retailers are, in general, lower than for the risk-averse case (Table 3 b). This is a counter-intuitive result. Our prior belief was for the risk-neutral players to be the most profitable. This is an instance of the “prisoner’s dilemma” from game theory. A priori, we would think, that for risk-neutral players it would be more profitable to trade more energy in the spot market. However, the individual incentives to do so are not that high (compared to a risk-averse case where players seek protection against demand and cost uncertainty) and a unilateral reduction of the futures trading may entail a significant decrease in the players’ market shares. Nevertheless, this is a favorable situation for the final consumers, increasing their expected utility through lower retail prices (last row in Table 3 b).



(a) Prices. (b) Total Traded Quantities.

Fig. 6. Cournot firms - risk neutrality.



(a) Prices. (b) Total Traded Quantities.

Fig. 7. Cournot generators, price-taking retailers - Risk aversion.

4.2.3. Cournot generators and price-taking retailers in both futures and spot markets ($V^S_{\forall g} = V^D_{\forall r} = Y^F_{\forall g} = 0$ and $V^S_{\forall r=-1} = Y^S_{\forall r} = -1$). Risk-averse generators and retailers ($\phi_g = \phi_r = 0$)

This configuration is similar to the market model in Section 4.2.1, but assumes that retailers behave as price takers (and not Cournot) in the futures and spot markets. This is an electricity market where the energy generation is controlled by a few dominant firms, while the retailing activity is open to a larger number of companies of relatively small size (with no individual market power).

When uncertainty is very small, for one month time horizon (with delivery time in January), the futures price equals the spot price (Fig. 7 a). However, as we move forward in time and cost and demand uncertainty increases, the market enters normal backwardation (futures price lower than expected spot price). Similar to Section 4.2.1, this effect is caused by a decrease in the futures trading level (Fig. 7b) as risk-averse generators and retailers prefer to postpone their sales and purchases until the elimination of cost and demand uncertainties, respectively. Compared to the Cournot retailers case (Section 4.2.1), market prices are similar with the exception that for the first months of the year, the expected spot prices are higher than the futures prices. Similarly, expected spot trading is also higher during the first months of the year than for the Cournot retailers case.

Again, compared to the Cournot case in Section 4.2.1, the price taker behavior of retailers entails a decrease for both their expected profit and CV@R (Table 4 a). Generators benefit from this and increase both their profit and CV@R. It is therefore evident from these simulations that firms' risk aversion is the major factor explaining contango markets, and is exacerbated by the retailers' market power.

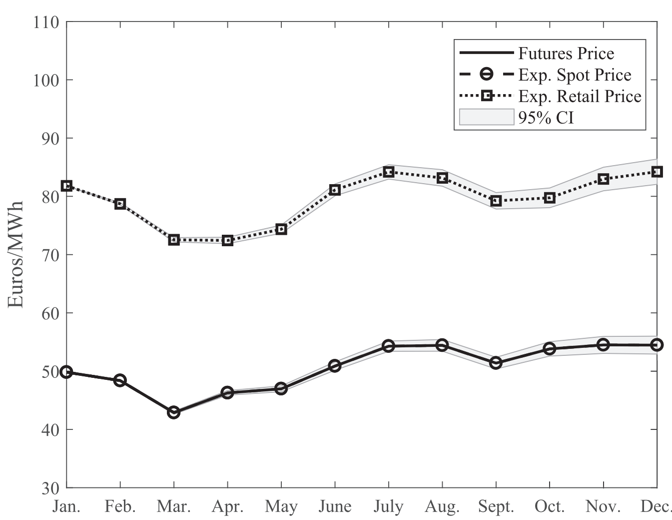
4.2.4. Cournot generators and price-taking retailers in both futures and spot markets ($V^S_{\forall g} = V^D_{\forall r} = Y^F_{\forall g} = 0$ and $V^S_{\forall r=-1} = Y^S_{\forall r} = -1$). risk-neutral generators and retailers ($\phi_g = \phi_r = 1$)

In this case we assume that both Cournot generators and price-taking retailers are risk-neutral in the futures market. Market prices are presented in Fig. 8a. Futures prices equal expected spot

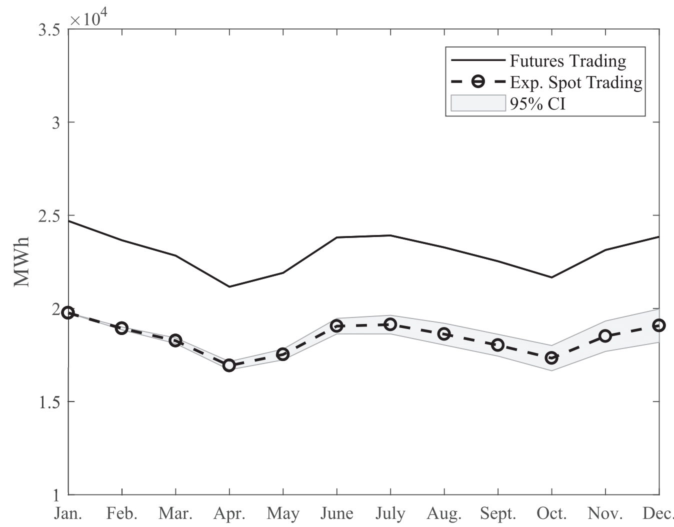
Table 4
Monthly profits and CV@R (000 €) for Cournot generators and price-taking retailers.

(a) Risk-averse generators and retailers.												
Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
E. Profit Generators	1467	1357	1272	1128	1215	1525	1573	1622	1510	1510	1784	1942
E. Profit Retailers	1274	1221	1232	1019	1094	1287	1222	1198	1101	970	1121	1227
CV@R Generators	1467	1246	1063	865	826	908	830	754	635	551	547	436
CV@R Retailers	1274	1096	1006	773	743	768	656	594	532	437	447	430
E. Consumers Utility	2736	2469	2325	1848	1931	2285	2143	2048	1851	1626	1896	2033

(b) Risk-neutral generators and retailers.												
Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
E. Profit Generators	1467	1347	1258	1088	1173	1402	1431	1379	1297	1231	1434	1542
E. Profit Retailers	1274	1178	1167	956	1022	1172	1141	1079	1045	937	1087	1173
E. Consumers Utility	2736	2497	2407	1948	2111	2508	2474	2327	2211	1981	2339	2536



(a) Prices.



(b) Total Trading Quantities.

Fig. 8. Cournot generators, Price-Taking Retailers - Risk neutrality.

prices for all months of the year. Generators exercise their market power over the retailers and establish the best selling price for their energy, which, due to their risk neutrality, is the same for both futures and spot markets. Similar to the Cournot case presented in Section 4.2.2, futures trading is always higher than spot trading (Fig. 8b), although the gap is smaller.

The expected profit and CV@R are presented in Table 4. The retail and generation profits are smaller in the risk-neutral case than under risk aversion. This is again a counter-intuitive result where risk-neutral players achieve lower expected profits than risk-averse firms.

Moreover, a third surprising result is that the expected consumer utility is higher in the scenarios in which retailers behave a la Cournot in the wholesale markets, and the largest when the firms are risk-neutral. This shows that double marginalization is actually worse when the retailers do not have market power in the futures and spot markets, as they need to pass the higher wholesale prices onto the consumers, which they are able to do as they behave as Cournot players in the retail market.

The numerical results reported above have been complemented with an extensive set of simulations, including different combinations of cost and demand parameters, and intermediate levels of competition and risk aversion. We conclude that the main market trends observed within Section 4.2 are general and robust against variations of the market data. Specifically, normal backwardation emerges as the dominant state of the futures market principally due to risk aversion (and mostly facilitated by the retailers' market power), whereas the contango is harder to observe and is only dominant when all firms are risk-neutral and the retailers' have market power in the wholesale markets. Normal backwardation was indeed observed for the last months of 2017 in the Spanish Market (OMIP, 2020), as shown by Fig. 3, which according to our model, may be explained by the presence of risk aversion.

5. Conclusions

In this article we have analyzed the relationship between futures and spot markets in the electricity supply chain, taking into consideration the interaction between risk aversion (using CV@R) and the market power held by multiple electricity generators and retailers. We have developed a model of the electricity supply chain that considers uncertainty in demand and generation costs, proposing a dynamic game to analyze the interaction between generators (who sell in futures and spot markets) and retailers (who buy from the generators in the wholesale markets and sell to the final consumers).

As a major methodological contribution to the study of electricity and futures markets, we derive the Nash equilibrium of the relationship between futures and spot markets in the context of the electricity supply chain, using CV@R as the risk measure and considering conjectural variations. We show that the futures price arises from the optimality conditions without requiring any assumptions regarding arbitrage between futures and spot prices and that, for the risk-neutral case, it is unique and can be derived analytically. Additionally, we use the concept of risk-adjusted expectation and consistent futures, spot and wholesale price derivatives to calculate the players' reaction functions. The resulting equilibrium enables the identification of the conditions in which firms pay or receive a premium to trade in the futures market.

The main theoretical insight of this article is the analysis of the relationship between spot and futures prices, explaining why the futures market is in normal backwardation or in contango, providing the first explanation for the inversion of the forward curve based on risk attitudes.

As a policy contribution, we have studied the Spanish electricity market, explaining how the interaction between market power and risk aversion impacts futures markets. We observed that normal backwardation appears under risk aversion when the generators behave a la Cournot, independently of the retailers' degree of market power. On the other hand, contango is only dominant when all the players are risk-neutral and the retailers (together with the generators) behave a la Cournot in the wholesale markets. Counter-intuitively our simulations illustrate that when all firms are risk-averse the profit increases and risk decreases. Moreover, consumer utility is higher if the retailers behave a la Cournot in the wholesale market.

There are several possible applications and extensions to the model and methodology proposed in this article. First, the profit equations of some of the firms could be modified to consider vertical integration, under which part of the production is traded between the generation and retail branches of the firm and any production margins are internalized. Second, the model could be used to study the impact of real-time metering on the optimal trading strategies of the firms. Third, the market setting can be modified to analyze the direct participation of large consumers in the wholesale markets. Fourth, this methodology is general enough to be applied to other types of supply chains, such as petroleum and gas, by including storage. Finally, the model provides the equilibrium solution for a very complex problem that incorporates the relationship between futures and spot markets under risk aversion, which can then be used as a test bed for empirical studies on the ability of people to deal with inter-temporal decision making and risk aversion.

Considering this, parameters $\mathcal{A}_\omega$, $\mathcal{B}$, $\mathcal{F}_\omega$, $\mathcal{G}$, $\mathcal{J}_\omega$, $\mathcal{K}$, $\mathcal{D}_{r\omega}$, $\mathcal{E}$, $\mathcal{H}$, $\mathcal{Q}_{g\omega}$ and $\mathcal{R}$ are computed as follows.

$$\mathcal{A}_\omega = \frac{R\sum_{g=1}^{G} c_{g\omega} + \left(1+V^S_{\forall g}\right)\left[R\nu_\omega + \beta\left(1+R+V^D_{\forall r}\right)\sum_{r=1}^{R}\bar{q}_r\right]}{1+GR+V^S_{\forall_g}} \quad \forall\omega \tag{39}$$

$$\mathcal{B} = -\frac{\beta\left(1+V^S_{\forall g}\right)\left(1+R+V^D_{\forall r}\right)}{1+GR+V^S_{\forall_g}} \tag{40}$$

$$\mathcal{F}_\omega = \frac{\left[R(\mathcal{A}_\omega-\nu_\omega)-\beta\left(1+R+V^D_{\forall r}\right)\sum_{r=1}^{R}\bar{q}_r\right]\left[\left(1+V^S_{\forall g}\right)\left(1+V^S_{\forall r}\right)-GR\right]}{GR\left(1+R+V^D_{\forall r}\right)} \quad \forall\omega \tag{41}$$

$$\mathcal{G} = \frac{\left[\left(1+V^S_{\forall g}\right)\left(1+V^S_{\forall r}\right)-GR\right]\left[\left(1+V^S_{\forall g}\right)(1-R)+GR\right]}{GR\left(1+GR+V^S_{\forall_g}\right)} \tag{42}$$

$$\mathcal{J}_\omega = \nu_\omega - \beta\mathcal{F}_\omega + \beta\sum_{r=1}^{R}\bar{q}_r \qquad \forall\omega \tag{43}$$

$$\mathcal{K} = -\beta\left[\frac{\left[\left(1+V^S_{\forall g}\right)\left(1+V^S_{\forall r}\right)-GR\right]\left[\left(1+V^S_{\forall g}\right)(1-R)+GR\right]}{GR\left(1+GR+V^S_{\forall_g}\right)}+1\right] \tag{44}$$

$$\mathcal{D}_{r\omega} = \frac{\left[-\beta\left(1+V^S_{\forall r}\right)\bar{q}_r-\mathcal{J}_\omega+\mathcal{A}_\omega\right]\left[\left(1+V^S_{\forall g}\right)\left(1+V^S_{\forall r}\right)-GR\right]}{R\beta\left[\left(1+V^S_{\forall g}\right)\left(1+V^S_{\forall r}\right)+G\left(1+V^D_{\forall r}\right)\right]} \quad \forall r,\forall\omega \tag{45}$$

$$\mathcal{E} = \frac{\left(1+V^D_{\forall r}\right)\left[\left(1+V^S_{\forall g}\right)\left(1+V^S_{\forall r}\right)-GR\right]}{R\left[\left(1+V^S_{\forall g}\right)\left(1+V^S_{\forall r}\right)+G\left(1+V^D_{\forall r}\right)\right]} \tag{46}$$

$$\mathcal{H} = \frac{(R-1)\left(1+V^D_{\forall r}\right)\left[\left(1+V^S_{\forall g}\right)\left(1+V^S_{\forall r}\right)-GR\right]\left[\left(1+V^S_{\forall g}\right)\left(1+V^S_{\forall r}\right)-GR\left(V^D_{\forall r}-V^S_{\forall r}\right)\right]}{R^2G\left(1+V^S_{\forall g}+GR\right)\left[\left(1+V^S_{\forall g}\right)\left(1+V^S_{\forall r}\right)+G\left(1+V^D_{\forall r}\right)\right]} \tag{47}$$

$$\mathcal{Q}_{g\omega} = \frac{(\mathcal{A}_\omega-c_{g\omega})\left[GR-(1+V^S_{\forall r})(1+V^S_{\forall g})\right]}{G\beta(1+V^S_{\forall g})(1+V^D_{\forall r}+R)} \qquad \forall g,\forall\omega \tag{48}$$

$$\mathcal{R} = \frac{\left(1+V^S_{\forall r}\right)\left(1+V^S_{\forall g}\right)-GR}{G\left(1+GR+V^S_{\forall g}\right)} \tag{49}$$

Appendix A. Derivation of the spot market outcomes (17)–(21)

- The linear expression for S_ω in (17), together with the definition of parameters $\mathcal{A}_\omega$ and $\mathcal{B}$ in (39) and (40), respectively, is directly obtained by rearranging the terms in (11).
- Similarly the expression for the total spot trading $\sum_{r=1}^{R} q^S_{r\omega} = \sum_{g=1}^{G} q^S_{g\omega}$ in (18), and the definition of parameters $\mathcal{F}_\omega$ and $\mathcal{G}$ in (41) and (42), respectively, are derived by replacing (17) with expression (5) and by solving for $\sum_{r=1}^{R} q^S_{r\omega}$ or $\sum_{g=1}^{G} q^S_{g\omega}$.
- The formulation of the retail price P_ω in (19), and the arameters $\mathcal{J}_\omega$ and $\mathcal{K}$ are obtained from Eq. (3) by replacing $\sum_{r=1}^{R} q^S_{r\omega}$ with expression (18).
- In order to derive $q^S_{r\omega}$ in (20), we replace S_ω by (17) and P_ω by (19) in (4), which renders (20) with the corresponding values for $\mathcal{D}_{r\omega}$, $\mathcal{E}$ and $\mathcal{H}$ indicated in (45), (46), (47), respectively.
- In equilibrium the generators' sales in the spot market, $q^S_{g\omega}$, and the respective $\mathcal{Q}_{g\omega}$ and $\mathcal{R}$, are derived from Eq. (7) by replacing S_ω by (17).

Appendix B. Concavity characterization and first-order optimality conditions for problems (22) and (23)

This appendix is devoted to mathematically characterizing the properties of the optimization problems (22) and (23). Within the following propositions, we show that the KKT conditions (24) and (25) are necessary and sufficient for the optimality of the two market configurations considered in the case study:

a) Both generators and retailers behave as Cournot players in the futures and spot markets, i.e., $V^D_{\forall r} = V^S_{\forall r} = V^S_{\forall g} = Y^F_{\forall r} = Y^F_{\forall g} = 0$.

b) Generators behave as Cournot and retailers behave as price-taker players in the futures and spot markets, i.e., $V^D_{\forall r} = V^S_{\forall g} = Y^F_{\forall g} = 0$ and $V^S_{\forall r} = Y^F_{\forall r} = -1$.

Proposition 9. *For the market configurations (a) and (b) above, in equilibrium, generator's g profit $\Pi_{g\,\omega}$ is a concave function of q_g^F and the retailer's r profit $\Pi_{r\omega}$ is a concave function of q_r^F.*

Proof. From (D.1) and (D.2) we can compute the second order derivatives:

$$\frac{\partial^2 \Pi_{r\omega}}{\partial {q_r^F}^2} = -2\frac{\partial W}{\partial q_r^F} + \mathcal{U}\left(1 + Y_{\forall r}^F\right) \tag{50}$$

$$\frac{\partial^2 \Pi_{g\omega}}{\partial {q_g^F}^2} = 2\frac{\partial W}{\partial q_g^F} + \mathcal{X}\left(1 + Y_{\forall g}^F\right) \tag{51}$$

By replacing $\frac{\partial W}{\partial q_r^F}$ and $\frac{\partial W}{\partial q_g^F}$ with (33) and (34), respectively, together with (35)–(37) and their associated inner dependencies (39)–(49), we derive the following second order derivatives.

Market configuration (a):

$$\frac{\partial^2 \Pi_{r\omega}}{\partial {q_r^F}^2} = -2\beta\left[\frac{R^6G^3(G^2-1) + GR^4(4G^2-3) + GR^2(6R-5) + R^2(2G^2R^3 + G^2R^2 + 3G^2R - 5) + \; + G^5R^5 + G^4R^5 + G^5R^4 + G^3R^3 + 2G^3R^5 + 2GR + 2R^3 + (4R-1)}{G^2R^3(GR-1)(GR+1)^2(G+1)}\right] \tag{52}$$

$$\frac{\partial^2 \Pi_{g\omega}}{\partial {q_g^F}^2} = -2\beta\left[\frac{G^2R^5(G^2-1) + G^2R^4(2G^2-1) + R^3(3G^2-1) + 6R(4R-1) + 4R(G^2R-1) + + G^4R^3 + G^3R^3 + G^3R^2 + G^2R + 2GR^4 + G + R^2 + 2}{GR^2(GR-1)(GR+1)^2(G+1)}\right] \tag{53}$$

Market configuration (b):

$$\frac{\partial^2 \Pi_{r\omega}}{\partial {q_r^F}^2} = 0 \tag{54}$$

$$\frac{\partial^2 \Pi_{g\omega}}{\partial {q_g^F}^2} = -2\beta\left[\frac{(R+1)(R(G-1)+1)}{(GR+1)^2}\right] \tag{55}$$

Note that $G \geq 1$ and $R \geq 1$ as they represent the number of generators and retailers in the market, respectively. Hence, it is easy to see that all the terms in the numerators and denominators of expressions (52)–(55) are always positive, which entails the concavity
of the profit functions as $\frac{\partial^2 \Pi_{r\omega}}{\partial {q_r^F}^2} \leq 0$ and $\frac{\partial^2 \Pi_{g\omega}}{\partial {q_g^F}^2} \leq 0$, for both market configurations (parameter β is always assumed positive in our model). □

Finally, we study the optimization problems (22) and (23) and their connection with the complementarity systems (24) and (25).

Proposition 10. *For the market configurations (a) and (b) above, optimization problems (22) and (23) are concave.*

Proof. As shown in Rockafellar and Uryasev (2000), the problem formulations (22) and (23) are linear in the variables ξ_g, $\eta_{g\omega}$ and $\Pi_{g\omega}$, and ξ_r, $\eta_{r\omega}$ and $\Pi_{r\omega}$, respectively. Moreover, $\Pi_{g\omega}$ and $\Pi_{r\omega}$ are present in both the objective functions and inequality constraints of problems (22) and (23). Thus, by Proposition 9 we conclude that both the objective and constraints of (22) and (23) are concave functions. Consequently, the optimization problems are concave. □

Corollary 1. *For the market configurations (a) and (b) above, the complementarity systems (24) and (25) are necessary and sufficient conditions for the optimality of problems (22) and (23), respectively.*

Proof. Problems (22) and (23) are concave maximization problems (Proposition 10) and both the objective functions and constraints are continuously differentiable functions, which ensures that the Karush–Kuhn–Tucker necessary conditions (24) and (25) are also sufficient conditions for optimality. □

Appendix C. Nonlinear formulation of the KKT system

Consider the following nonlinear optimization problem:

$$\begin{aligned}\underset{\substack{W, q_g^F, q_r^F, \xi_g, \eta_{g\omega}, \delta_{g\omega}, \lambda_{g\omega} \\ \delta_{g\omega}, \xi_r, \eta_{r\omega}, \delta_{r\omega}, \lambda_{r\omega}, \delta_{r\omega}}}{Minimize} \quad & FO = \sum_{g=1}^{G}\sum_{\omega=1}^{\Omega}\left(\Pi_{g\omega} - \xi_g + \eta_{g\omega}\right)\lambda_{g\omega} \\ & + \sum_{r=1}^{R}\sum_{\omega=1}^{\Omega}(\Pi_{r\omega} - \xi_r + \eta_{r\omega})\lambda_{g\omega} \\ & + \sum_{g=1}^{G}\sum_{\omega=1}^{\Omega}\eta_{g\omega}\delta_{g\omega} + \sum_{r=1}^{R}\sum_{\omega=1}^{\Omega}\eta_{r\omega}\delta_{r\omega}\end{aligned} \tag{57a}$$

subject to:

$$-\phi_g\sum_{\omega=1}^{\Omega}\sigma_{g\omega}\frac{\partial \Pi_{g\omega}}{\partial q_g^F} - \sum_{\omega=1}^{\Omega}\lambda_{g\omega}\frac{\partial \Pi_{g\omega}}{\partial q_g^F} = 0 \qquad \forall g \tag{57b}$$

$$(1-\phi_g)\frac{1}{1-\alpha_g}\sigma_{g\omega} - \lambda_{g\omega} - \delta_{g\omega} = 0 \qquad \forall g, \forall \omega \tag{57c}$$

$$\sum_{\omega=1}^{\Omega}\lambda_{g\omega} = (1-\phi_g) \qquad \forall g \tag{57d}$$

$$\Pi_{g\omega} - \xi_g + \eta_{g\omega} \geq 0 \qquad \forall g, \forall \omega \tag{57e}$$

$$-\phi_r\sum_{\omega=1}^{\Omega}\sigma_{r\omega}\frac{\partial \Pi_{r\omega}}{\partial q_r^F} - \sum_{\omega=1}^{\Omega}\lambda_{r\omega}\frac{\partial \Pi_{r\omega}}{\partial q_r^F} = 0 \qquad \forall r \tag{57f}$$

$$(1-\phi_r)\frac{1}{1-\alpha_r}\sigma_{r\omega} - \lambda_{r\omega} - \delta_{r\omega} = 0 \qquad \forall r, \forall \omega \tag{57g}$$

$$\sum_{\omega=1}^{\Omega}\lambda_{r\omega} = (1-\phi_r) \qquad \forall r \tag{57h}$$

$$\Pi_{r\omega} - \xi_r + \eta_{r\omega} \geq 0 \qquad \forall r, \forall \omega \tag{57i}$$

$$\eta_{g\omega}, \lambda_{g\omega}, \delta_{g\omega}, \eta_{r\omega}, \lambda_{r\omega}, \delta_{r\omega} \geq 0 \qquad \forall g, \forall r, \forall \omega \tag{57j}$$

$$\sum_{g=1}^{G} q_g^F = \sum_{r=1}^{R} q_r^F \tag{57k}$$

Under the optimization problem (57) Proposition 11 holds.

Proposition 11. *A solution to the NLP problem (57) that meets FO = 0 is also a solution to the complementarity system (24) and (25).*

Proof. The proof is straightforward noting that conditions (57b)–(57j) are almost the same as conditions (24) and (25), but without including the products of the complementarity constraints (24d), (24e), (25d) and (25e). However, the objective function (57a) is the sum of these positive terms so that if a solution to (57) meets $FO = 0$, it then implies that every single term equals zero, which is equivalent to enforcing the complementarity conditions (24d), (24e), (25d) and (25e). □

Appendix D. Proof for Proposition 7

Proof. First we compute explicit formulations of the partial derivatives $\frac{\partial \Pi_{r\omega}}{\partial q_r^F}$ and $\frac{\partial \Pi_{g\omega}}{\partial q_g^F}$ as a function of the futures trading q_r^F and q_g^F. To this end, we replace the spot market equilibrium outcomes (17)–(21), together with the spot partial derivatives (28)–(32), in (27a) and (27b). After some term rearrangements we obtain the following linear expressions:

$$\frac{\partial \Pi_{r\omega}}{\partial q_r^F} = -W + \mathcal{S}_{r\omega} + q_r^F\left(-\frac{\partial W}{\partial q_r^F} + \mathcal{T}\right) + \mathcal{U}q^F \quad (1)$$

$$\frac{\partial \Pi_{g\omega}}{\partial q_g^F} = W + q_g^F\frac{\partial W}{\partial q_g^F} + \mathcal{V}_{g\omega} + \mathcal{X}q^F \quad (2)$$

where parameters $\mathcal{S}_{r\omega}$, $\mathcal{T}$, $\mathcal{U}_{r\omega}$, $\mathcal{V}_{g\omega}$ and $\mathcal{X}$ are defined as follows:

$$\mathcal{S}_{r\omega} = \mathcal{K}\left(1 + Y_{\forall r}^F\right)(\mathcal{D}_{r\omega} - \bar{q}_r) + \left[\mathcal{H}\left(1 + Y_{\forall r}^F\right) + \mathcal{E} + 1\right]\mathcal{J}_\omega - \left[\mathcal{H}\left(1 + Y_{\forall r}^F\right) + \mathcal{E}\right]\mathcal{A}_\omega - \mathcal{B}\left(1 + Y_{\forall r}^F\right)\mathcal{D}_{r\omega} \qquad \forall r, \forall \omega$$

$$\mathcal{T} = \left(1 + Y_{\forall r}^F\right)[\mathcal{K}(\mathcal{E}+1) - \mathcal{B}\mathcal{E}]$$

$$\mathcal{U} = 2\left(1 + Y_{\forall r}^F\right)\mathcal{H}(\mathcal{K} - \mathcal{B}) + \mathcal{K}(\mathcal{E}+1) - \mathcal{B}\mathcal{E}$$

$$\mathcal{V}_{g\omega} = \left(1 + Y_{\forall g}^F\right)(\mathcal{R}\mathcal{A}_\omega + \mathcal{B}\mathcal{Q}_{g\omega} - c_{g\omega}\mathcal{R}) - c_{g\omega} \qquad \forall g, \forall \omega$$

$$\mathcal{X} = 2\mathcal{R}\mathcal{B}\left(1 + Y_{\forall g}^F\right)$$

Section 3.2.1 shows that the risk-averse equilibrium for the futures market is equivalent to solving the system of equations $\mathbb{E}_r\left[\frac{\partial \Pi_{r\omega}}{\partial q_r^F}\right] = 0$ and $\mathbb{E}_g\left[\frac{\partial \Pi_{g\omega}}{\partial q_g^F}\right] = 0$, $\forall r$, $\forall g$, where $\mathbb{E}_r$ and $\mathbb{E}_g$ represent risk-adjusted expectations. In this setting, the probability associated with each scenario is $\sigma_{r\omega}^* = \phi_r\, \sigma_{r\omega} + \lambda_{r\omega}$ and $\sigma_{g\omega}^* = \phi_g\, \sigma_{g\omega} + \lambda_{g\omega}$, respectively. By imposing these equilibrium conditions to the partial derivatives (D.1) and (D.2), and realizing that some terms are scenario independent, we obtain expressions (D.3) and (D.4).

$$-W + \sum_{\omega=1}^{\Omega}\sigma_{r\omega}^*\mathcal{S}_{r\omega} + q_r^F\left(-\frac{\partial W}{\partial q_r^F} + \mathcal{T}\right) + \mathcal{U}\sum_{r=1}^{R}q_r^F = 0 \qquad \forall r \quad (3)$$

$$W + q_g^F\frac{\partial W}{\partial q_g^F} + \sum_{\omega=1}^{\Omega}\sigma_{g\omega}^*\mathcal{V}_{g\omega} + \mathcal{X}\sum_{g=1}^{G}q_g^F = 0 \qquad \forall g \quad (4)$$

By using expressions (D.3) and (D.4), and aggregating all the energy bought $(\sum_{r=1}^{R} q_r^F)$ and sold $(\sum_{g=1}^{G} q_g^F)$, we can derive the inverse aggregate demand for retailers (D.5) and the aggregate supply curve for generators (D.6) in the futures market.

$$W = \frac{1}{R}\left[\left(-\frac{\partial W}{\partial q_r^F} + \mathcal{T} + R\mathcal{U}\right)\sum_{r=1}^{R}q_r^F + \sum_{r=1}^{R}\sum_{\omega=1}^{\Omega}\sigma_{r\omega}^*\mathcal{S}_{r\omega}\right] \quad (5)$$

$$W = \frac{1}{G}\left[-\left(\frac{\partial W}{\partial q_g^F} + G\mathcal{X}\right)\sum_{g=1}^{G}q_g^F - \sum_{g=1}^{G}\sum_{\omega=1}^{\Omega}\sigma_{g\omega}^*\mathcal{V}_{g\omega}\right] \quad (6)$$

Therefore, the generators face the inverse demand curve (D.5) which, considering that supply equals demand in the futures market (26), i.e., $q^F = \sum_{r=1}^{R} q_r^F = \sum_{g=1}^{G} q_g^F$, can be used to evaluate the impact of their sales q_g^F in the futures price W, as represented by (D.7).

$$\frac{\partial W}{\partial q_g^F} = \frac{1}{R}\left(-\frac{\partial W}{\partial q_r^F} + \mathcal{T} + R\mathcal{U}\right)\left(1 + Y_{\forall g}^F\right) \quad (7)$$

Similarly, the retailers face the supply curve (D.6) which, noting that $q^F = \sum_{g=1}^{G} q_g^F = \sum_{r=1}^{R} q_r^F$, can be used to evaluate the impact of their purchases q_r^F in the futures price W, as summarized in (D.8).

$$\frac{\partial W}{\partial q_r^F} = -\frac{1}{G}\left(\frac{\partial W}{\partial q_g^F} + G\mathcal{X}\right)\left(1 + Y_{\forall r}^F\right) \quad (8)$$

Note that expressions (D.7) and (D.8) do not depend on the risk-adjusted probabilities $\sigma_{r\omega}^*$ or $\sigma_{g\omega}^*$. From the solution of the linear system (D.7) and (D.8), we can derive the explicit expressions for $\frac{\partial W}{\partial q_r^F}$ and $\frac{\partial W}{\partial q_g^F}$ in (33) and (34), respectively. □

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