MURPHY, F. and OLIVEIRA, F.S. 2013. Pricing option contracts on the strategic petroleum reserve. *Energy economics* [online], 40, pages 242-250. Available from: https://doi.org/10.1016/j.eneco.2013.06.016

Pricing option contracts on the strategic petroleum reserve.

MURPHY, F. and OLIVEIRA, F.S.

2013

F. Murphy and F. S. Oliveira, 2013. Pricing Option Contracts on the Strategic Petroleum Reserve. *Energy Economics*, 40: 242-250. http://authors.elsevier.com/sd/article/S0140988313001436

Pricing Option Contracts on the Strategic Petroleum Reserve

Frederic H. Murphy, Fox School of Business, Temple University

Fernando Oliveira, ESSEC Business School, Oliveira@essec.fr

Abstract: In this article we examine the pricing of option contracts on the Strategic Petroleum Reserve (SPR) and how consider these can be used by both the government and refiners. We analyze the interaction between the call and put option contracts, taking into account the underlying game, in the infinite Markov decision process with discounting, explaining the relationship between the valuation of options on the SPR by refiners and the valuation of financial options on a marker crude in financial markets. We conclude that the values of both call and put options on the SPR increase with oil prices and decrease with total inventory. Furthermore, our analysis shows that a more active management of the SPR creates higher social welfare (although refiners profit less from inventories) and larger volatility in inventory profits, decreasing private investment in petroleum stocks.

Keywords: Finance; Inventory; Markov Processes; Option Pricing; Petroleum markets; Risk Management.

JEL classifications: C6, C7, Q4, G1.

1. Introduction

The actions of refiners (and, sometimes, the security of countries) are affected by important risk factors such as oil price volatility and supply disruptions. These may be caused, for example, by poor weather conditions, accidents, political events in unstable producing countries, and terrorist attacks. Given these important risk factors, both for the country and for the operations of refiners, some governments have decided to build their own reserves of petroleum and crude products (e.g., Switzerland, USA, Japan, South Korea, Russia, Singapore, China and India) and, in some cases, governments have passed legislation imposing on refiners the obligation to keep a compulsory level of reserves as a function of petroleum imports (e.g., European Union and China). See the International Energy Agency, IEA (2007), for a description of the petroleum security policies by IEA member countries. Other countries that reportedly have a strategic petroleum reserve include Iran, Kuwait, Jordan and Israel (possibly the largest one in terms of days of reserve). Interestingly, New Zealand holds a strategic petroleum reserve (SPR) based on option contracts ("ticket reservations") with Australia, Japan, the UK, and the Netherlands, which gives the country guaranteed purchases of petroleum in the case of an emergency (IEA, 2010).

Even though the use of option contracts, as implemented by New Zealand, seems to have the benefits of avoiding the SPR management costs and the use of a diversified portfolio of potential reserves, the original debates on setting up the SPR, and the respective literature on buildup and drawdown policies for the SPR (e.g., Teisberg, 1982; Chao and Manne, 1983; Hogan, 1983; Samouilidis and Magirou, 1985; Oren and Wan, 1986; Murphy et al., 1986, 1987, 1989) did not include the possibility of option contracts as instruments for holding reserves. This omission may be because financial derivatives markets were just forming as the SPR was being implemented in the USA (see Taylor and van Doren, 2005; Hogan, 2002) and to the mindset that still views the reserves as instruments to be used only in very extreme conditions of market failure. At the time of the inception of the SPR, nonetheless, Devarajan and Hubbard (1983) recommended selling futures on the SPR during periods with a high risk of disruption to increase the certainty that the SPR would be used and to forestall private inventory builds.

More recently, the debate around the SPR has focused on the ability of the reserve to address market disruptions and private risks (e.g., Gordon, 1992; Hogan, 2002; Taylor and van Doren, 2005; Considine and Dowd, 2005; Considine, 2006; Murphy and Oliveira, 2010) and on the buildup and drawdown strategies for strategic petroleum reserves in different regions of the world. Fan and Zhang (2010) have modeled the buildup of the SPR in China and India, studying the interactions between these countries' policies, based on Murphy et al. (1987). Bai et al. (2012) have used a dynamic programming model to analyze the buildup of the SPR in China between 2008 and 2020. Wu et al. (2012) have analyzed the stockpiling of oil in China, aiming to protect the security of energy supply and taking into consideration different scenarios for disruptions.

Murphy and Oliveira (2010) have proposed the use of option contracts as instruments to manage the SPR, as they signal the government commitment to act during a disruption, provide more risk-management opportunities to the refinery industry, and defray some of the costs of maintaining the reserve. In their analysis these option contracts give the right to its holders to buy or sell from the strategic reserve based in the country and, for this reason, there is no risk of the petroleum not being delivered due to political issues. They modeled the interaction between the refiners and the government as an infinite-horizon Markov decision process, where the government maximizes social welfare subject to the equilibrium conditions on private inventory, in a Markov game. They were able to calculate the buildup and drawdown patterns that would emerge from these option contracts, and to capture their impact on market prices. Most importantly, the proposed model was able to compute the number of different types of contracts that would be required by the Nash equilibrium of the dynamic game.

In this article we extend Murphy and Oliveira (2010) by showing how to price option contracts on SPR oil, as this issue was not addressed in their article. Furthermore, our contributions include: a) an analysis of the interactions between SPR and private management of crude oil inventory; b) a description of the relationship between the valuation of these option contracts and the inventory levels, the degree of market disruption, and the spot and futures prices; c) a study of the interactions between financial options on oil and options on the SPR; d) a proof that the government can use the options trading scheme to pay for part of the costs of running the SPR.

The article proceeds as follows. In section 2 we present the general model. In section 3 we describe the use of options on the SPR's oil, their valuation by refiners and speculators, and the viability of such a trading scheme from the government's perspective. In section 4 we present the computational results of a case study in the management of the USA's SPR. In section 5 we conclude the article.

2. Modeling the Interactions Between the Public Sector and Refiners in Oil Markets

The model used in this article is very similar to the one presented in Murphy and Oliveira (2010). For this reason we keep the model description short. The model is based on an infinite-horizon Markov decision game between the government and the refiners, both of whom hold inventory, and includes financial speculators who hold no inventory but are used to explain the equilibrium properties of the option contracts. The choice of an infinite-horizon Markov model allows for the consideration of a strategic reserve that can have a very long life and keeps the model solvable because of the availability of good solution algorithms for infinite-horizon models. Moreover, the use of a Markov-decision process has the ability to represent dynamic processes in a compact way, as the information in the current state is all you need to define the probabilities of moving to other states. At the same time, stochastic processes without the memoryless property can be approximated arbitrarily closely by a Markov process.

The public sector's decisions are the amount to add to or withdraw from the SPR. Refiners manage their stocks of crude oil in order to balance the cost of holding stocks with the convenience yield of inventories and the expected price of oil, leading to equilibrium conditions on private inventories and oil markets. The convenience yield in oil markets has been estimated by Pindyck (2001) and by Considine and Larson (2001). In the original literature, Kaldor (1939) and Working (1949), the convenience yield is used to justify situations in which the futures price could be lower than the current price. In this case, they would argue, there was a negative price of storage (Working, 1949). Nonetheless, Kaldor (1939) clearly explains why costs of storage and convenience yield are separate parameters. As in Murphy and Oliveira (2010), we separate convenience yield and storage costs so that the influence of both in the inventory management policy is evident.

We model refinery and SPR inventories as discrete variables. Each state of the model is defined by the size of the SPR, S, the size of refinery stocks, X, and the degree of disruption of the oil market (indexed by i), which are caused by exogenous shocks to the supply side of the market.

The level of disruption in oil markets is exogenous: disruptions can be caused by shortages of supply due to political unrest, bad weather conditions, wars, or possibly deliberate actions by oil producing counties. A disruption state represents how the supply is affected by these risk factors in a given state of the world. Therefore, each state in the model is represented by the triple (*S*,*X*,*i*) with $S \in \{S_1,...,S_{\bar{S}}\}$, $X \in \{X_1,...,X_{\bar{X}}\}$, and $i \in \{1,...,\bar{I}\}$.

The transitions between the different market and inventory states are determined by increases (decreases) in the SPR, in private reserves, and by the transitions between the different levels of disruption. Let $pr(j|i)$ stand for the conditional transition probability from state *i* to state *j* and $pr(S_p,X_p,j|S,X,i)$ be the conditional transition probability from state (*S*, *X*, *i*) to state (S_p, X_p, *j*). Then, for any state (*S*, *X*, *i*) the next state is (S_p, X_p, *j*) with probability $pr(S_p,X_p,j \mid S,X,i) = pr(j\backslash i)$. In this model we assume that these parameters are known to the public and private players and can be estimated from reality. Here we have estimated these parameters from observations in pricing data.

The oil supply is set at the level $q_i = q - w(i)$, in which *q* is the intercept of the supply function and *w*(*i*) represents the total reduction of supply determined by the market disruption associated with state *i*.

The imports of petroleum are represented by $D = kP^{-e}$, an isoelastic demand function, in which *P* is the petroleum price, *e* is the price elasticity of demand, and *k* is a parameter that scales the level of imports. The isoelastic demand function follows Teisberg (1982), Murphy and al. (1987) and Murphy and Oliveira (2010), and is the standard first-order approximation to a general demand function. In equilibrium, the supply of oil in the market equals demand at a market-clearing price.

Therefore, in equilibrium, we have the following relationship among petroleum demand, supply disruptions, and inventory changes: $D = q - w(i) - (S_p - S) - (X_p - X)$. It then follows that the equilibrium oil price can be expressed as a function of demand and changes in stocks as represented in (1). This means that the larger the level of disruption *w*(i) the lower the oil supply to the market and the larger the price spike, everything else constant. The government can release petroleum from the SPR when it wants to compensate for reductions on the supply of petroleum.

$$P(S, X, i, S_p, X_p) = \left[\frac{k}{q - w(i) - (S_p - S) - (X_p - X)} \right]^{\frac{1}{e}}. \tag{1}$$

Next, we describe the objective function that the government minimizes. We use consumer surplus as the main component of the social welfare function that the government maximizes, as this is the standard utility function used in industrial economics and was used before by Teisberg (1982), Murphy and al. (1987) and Murphy and Oliveira (2010) to analyze this same problem. (An alternative option would have been to include the implications of increases in petroleum prices on the macroeconomic output; however this function is also very hard to estimate accurately, given the wide range of factors affecting GDP.) Therefore, the government's goal is to maximize the consumer surplus and to minimize the costs associated to holding private and public stocks, in order to maximize the social surplus.

Let h stand for the SPR's marginal holding cost. Let $l(X)$ and $b(X)$ represent, respectively, the holding cost and convenience yield of private inventories (these are a function of the level of stocks, as found in practice, e.g., Milonas and Henker, 2001). Then maximizing social surplus is equivalent to minimizing the cost function $c\left(S, X, i, S_p, X_p\right)$ presented in (2).

$$\begin{aligned} &c\left(S, X, i, S_p, X_p\right) = -\frac{kP^{1-e}}{1-e} + P \times (S_p - S) + P \times (X_p - X) + \\ &h \times S_p + l(X_p) - b(X_p) \end{aligned} \tag{2}$$

In this function (2) the first term $\frac{kP^{1-e}}{1-e}$ is the consumer surplus derived from the isoelectric demand function and representing the value to all the consumers of petroleum product have when buying the equilibrium quantity at price P. The second term stands for the purchase costs, sales revenues, and holding costs of the SPR $P \times \left(S_p - S\right) + h \times S_p$. Finally, the third term is the costs of holding private reserves minus the convenience value associated with them $P \times (X_p - X) + l(X_p) - b(X_p)$.

Therefore, in this game the government maximizing social welfare taking into consideration the behavior of the refiners in determining equilibrium inventories and the resulting equilibrium oil prices. Besides the government and refiners, we have a third group to consider, financial speculators, that buy and sell positions in petroleum futures, and respective option contracts. One of the activities of speculators is arbitrage: if possible they would try to buy cheap and sell expensive at no risk (this in theory would be possible if there were market imperfections). From there relentless activity to make a profit from speculation and arbitrage, in equilibrium, there are no arbitrage opportunities: it has been shown in a similar context, e.g., Allaz (1992), that the presence of a single speculator is sufficient for the no-arbitrage condition to hold. For this reason, in this article, we work with aggregations of refiners and speculators.

Let $l'(X)$ and $b'(X)$ represent the industry marginal holding cost and marginal convenience yield respectively. We assume constant, or increasing, marginal inventory costs and constant, or decreasing, marginal convenience yields, this captures the aggregate of the diseconomies of scale that may arise, at the individual level, from managing inventory by allowing the aggregate curves to be constructed through summing the inverse functions for the individual players and then taking the inverse of the summed functions. Then, the no-arbitrage condition (3), in which g is a discount parameter, describes how the discounted expected price of the marginal barrel, $gE[P(S_p^*, X_p^*) \mid S, X, i]$, equals the current price and marginal holding costs minus the marginal benefits of holding the marginal barrel. In (3) the price in state (S, X, i) is represented by $P(S, X, i, S_p^*, X_p^*)$ and takes into consideration the change in inventory decided in the current state, i.e., $S_p^* - S$ and $X_p^* - X$. The term $E[P(S_p^*, X_p^*) \mid S, X, i]$ represents the expected spot price in state (S_p^*, X_p^*) conditional on the current state of the system being (S, X, i).

$$P(S, X, i, S_p^*, X_p^*) + l'(X_p^*) - b'(X_p^*) = gE[P(S_p^*, X_p^*) \mid S, X, i], \tag{3}$$

The expected price, $E[P(S_p^*, X_p^*) \mid S, X, i]$, given the optimal policies (identified by the superscript '*') in the next state, is computed by (4). Note that (S_{pp}^*, X_{pp}^*) is the equilibrium of the dynamic game in the state that follows, (S_p^*, X_p^*, j). This means that the expected value computed in (4) takes into account the current state and all the possible states in the next period, together with the optimal actions in those states, i.e., $S^*_{pp} - S^*_p$ and $X^*_{pp} - X^*_p$.

$$E[P(S^*_p, X^*_p) \mid S, X, i] = \sum_{j, S^*_{pp}, X^*_{pp}} pr\left(S^*_p, X^*_p, j \mid S, X, i\right) P(S^*_p, X^*_p, j, S^*_{pp}, X^*_{pp}). \qquad (4)$$

Likewise, in financial markets the futures and options contracts on oil do not require the traders to hold any stock of oil and, therefore, they do not incur holding costs and do not benefit from the convenience of holding crude oil. In their case, by the same argument used for the refiners, we can represent the market consequences of their behavior with the following function, $F(S, X, i, S^*_p, X^*_p) = gE[P(S^*_p, X^*_p) \mid S, X, i]$, in which $F(S, X, i, S^*_p, X^*_p)$ is the price of a futures contract in state (*S*,*X*,*i*), for the SPR's and refiners' optimal policy. Given the optimal inventory in the economy, the traders in financial markets compute the price in the futures markets, based on their expectations about market behavior. This means that we can derive a relationship between the activities of the refiners and financial markets. From the no-arbitrage equation and the definition of the futures price we get (5), which shows that the no-arbitrage condition establishes a relationship between the spot and the futures prices.

$$P(S, X, i, S^*_p, X^*_p) + l'(X^*_p) - b'(X^*_p) = F(S, X, i, S^*_p, X^*_p). \qquad (5)$$

We solve the infinite horizon Markov game by using an iterative process in which the government maximizes social welfare, the refiners solve the no-arbitrage equation, given the government actions, and the speculators set the futures price, taking into account the optimal policies of the government and speculators, see Table 2.1 (adapted from Murphy and Oliveira, 2010). The procedure to compute the Nash equilibrium of this Markov game starts by initializing the prices and the public cost function in every state (steps 1 and 2), the probability matrix, and the refiners' and the government's initial policies (steps 3-5). In step 6.A) the refiners solve the no-arbitrage condition and in step 6.B) the government optimizes given the refiners' inventory equilibrium. In step 6.C), if the policies do not change from the previous iteration, the loop ends, returning the optimal policies for government and refiners (if convergence has not been reached, a new iteration of policy updating is done). Finally, in step 7 we compute the futures prices.

TABLE 2.1: Solution Procedure for the Markov Game

1. Initialize the equilibrium oil prices, for every state:

$$P(S,X,i,S_p,X_p)=\left[\frac{k}{q-w(i)-(S_p-S)-(X_p-X)}\right]^{\frac{1}{e}}$$

2. For every state (S,X,i) initialize the public cost function

$$c\left(S,X,i,S_p,X_p\right)=-\frac{kP^{1-e}}{1-e}+P\times(S_p-S)+P\times(X_p-X)+$$
$$h\times S_p+l(X_p)-b(X_p)$$

3. Initialize the probability matrix $pr(S_p,X_p,j\,|S,X,i)=pr(j|i)$.

4. Initialize the refiners' strategy: $X_p=X$

5. Initialize the starting policy for the SPR, $S_p=S$.

6. For every iteration k :

   A) The refiners solve the no-arbitrage equation

   $$P(S,X,i,S_p^*,X_p^*)+l'(X_p^*)-b'(X_p^*)=gE[P(S_p^*,X_p^*)\,|\,S,X,i]$$

   obtaining X_p^k.

   B) The government maximizes expected social welfare, given $X_p=X_p^k$, obtaining S_p^k.

   C) If $X_p^k=X_p^{k-1}$ and $S_p^k=S_p^{k-1}$, stop and, for every state, return $S_p^*=S_p^k, X_p^*=X_p^k$.

   Otherwise, return to step 6.A).

7. Compute futures prices for the optimal policies

   $$F(S,X,i,S_p^*,X_p^*)=gE[P(S_p^*,X_p^*)\,|\,S,X,i]$$

In the context of our model, as the transition probabilities and actions, in each state, are common knowledge, the equilibria found are Nash. In Murphy, Toman and Weiss (1989) it has been shown that, in these conditions, there is only one monotonic solution to this game.

3. Pricing Option Contracts on the SPR

In this section we analyze how to price such option contracts. We begin by explaining the basic workings of these contracts, which, as explained in Murphy and Oliveira (2010), are special, as the conditions for their exercise are a function not only of the market price but also of the level of private and public inventories.

Let us summarize how these contracts work. In order for the contracts to be able to replicate the optimal public policy described in section 2, for a disruption scenario *j*, the number of options issued in advance is equal to the incremental barrels to be released over a less severe disruption, *j*-1. The options for all disruption scenarios less severe than *j* are also exercised and the total matches the government policy. (We compute the value of these options for different maturity periods, *n*.) For this reason, the conditions under which these options are exercised depend not only on the exercise price but also on the levels of inventory held by private firms and the SPR.

Therefore, the value of a call or put option is a function of the probability that the market reaches the triggering state (S_p^*, X_p^*, j) by the maturity date. In this case, the put option is in the money when the inventory levels are S_p^*, X_p^* and its exercise price exceeds the market price. The call option is in the money when the inventory levels are S_p^*, X_p^* and the market price is above its exercise price. It is optimal to exercise an option when the profit is higher from exercising the option now rather than later. An option contract can be for any amount, we use 1 MMB of the SPR. The exercising of call options adds oil to the market, thus lowering the market-clearing price. The exercising of put options decreases the amount of oil available in the market and raises the price. The values of the put and call options need to take into account that these options can be exercised only for inventory levels in an interval around a discrete set of quantities, say, S_p^*, X_p^*. A general introduction to financial option pricing can be found, for example, in Hull (2007).

Moreover, we need to set the strike prices to ensure that the options are exercised as planned under the optimal policy. To calculate the strike prices we begin with the most disrupted state, as there is no higher price that has to be factored in. Let $pr(S_p, X_p, j \mid S, X, i, n)$ represent the probability of starting in state (*S*, *X*, *i*) and arriving in state (S_p, X_p, *j*) for the first time *n* steps after departing from state (*S*, *X*, *i*). Let the state with the most severe disruption be indexed by *I*. Consider the exercise price of $P(S, X, I, S_p^*, X_p^*)$ for call options that cover the most disrupted state *I*. If some options are exercised, all options for state *I* will be exercised simultaneously, because if some are not, the market price would be above the strike price as a result of the reduced amount oil on the market. To ensure that the options are exercised, given that the options are not necessarily exercised at the same time and there is noise in the price, we can set the exercise price for the call, $P_c^e(S_p^*, X_p^*, I)$ at the target price minus a small amount $\delta > 0$, $P_c^e(S_p^*, X_p^*, I) = P(S, X, I, S_p^*, X_p^*) - \delta.$ We now perform the calculation for the next level of disruption *I*-1 with an exercise price of $P_c^e(S_p^*, X_p^*, I-1)$. For the options to be exercised, the exercise price has to make exercising immediately in state *I*-1 more valuable than holding the option in anticipation of a greater disruption. For the last option targeted to be exercised in state *I*-1 rather than held, we need (6) to hold.

$$\begin{aligned} &P(S, X, I-1, S_p^*, X_p^*) - P_c^e(S_p^*, X_p^*, I-1) > \\ &\sum_n pr\left(S_p^*, X_p^*, I \mid S, X, I-1, n\right) g^n \left(P(S, X, I, S_p^*, X_p^*) - P_c^e(S_p^*, X_p^*, I-1)\right) \end{aligned} \tag{6}$$

Consequently, the strike price is lower than the market-clearing price resulting from the optimal policy. Generalizing, we need (7) to hold. To set the strike price, for *i*, we find the value of the right-hand-side of (6), then we subtract δ and repeat this process for all *i* with call options. By requiring (8) we guarantee that the option is not exercised during less severe disruptions. Note that there is always a value for the strike price that satisfies both (7) and (8).

$$\begin{aligned} &P(S, X, i, S_p^*, X_p^*) - P_c^e(S, X, i) > \\ &\max_{j>i} \sum_n pr\left(S_p^*, X_p^*, i \mid S, X, j, n\right) g^n \left(P(S, X, j, S_p^*, X_p^*) - P_c^e(S_p^*, X_p^*, i)\right) \end{aligned} \tag{7}$$

$$\begin{aligned} &P(S, X, i, S_p^*, X_p^*) - P_c^e(S_p^*, X_p^*, i) > \\ &\max_{j>i-1} \sum_n pr\left(S_p^*, X_p^*, i \mid S, X, j, n\right) g^n \left(P(S, X, j, S_p^*, X_p^*) - P_c^e(S_p^*, X_p^*, i)\right) \end{aligned} \tag{8}$$

For the put option contracts we follow a similar process. We start at the least disrupted state, state 1, and set the price of the put as follows: $P_p^e(S_p^*, X_p^*, 1) = P(S, X, 1, S_p^*, X_p^*) + \delta.$ We then continue into states with successively higher levels of disruption using the analogue of (6).

We start by representing the relationship between the expected values of the option contracts in financial markets (say on Brent) as a function of the inventory policies by the government, refiners, and the market level of disruption. Let $op\left(S, X, i, S_p^*, X_p^*, j, n\right)$ stand for the value of put options in the financial markets and let $oc\left(S, X, i, S_p^*, X_p^*, j, n\right)$ represent the value of the call options in the financial markets. Their respective values are represented by (9) for puts and (10) for calls, respectively. In general, these options are valuable under the same conditions as any other financial option: if the exercise price is larger than the market price (for the put option) or if the exercise price is below the market price (e.g., Hull, 2007). The particularity of these options is that their value depends on the state of the Markov decision process during the lifetime of the options. The put option is in the money when the exercise price is larger than the futures price. Speculators in financial markets do not take physical delivery and they do not benefit from holding inventory, or pay for their holding cost, nonetheless, their valuations of financial options are influenced by the changes in the levels of the SPR and private inventory through the no-arbitrage condition.

$$op\left(S, X, i, S_p^*, X_p^*, j, n\right) = \sum_{n' \le n} \sum_{j'} pr\left(S_p^*, X_p^*, j' \mid S, X, i, n'\right) g^{n'} \left[P_p^e(S_p^*, X_p^*, j) - F(S_p^*, X_p^*, j', S_{pp}^*, X_{pp}^*)\right]^+ \tag{9}$$

$$oc\left(S, X, i, S_p^*, X_p^*, j, n\right) = \sum_{n' \le n} \sum_{j'} pr\left(S_p^*, X_p^*, j' \mid S, X, i, n'\right) g^{n'} \left[F(S_p^*, X_p^*, j', S_{pp}^*, X_{pp}^*) - P_c^e(S_p^*, X_p^*, j)\right]^+ \tag{10}$$

Using the aggregate marginal holding cost and the aggregate marginal benefit (convenience value) of having the inventory we price the option from the perspective of refiners, with $opr\left(S, X, i, S_p^*, X_p^*, j, n\right)$ and $ocr\left(S, X, i, S_p^*, X_p^*, j, n\right)$ standing for the refiners' valuations, respectively, of the put and call contracts. The values of the put and call options are respectively (11) and (12). Note that (11) and (12) differ from the valuation of financial options as the options on the SPR oil are associated with physical delivery and the refiners have a convenience benefit from holding oil and need to support the inventory cost of holding the product: for this reason, when computing the option value, they need to consider not just the current oil price but also the net holding cost associated with it.

$$\begin{aligned} &opr\left(S, X, i, S_p^*, X_p^*, j, n\right) = \\ &\sum_{n' \le n} \sum_{j'} pr\left(S_p^*, X_p^*, j' \mid S, X, i, n'\right) g^{n'} \left[P_p^e(S_p^*, X_p^*, j) - P(S_p^*, X_p^*, j', S_{pp}^*, X_{pp}^*) - l'(X_{pp}^*) + b'(X_{pp}^*)\right]^+ \end{aligned} \tag{11}$$

$$ocr\left(S,X,i,S_p^*,X_p^*,j,n\right)= \sum_{n'\le n}\sum_{j'} pr\left(S_p^*,X_p^*,j'|\,S,X,i,n'\right)g^{n'}\left[P(S_p^*,X_p^*,j',S_{pp}^*,X_{pp}^*)+l'(X_{pp}^*)-b'(X_{pp}^*)-P_p^e(S_p^*,X_p^*,j)\right]^+ \tag{12}$$

In Proposition 3.1 we analyze the relationship between the valuations of option contracts on the SPR and option contracts in financial markets for marker crudes: we show that, in equilibrium, these valuations are equal. This is an interesting result, as the refiners hold inventory (benefiting from the convenience value and having to support the holding costs) and the speculators in financial markets do not.

Proposition 3.1: $ocr\left(S,X,i,S_p^*,X_p^*,j,n\right)=oc\left(S,X,i,S_p^*,X_p^*,j,n\right)$ and $opr\left(S,X,i,S_p^*,X_p^*,j,n\right)=op\left(S,X,i,S_p^*,X_p^*,j,n\right)$. [Proof is in the Appendix.]

These options on the SPR provide a guarantee for refiners (as shown in Proposition 3.2), as a refiner that is caught short, if it has purchased the options, has access to crude. Let *r* represent a specific refiner: X_r^+, X_r^*, l_r', b_r', stand, respectively, for *r*'s disrupted inventory, optimal inventory, marginal holding cost, and marginal convenience yield.

Proposition 3.2: *A refiner that is short because of a disruption is willing to pay a premium of* $l_r'(X_r^+)-b_r'(X_r^+)-[l_r'(X_r^*)-b_r'(X_r^*)]$ *over the market value for refiners with secure supplies to acquire options to buy crude.* [Proof is in the Appendix.]

Therefore, with a disruption, as the disrupted refiner does not have its arbitrage condition satisfied, it is willing to pay a premium to acquire the option either when first auctioned (because it sees itself at greater risk) or when the disruption occurs. Consequently, the purchase price of the option is above the option prices in (11) and (12). Hence, the government can make a profit by issuing options on the SPR while assisting disrupted refiners, which is proved in Proposition 3.3. This profit mitigates some of the cost of managing the SPR. Moreover, these option contracts increase both social welfare and the refiners' inventory management profits. In Proposition 3.4 we show that refiners with secure supplies can manage inventory to profit from disruptions. This shows that market price volatility, and the presence of supply disruptions, benefits the non-disrupted refiners.

Proposition 3.3: *On average, for an option issued in state* (S,X,i)*, with a maturity of n periods, the government makes a profit of*

$\sum_{n'\leq n}\sum_{j'} pr\left(S_p^*, X_p^*, j' | S, X, i, n'\right) g^{n'}\left[-l_r'(X_{pp}^*) + b_r'(X_{pp}^*) + l'(X_{pp}^*) - b'(X_{pp}^*)\right]^+$ *in the put* option *and of*

$\sum_{n'\leq n}\sum_{j'} pr\left(S_p^*, X_p^*, j' | S, X, i, n'\right) g^{n'}\left[l_r'(X_{pp}^*) - b_r'(X_{pp}^*) - l'(X_{pp}^*) + b'(X_{pp}^*)\right]^+$ *in the call option.* [Proof is in the Appendix.]

Proposition 3.4: *Refiners that are not disrupted profit from inventory management due to the volatility in crude oil prices*. [Proof is in the Appendix.]

Next, in section 4, we present an application of our pricing model to the management of the SPR in the United States.

4. Analysis of the SPR in the United States

In this section we apply the option pricing model to the case of the U.S. strategic petroleum reserve. We use the same model parameters as in Murphy and Oliveira (2010), which was based on Department of Energy (2009) and Energy Information Administration (2009). In the simulations presented in this case we set the increments (and reductions) to the SPR at 25 MMB. We allow the size of the reserve to change from 0 to 1400 MMB (almost twice its size in 2010). The private inventories are also segmented in blocks of 25 MMB and they are allowed to change from 200 MMB to 500 MMB (in 2010 private inventories were about 300 MMB). The different possible states considered for the oil market, with the corresponding oil prices, are Normal ($60/bbl), Disrupted ($90/bbl) and Very Disrupted ($120/bbl), which we model by using the following parameters for the demand and supply functions: q equals 12,800, k equals 15,700 and $w(i)$ is zero in the Normal state, 270 in the Disrupted state, and 440 in the Very-Disrupted state.

Furthermore, the price elasticity of demand used is 5% (estimated using data for petroleum prices and imports in the USA from the Department of Energy, 2009; and Energy Information Administration, 2009) and we assume a discount factor of about 0.98 per quarter. We use $1.2/bbl/quarter for the SPR storage costs, which includes the opportunity costs associated with the investment in inventories, i.e., given that for the strategic reserve the marginal cost of physical storage is approximately zero, the opportunity cost, is estimated at about 2% of the normal state price ($60/bbl).

The private inventory costs and the private convenience yield are modeled so that the inventory cost is approximately equal to the convenience yield at about 300 MMB. The difference between the private and public cost functions is due to the technologies used for storage. We assume that the marginal cost of private inventories increases with capacity, whereas the public marginal storage costs are independent of capacity, as the oil is stored in underground caverns.

The transition probabilities between these states are summarized in Table 4.1. This analysis was based on the crude prices since 1861, published by BP (2012). We have classified prices into quintiles. The bottom 67% were classified as Normal states, prices in the quintiles between 67% and 85% were classified as Disrupted and the top 15% prices were classified as Very-Disrupted. This is a very simple analysis that aims to capture the main features of conditional probabilities in the transition between states.

TABLE 4.1 Transition probabilities between states (%), Normal (N), Disrupted (D) and Very-Disrupted (V)

		Disruption Level		
		N	D	V
Disruption Level	N	90.1	7.9	2.0
	D	24.6	53.8	11.5
	V	0.0	22.2	77.8

Our analysis is structured as follows. In sub-section 4.1 we start by analyzing the relationship between the pricing of option contracts and the different factors influencing their value: petroleum prices and inventory, duration of the options and the marginal convenience yield. In sub-section 4.2 we present the interaction between the management of the SPR using option contracts and the refiners' inventory policies and profits.

4.1. Factors Influencing the Pricing of Option Contracts on the SPR

As can be seen from equations (9) and (10) the pricing of option contracts depends on the futures prices. For this reason, we start by analyzing the prices of the futures contracts as a function of the level of disruption and inventory. Then, as it follows from equations (11) and (12) that the pricing of the options contracts depends on the spot prices, we analyze how the latter depend on the levels of disruptions and inventory.

In Figure 4.1 we present the prices of the futures contracts (with maturity within 8 quarters) as a function of the level of disruption of the current state, and the total public and private inventory (the final level of inventory in each state). These prices indicate the value of the SPR in ameliorating a disruption. The prices of the futures contracts increase monotonically with the degree of disruption and decrease monotonically with the level of inventory. The impact of the level of inventory on futures prices is stronger in Normal states (in which the prices range from about \$55 to \$62) and weaker in the Very Disrupted states (in which the prices are stable around \$73). This is because once a disruption occurs the public inventory is considerably reduced and the withdrawals diminish under the optimal withdrawal policy, in subsequent periods.

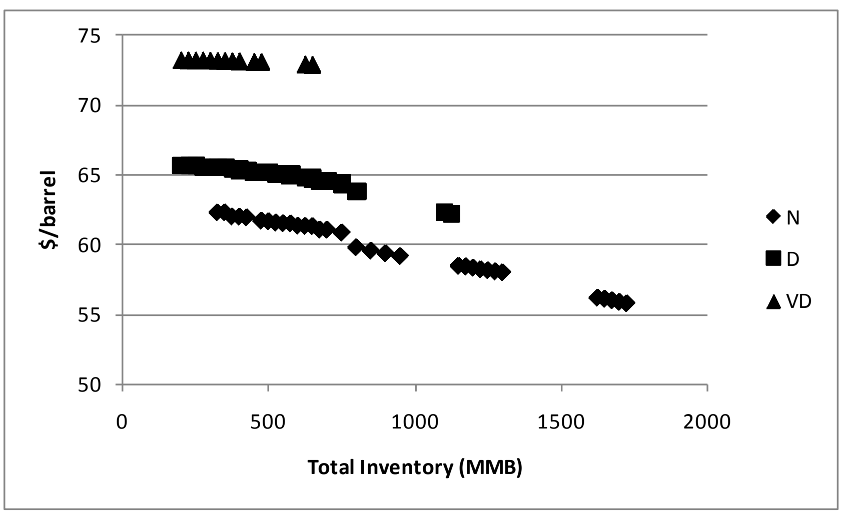

FIGURE 4.1: Futures prices (8 quarters ahead) as a function of total inventory (MMB) and level of disruption in the current state (Normal – N, Disrupted – D, and Very Disrupted - VD).

In Figure 4.2 we plot the relationship between the crude-oil spot price and the level of total inventory. It is evident that prices decrease with the level of inventory and increase with the degree of disruption. Moreover, the relationship between price and inventory is non-linear as prices increase faster when inventories approach the minimum levels. This means that the impact of supply disruptions on market prices increases when the total level of inventories is low, as the SPR inventory cannot supply as much crude oil (in the case of high prices).

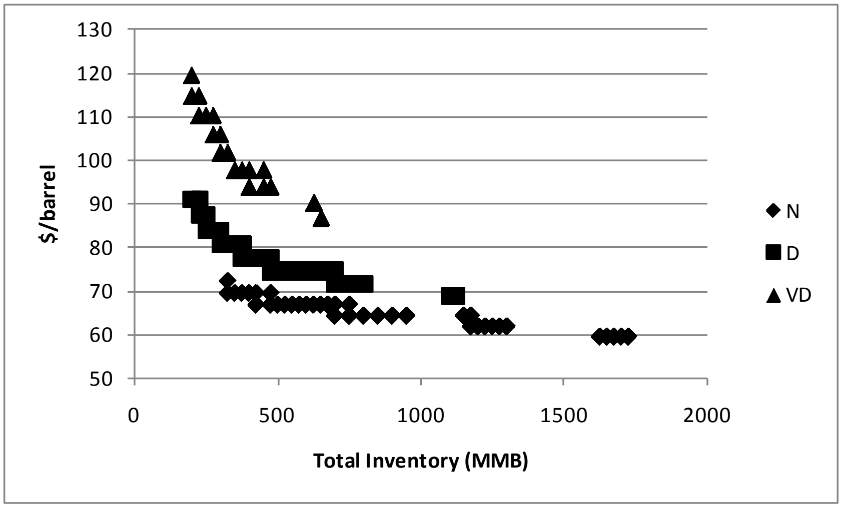

FIGURE 4.2: Spot price as a function of total inventory (MMB) and the level of disruption in the current state.

Figure 4.3 describes the interaction between the type of option contract and its duration. In this figure we plot the value of the put and call options as a function of duration, for a scenario with an exercise price of $90/bbl. For all the durations the value of the put options are always higher than the value of the corresponding call options (these values are consistent with the state of the world plotted as there are no starting reserves in the SPR). The value of both options increases with the duration, as is typical of option pricing, given that uncertainty increases with duration.

We now focus on options with duration of 8 quarters and analyze the different factors affecting their value. In Figures 4.4 and 4.5 we plot the value of all the call and put options issued by the SPR and relate them to the spot and futures crude prices, total inventory and convenience yield. The contracts are classified taking into account the level of disruption of the state in which they are exercised, and the level of the exercise price. (As we are plotting many contracts with very different exercise prices, we have aggregated them into three categories, low, high, and very high, corresponding, respectively, to the typical price in a

Normal, Disrupted and Very Disrupted state.) We look at the value of put and call options issued during normal periods.

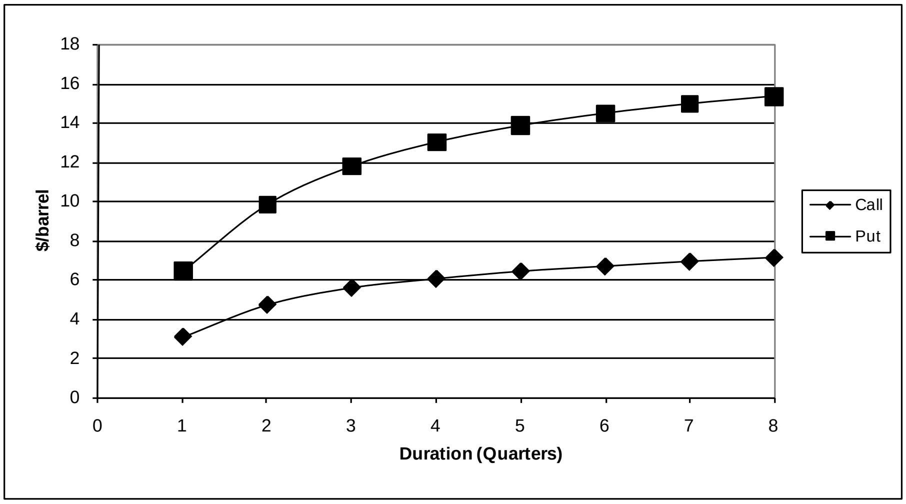

FIGURE 4.3: Value of the put and call option contracts as function of duration (in quarters).

Figure 4.4 shows that the value of a put option, issued in a normal state with a high exercise price, increases roughly linearly with the crude spot price and with the convenience yield, and increases nonlinearly with the futures price. This increase in the rate of growth is related to the correlation between futures prices, spot prices and inventory: as the SPR is depleted, total inventory approximates the minimum and spot and futures prices increase (as illustrated in Figures 4.1 and 4.2). The spot prices increase at a faster rate than do the futures prices when inventory is low. This result is consistent with the nature of the put option (which gives the holder the right to sell to the SPR): when the inventory is scarce the put option is more valuable.

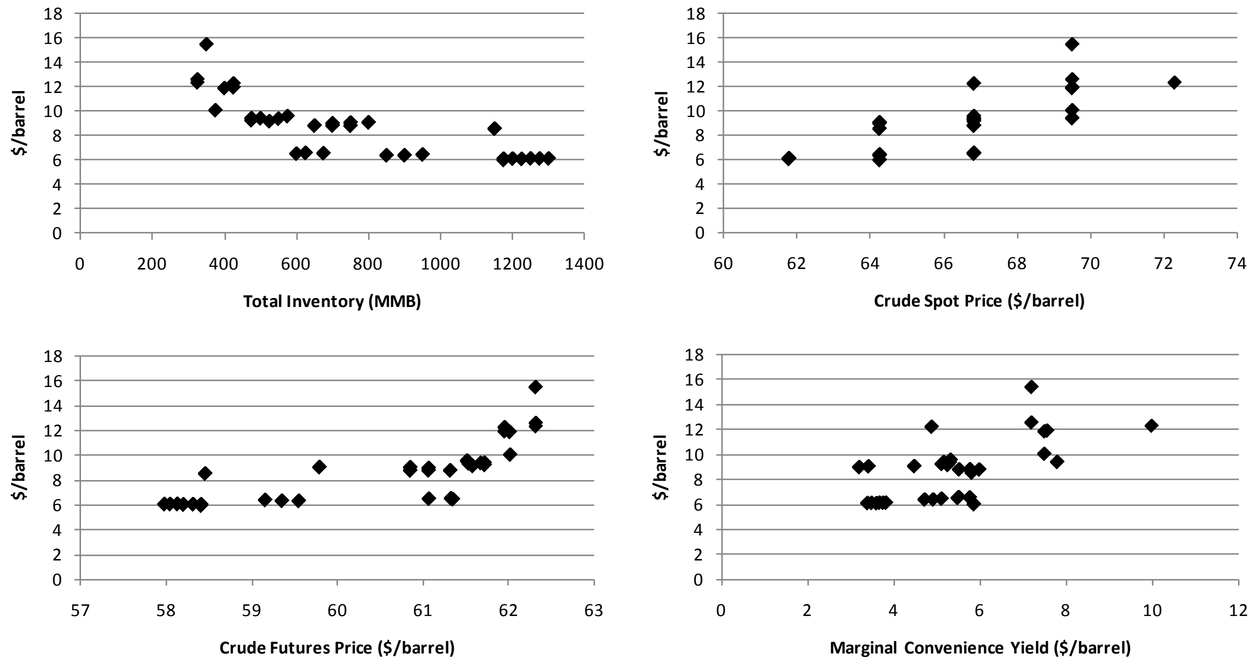

FIGURE 4.4: Values of the put options issued in Normal states, with a high exercise price, as a function of total inventory (MMB), spot price, futures price and marginal convenience yield.

Furthermore, the values of the put option contracts are asymptotic to the minimum inventory. This surprising result is peculiar to these options. As the level of inventory in the disrupted state decreases, the exercise prices in equations (8) and (10) increase more than do the futures prices in those states. Hence, the value of the options to be exercised in states with lower levels of inventory tends to be higher (this is the reason why the value of these options has a linear relationship with the convenience yield). Moreover, in states with low inventory the value of the put option increases linearly with the crude spot price (the increase of which leads to larger futures prices and larger values for put prices.)

The values of the call options issued in the normal states with low exercise price are presented in Figure 4.5. In general, the call prices tend to increase linearly with the crude spot and future prices and with the convenience yield; they also tend to linearly decrease with the level of inventory. As the SPR is depleted, inventory becomes scarce and spot and futures crude prices increase. This lack of inventory strongly restricts the ability of the market, still in a normal state, to mitigate the potential price spikes caused by a supply disruption. For this reason, when inventory decreases the value of a call option increases as well.

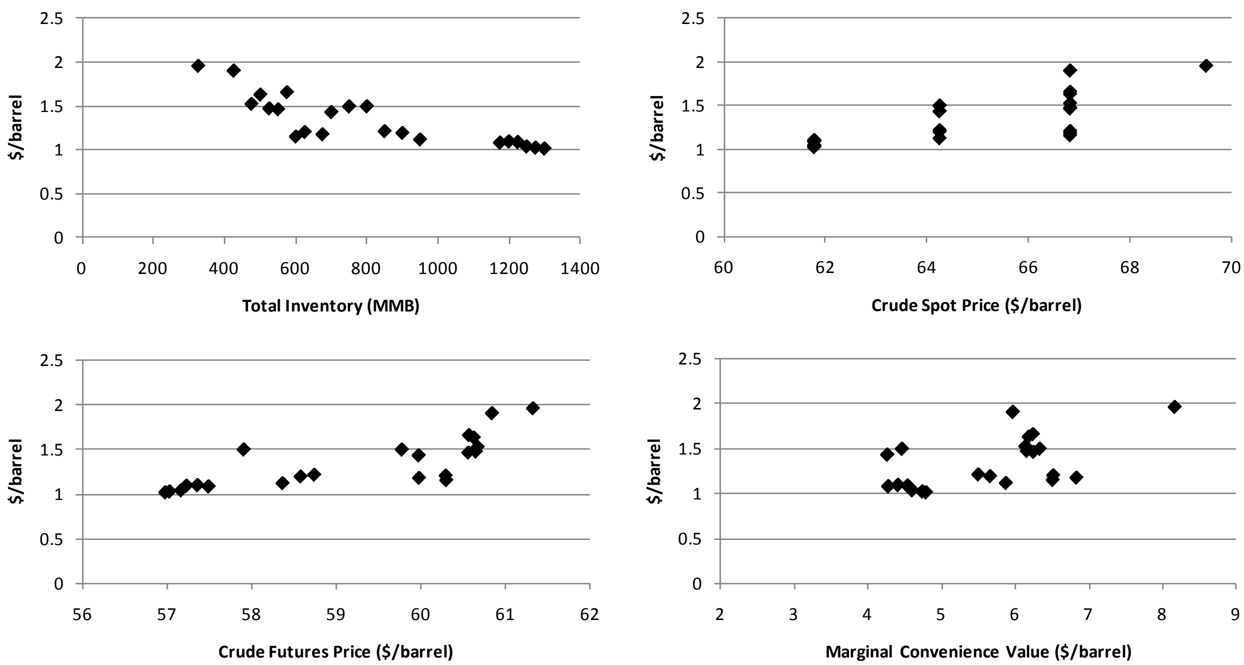

FIGURE 4.5: Values of the call option issued in Normal states, with a low exercise price, as a function of total inventory (MMB), spot price, futures price and marginal convenience yield.

4.2. The Effect of SPR Management on the Refiners' Profits and Inventories

In this sub-section we analyze the impact of the options contracts, and their pricing scheme, on the refiners' inventory policies and profits. Figure 4.6 plots the expected profit and standard deviation of the refiners' profits as a function of the SPR management policy (i.e., how much the government is willing to change the level of the SPR in response to disruptions). Using Figure 4.6 we can analyze the interaction between the degree of flexibility of the SPR management policy and the refiners' profits.

We say that a policy is flexible if the government allows the SPR to adjust its level as much as required to meet the social optimum (from 0 to 1400 MMB in our case). A restrictive policy imposes a minimum and/or maximum allowed capacity in the SPR which cannot be sold as option contracts, assuring that the government maintains a part of the reserve which is accessible to the economy only upon direct order from the government. For example, the most restrictive policy is to keep the SPR at the level of 700 MMB, as in this case it is not used.

As shown in Figure 4.6, the greater the operating range of the SPR the lower the average inventory profits and the larger their volatility. The effect on average inventory profits is understandable. The price band between the normal-market and disrupted-market prices is narrowed by the SPR: this reduces the average avoided cost per barrel subtracted and increases the average price of inventory additions.

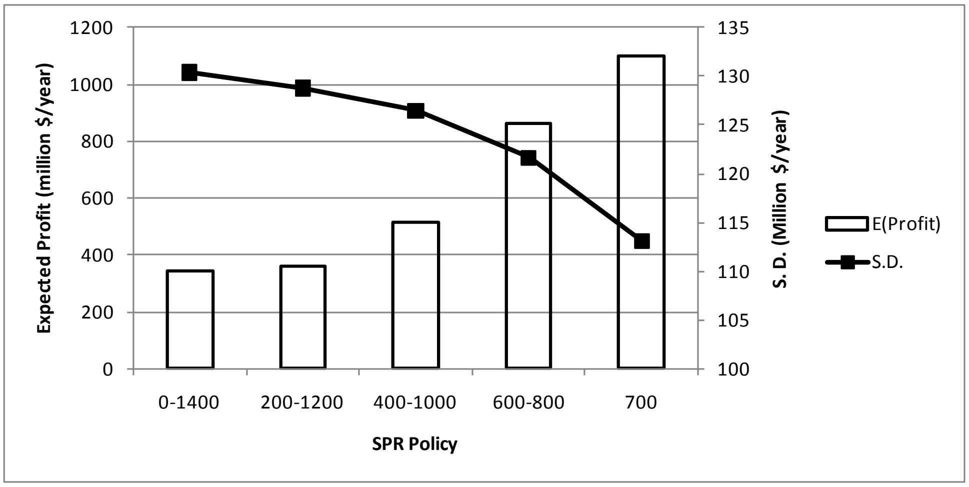

FIGURE 4.6: Refiners' Expected Profit and Profit Volatility as a function of the SPR management policy.

Most noteworthy is that in a disruption that persists and the SPR inventory is eventually mostly drawn down refiners increase their inventories early in the disruption (in anticipation of reduced draws from the public inventory and the resultant higher prices). This means they are spending on increasing inventories at the beginning of a disruption in anticipation of even higher prices from the disruption continuing, instead of making guaranteed profits by drawing down inventory. However, there is a significant probability of the disruption ending, and creating losses for the private player, increasing the variability of returns.

Hence, this result goes against the idea that increased access to oil from the SPR, when managing private inventory, would necessarily increase profits and decrease risks faced by firms (e.g., Tang and Tomlin, 2008). In our analysis, increased flexibility translates into lower profits and higher uncertainty in inventory profits to the refiners not affected by the market disruption. Table 4.2 illustrates the impact of the SPR policy on the steady state and average refiners' inventory levels by degree of disruption. It shows that the steady state and average level of inventories tend to decrease with the degree of flexibility of the SPR policy for all levels of disruption.

TABLE 4.2: Refiners' Steady-State and Average Inventories as a Function of the SPR policy and Degree of Disruption

Steady-State Refiners' Inventory (MMB)					
SPR-Policy	**700**	**600-800**	**400-1000**	**200-1200**	**0-1400**
Normal	450	450	413	363	290
Disrupted	200	200	200	200	200
Very Disrupted	200	200	200	200	200
Average Refiners' Inventory (MMB)					
SPR-Policy	**700**	**600-800**	**400-1000**	**200-1200**	**0-1400**
Normal	381	325	291	283	274
Disrupted	283	263	258	250	250
Very Disrupted	281	260	258	257	249

5. Conclusions

The use of option contracts to manage the SPR is now a reality with countries such as New Zealand using them as instruments to secure access to oil in case of an emergency. In this article we complement Murphy and Oliveira (2010) by proposing a model for setting strike prices and the pricing of option contracts on the SPR's oil, taking into account the interactions between the government and refiners, in a Markov game. Our main results are the following: a) We establish the relationship between the value of these options for refiners and the value of financial options on futures contracts on a marker crude oil. b) The value of options contracts on the SPR's oil is an increasing function of spot and future crude oil prices and a decreasing function of the total level of inventory. c) A more flexible management of the SPR decreases private investment in crude oil stocks. d) We prove that the options mechanism is profitable from the government's perspective and can be used to finance a part of the cost of keeping the reserve, lowering the bill for taxpayers while increasing the value of the reserve to refiners at risk of disruption. e) The computational results reinforce the idea that there is a trade-off between social welfare and the refiners' performance (in terms of expected profit and its uncertainty).

In our analysis we have used a Markov decision process to model the relationship between petroleum prices, supply disruption and the level of reserves. The model, even though attractive from its mathematical simplicity, can be very hard to use in practice due to the very difference sources of possible supply disruptions. Indeed, it would seem that a better valuation of the options would be dependent on good estimates of the different states and transition probabilities between them (which, furthermore, may be non-stationary if the sources of uncertainty change overtime).

Nonetheless, the increased availability of data, and cheaper computer processing power, makes it easier for governments to exchange data between themselves, within the context of the International Energy Agency, and with the different refiners importing oil to the country. This data exchange will make the estimates of the different levels of disruption, and transition probabilities between states, more reliable. Cheaper computer processing power has translated into the ability to run very large models within the required time frame to design and revise policies on the strategic petroleum reserve as compared to the past.

Finally, we envisage that, in general, the framework we proposed for the petroleum market and analyzed for the specific case of the USA can be used at the IEA level if the member countries try to better coordinate their policies.

Appendix

Proof of Proposition 3.1: For the financial markets, in the case of the put option, it follows from $P(S,X,i,S_p^*,X_p^*)+l'(X_p^*)-b'(X_p^*)=F(S,X,i,S_p^*,X_p^*)$ that

$$op\left(S,X,i,S_p^*,X_p^*,j,n\right)=$$
$$\sum_{n'\le n}\sum_{j'} pr\left(S_p^*,X_p^*,j'|S,X,i,n'\right)g^{n'}\left[P_p^e(S_p^*,X_p^*,j)-P(S_p^*,X_p^*,j',S_{pp}^*,X_{pp}^*)-l'(X_{pp}^*)+b'(X_{pp}^*)\right]^+$$

which is equal to (11). In the case of the call option, similarly we obtain

$$oc\left(S,X,i,S_p^*,X_p^*,j,n\right)=$$
$$\sum_{n'\le n}\sum_{j'} pr\left(S_p^*,X_p^*,j'|S,X,i,n'\right)g^{n'}\left[P(S,X,i,S_p^*,X_p^*)+l'(X_{pp}^*)-b'(X_{pp}^*)-P_c^e(S_p^*,X_p^*,j)\right]^+$$

which is equal to (12). ■

Proof of Proposition 3.2: In a disruption at least one refiner *r* loses the delivery of a shipload and has an inventory of $X_r^+ < X_r^*$ and

$P(S,X,i,S_p^*,X_p^*)+l_r'(X_r^+)-b_r'(X_r^+)<gE[P(S_p^*,X_p^*)]$. Thus, if *r* has the opportunity to buy

and exercise a call option at $P(S, X, j, S_p^*, X_p^*)$, this refiner is willing to pay up to a premium represented by $l_r'(X_r^+) - b_r'(X_r^+) - [l_r'(X_r^*) - b_r'(X_r^*)] > 0$. ■

Proof of Proposition 3.3: If a refiner *r* is disrupted and unable to change its levels of inventory then the arbitrage condition does not hold and, for *r*, $P(S, X, i, S_p^*, X_p^*) + l_r' - b_r' < gE[P(S_p^*, X_p^*)]$, which means that the convenience yield of the oil for this specific refiner is above the one set for the marginal plant trading normally in the market. We can compute the value of the options for this refiner using (11) and (12) and $l_r' - b_r'$. Then subtracting from these equations the value of the marginal non-disrupted refiner we obtain a difference of

$\sum_{n' \leq n} \sum_{j'} pr\left(S_p^*, X_p^*, j' \mid S, X, i, n'\right) g^{n'} \left[-l_r'(X_{pp}^*) + b_r'(X_{pp}^*) + l'(X_{pp}^*) - b'(X_{pp}^*)\right]^+$ in the put option and of $\sum_{n' \leq n} \sum_{j'} pr\left(S_p^*, X_p^*, j' \mid S, X, i, n'\right) g^{n'} \left[l_r'(X_{pp}^*) - b_r'(X_{pp}^*) - l'(X_{pp}^*) + b'(X_{pp}^*)\right]^+$ in the call option, which are the values appropriated by the SPR by providing the risk hedging service to disrupted refiners that are willing to pay a premium to acquire and exercise the options. Since we presume financial markets operate competitively speculators capture none of the profit. This leads to higher prices on the left of (11) and (12) and the government sells above the expected price of oil. ■

Proof of Proposition 3.4: From the refiners' arbitrage equation we have $P(S, X, i, S_p^*, X_p^*) - gE[P(S_p^*, X_p^*) \mid S, X, i] = b'(X_p^*) - l'(X_p^*)$. Note that $b'(X_p^*) - l'(X_p^*)$ is monotonically nonincreasing since $b(X_p^*) - l(X_p^*)$ is concave. We treat two cases, the refiner increasing and decreasing inventory. If the refiner increases inventory from *X* to X^* it has revenues of $(P(S, X, i, S_p^*, X_p^*) - gE[P(S_p^*, X_p^*) \mid S, X, i])(X_p^* - X)$ and has a change in convenience benefit of $\int_X^{X^*} [b'(z) - l'(z)]dz$. Since $b'(X_p^*) - l'(X_p^*)$ is monotonically nonincreasing, and $P(S, X, i, S_p^*, X_p^*) - gE[P(S_p^*, X_p^*) \mid S, X, i] = b'(X_p^*) - l'(X_p^*)$ holds at the upper limit of the integral, *X**, for each *z* the following integrand is positive and

$\int_{X}^{X^*} \left[b'(z) - l'(z) - P(S, X, i, S_p^*, X_p^*) + gE[P(S_p^*, X_p^*) \mid S, X, i]\right] dz > 0$. If the refiner decreases its inventory and takes a forward position to replace the inventory reduction, this equation becomes $\int_{X}^{X^*} \left[-b'(z) + l'(z) + P(S, X, i, S_p^*, X_p^*) - gE[P(S_p^*, X_p^*) \mid S, X, i]\right] dz > 0$. The inequality holds here because $P(S, X, i, S_p^*, X_p^*) - gE[P(S_p^*, X_p^*) \mid S, X, i] = b'(X_p^*) - l'(X_p^*)$ holds at X^* and the sign of the integrand is reversed. Thus, refiners can take advantage of the relative moves between spot and forward prices by changing inventory levels and increasing profitability. ■

References

Allaz, B., 1992. Oligopoly, Uncertainty and Strategic Forward Transactions. *International Journal of Industrial Organization*, 10 (2): 297-308.

Bai, Y., D. Q. Zhou, P. Zhou, L. B., Zhang, 2012. Optimal Path for China's Strategic Petroleum Reserve: A Dynamic Programming Analysis. *Energy Economics*, 34, 1058-1063.

BP, 2012. BP Statistical Review of World Energy. www.bp/com/statisticalreview.

Chao, H-P and A. Manne. 1983. Oil Stockpiles and Import Reductions: a dynamic programming approach, *Operations Research*, 31(4) 632-651.

Considine, T.J, and D.F. Larson. 2001. Risk Premiums on Inventory Assets: The Case of Crude Oil and Natural Gas, *The Journal of Futures Markets*, 21(2), 109–126.

Considine, T.J., and K.M. Dowd. 2005. A Superfluous Petroleum Reserve? *Regulation*, Summer, 18-25.

Considine, T.J. 2006. Is the Strategic Petroleum Reserve our Ace in the Hole? *Energy Journal*, 27(3), 91-112.

Department of Energy, 2009. http://energy.gov/data/open-energy-data

Devarajan, S., and R. G. Hubbard. 1983. Drawing down the strategic petroleum reserve: the case for selling futures contracts, in *Oil Shock: Report of the Harvard Energy Security Program*, ed. A. L. Alm, R. J. Weiner, pp. 187-96. Cambridge, Mass, Ballinger.

Energy Information Administration, 2009. http://www.eia.gov/petroleum/.

Fan, Y., X.-B. Zhang, 2010. Modelling the Strategic Petroleum Reserves of China and India by a Stochastic Dynamic Game. *Journal of Policy Modeling*, 32: 505-519.

Gordon, R.L., 1992. Energy intervention after Desert Storm: Some unfinished tasks, *Energy Journal*, (13) 4 1-15.

Hogan, W.W. 1983. “Oil Stockpiling: Help Thy Neighbor,” *Energy Journal*, 4, 49-72.

Hogan, W.W. 2002. Energy modelling for policy studies, *Operations Research*, 50(1), 89-95.

Hull, J. C., 2007 “Options, Futures, and Other Derivative Securities,” Seventh Edition, Prentice Hall Inter. Editions, New Jersey.

International Energy Agency, 2010. Oil & Gas Security – Emergency Response of IEA Countries – New Zealand. http://www.iea.org/publications/freepublications/publication/name,3942,en.html

International Energy Agency, 2007. Oil Supply Security. Emergency Response of IEA Countries. http://www.iea.org/publications/freepublications/publication/name,3942,en.html

Kaldor, N., 1939. Speculation and Economics Stability. *The Review of Economic Studies*, 7 (1): 1-27.

Milonas, N.T. and T. Henker. 2001. Price spread and convenience yield behaviour in the international oil market, *Applied Financial Economics*, 11, 23-36.

Murphy, F.H., M. Toman and H. Weiss. 1986. An Integrated Analysis of U. S. Oil Security Policies, *Energy Journal*, 7(3) 67-82.

Murphy, F.H., M. Toman and H. Weiss. 1987. A Nash Model of Consuming Nations' Strategic Petroleum Reserves, *Management Science*, 33(4), 484-499.

Murphy, F.H., M. Toman and H. Weiss, 1989. A Dynamic Nash Game Model of Oil Market Disruptions and Strategic Stockpiling. *Operations Research*, 37 (6): 158-171 1989.

Murphy, F.H., and F. S. Oliveira, 2010. Developing a Market-Based Approach to Managing the US Strategic Petroleum Reserve. *European Journal of Operational Research*, 206 (2), 488-495.

Oren, S. and S.H. Wan, 1986. Optimal Strategic Petroleum Reserve Policies: a steady State Analysis, *Management Science*, 32(1) 14-29.

Pindyck, R.S. 2001. The Dynamics of Commodity Spot and Futures Markets: A Primer, *Energy Journal*, 22(3) 1-29.

Samouilidis, J. E. and V. F. Magirou, 1985. On the optimal level of a small country's strategic petroleum reserve, *European Journal of Operational Research*, *20*(*2*): *190-197.*

Tang, C., and B. Tomlin, 2008. The power of flexibility for mitigating supply chain risks. *International Journal of Production Economics*, 103: 451-488.

Taylor, J. and P. Van Doren. 2005. The Case Against the Strategic Petroleum Reserve, Cato Institute, No. 555.

Teisberg, T.G., 1982. A Dynamic Programming Model of the U.S. Strategic Petroleum Reserve. *Bell Journal of Economics*, 13, 341-353.

Working, H., 1949. The Theory of Price Storage. *The American Economic Review*, 39 (6): 1254-1262.

Wu, G., Y.-M. Wei, C. Nielsen, X. Lu, M. B. McElroy, 2012. A Dynamic Programming Model of China's Strategic Petroleum Reserve: General Strategy and the Effect of Emergencies. *Energy Economics*, 34, 1234-1243.