ANSARIPOOR, A.H. and OLIVEIRA, F.S. 2018. Flexible lease contracts in the fleet replacement problem with alternative fuel vehicles: a real-options approach. *European journal of operational research* [online], 266(1), pages 316-327. Available from: https://doi.org/10.1016/j.ejor.2017.09.010

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ANSARIPOOR, A.H. and OLIVEIRA, F.S.

2018

A. H. Ansaripoor and F. S. Oliveira, 2018. Flexible Lease Contracts in the Fleet Replacement Problem with Alternative Fuel Vehicles: A Real-Options Approach. *European Journal of Operational Research,* 266 (1): 316 – 327.

Flexible Lease Contracts in the Fleet Replacement Problem with Alternative Fuel Vehicles: A Real-Options Approach

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Abstract: We study the fleet portfolio management problem of a firm that aims to minimize its cost and risk (Recursive Expected Conditional Value at Risk), simultaneously, in a stochastic multi-period setting by deciding which technologies to use in its fleet. We propose a model using real options (return and swap) to account for different uncertainties (technological and regulatory change and demand shifts). We analyze how changes in fuel prices and technological change influence the value of the options. We validate the results using a real-world case study conducted in the UK.

Keywords: Transportation, Fleet Replacement, Real Options, Risk Management, Sustainability.

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1. Introduction

Leasing contracts are an important component of fleet management decisions. Leasing companies and (large) firms negotiate lease agreements and their conditions. Then, after negotiating with various leasing companies, a firm decides which lease company wins the contract.

An important development in this market is the increased importance of electrical vehicles (EVs) as batteries become cheaper and better. Book et al. (2009) have predicted that in China, Japan, North America, and Western Europe, 1.5 million EVs will be sold in 2020, accounting for some 2.7% of the total automotive market in these regions. In terms of market segments, EVs are most likely to be introduced in the city car segment, where they will take the form of small city cars used mainly for commuting within the city. Dinger et al. (2010) have forecasted that battery costs will decline steeply as production volumes increase. Individual parts will become less expensive thanks to the effects of experience and scale. Equipment costs will also drop, lowering depreciation. Their analysis suggests that from 2009 to 2020, the price of NCA (Nickel Cobalt Aluminium) batteries, which are those mostly used in EVs, will have decreased by 60 to 65 percent. In addition, (Nykvist and Nilsson, 2015) show that industry-wide cost estimates decreased by approximately 14% annually between 2007 and 2014, from above US$1,000 per kWh to around US$410 per kWh, and that the cost of battery packs used by market-leading EV manufacturers are even lower, at US$300 per kWh, and have decreased by 8% annually. On the other hand, other technologies such as diesel and petrol may not be economically efficient in the future. Moreover, these are mature technologies with highly negotiable lease prices, even for shorter lease contracts. Therefore, in order to better manage the replacement policy for a fleet of vehicles, the decision makers need to understand the impact of technological evolution on the performance of the leased vehicles.

Moreover, besides technological change, there are other factors that may affect the value of a technology such as a regime shift in fuel prices, a modification in the regulatory environment, or changes in the demand profiles for the services of the fleet vehicles (e.g., G´omez et al., 2011). Regime changes (e.g., Janczura and Weron, 2010) in fuel prices seem to have become ever more frequent with strong shifts in natural gas prices and petroleum prices, moving significantly within a very short time, and remaining at the new levels for a long and unpredictable time. Regulatory uncertainty also plays a role in increasing the risk inherent in lease contracts and investment in general. For example, the increasing pro-EV legislation in several countries (e.g., Jensen et al., 2014), by banning older polluting vehicles from the centre of cities, by restricting fossil fuel vehicles to be used during a limited number of days per week, or by placing a ban on these types of vehicles altogether (as was the case recently in several Scandinavian countries), has a strong impact on the value of a real option by increasing contract flexibility. A more mundane and simple example where these options may create value is when a firm is able to better deal with an unforeseen demand shift as a result of which a van that was perfect for a given task at the time of the lease may not be optimal for the duration of the contract, either due to little or excessive use.

Currently, companies tend to deal with these issues by continuing to use vehicles that are not as efficient as they could be (in terms of technological change) or buy transferring the inappropriate vehicles to other departments to be used for tasks for which they were not designed in the first place. For these main reasons, it is important to analyse how contract design can be used as an important management tool, and to measure the value that a firm can derive from having flexible contracts. This important issue of contract flexibility enables a firm to better adapt to technological and regulatory changes, and to unforeseen shifts in demand conditions.

The major contribution of this article is that it provides contract flexibility by incorporating various options in the lease contracts, whereby contracts can be changed if the lessees pay a price to exercise the option. These options allow a firm to better adapt to changes in what is required of the leased vehicles and to take into account the technological life-cycle of EVs. We study how real options can be used to provide flexibility for the leasing contracts in the fleet system, taking into account the technological development of EVs and the possibility of changes to what is required of the leased vehicles. Moreover, we analyse how the use of options not only affects the expected cost, but also the risk of the leasing contract. In this article, we use the Recursive Expected Conditional Value at Risk (RECVaR) developed by Ansaripoor et al. (2016) as a risk measure.

This article is structured as follows. In Section 2, we present a broad review of the literature pertaining to fleet replacement problems and real options, and their different applications, specifically in terms of environmental and sustainability development issues. In Section 3, we develop a general model for flexible lease contracts comprising different options. In Section 4, we derive the analytical results. In Section 5, we present the computational results and develop insights based on the solved model. In addition, in Section 5, we extend our analysis by considering the technological changes that have affected EVs. Section 6 concludes the article.

2. Literature Review on the Fleet Replacement and Real Options

Decision support systems for fleet operations, capacity decisions, routing problems, and humanitarian operations are well represented in the logistics literature (e.g., Couilard, 1993; Lau et al., 2003; Ghiani et al., 2004; Pedraza-Martinez et al., 2011, 2013; Eftekhar et al., 2013). For example, the literature on humanitarian operations has studied fleet management in the context of both relief operations and development programs. The focus of humanitarian operations varies to reflect the different transportation requirements of the three objectives: maximizing responsiveness and demand coverage for relief operations; reducing costs; and increasing fleet utilization for development programs (Pedraza Martinez et al., 2011). We focus our review on parallel replacement models and real options.

We also give a brief introduction to risk measures, all of which are central to the issues discussed in this article.

2.1. Parallel Replacement Models and Real Options

These models can generally be categorised into two main groups based on different fleet (asset) characteristics: homogenous and heterogeneous. In the homogeneous replacement model, a group of similar vehicles in terms of type and age that form a cluster (each cluster or group cannot be decomposed into smaller clusters) have to be replaced simultaneously. In the heterogeneous model, multiple heterogeneous assets, such as fleets with different vehicle types, have to be optimised simultaneously. For instance, vehicles of the same type and age may be replaced at different periods (years) because of a firm's restricted budget for the procurement of new vehicles. The heterogeneous models are closer to the real-world commercial fleet replacement problem. The problems associated with such models are solved by integer programming and, generally, the input variables are assumed to be deterministic (e.g., Simms et al., 1984; Hartman, 1999, 2000, 2004). Generally, dynamic programming is the methodology applied to solve homogenous models. The advantage of the homogenous model is that it assumes probabilistic distributions for input variables in the optimisation model (e.g., Bellman, 1955; Bean et al., 1984; Oakford et al., 1984; Hartman, 2001; Hartman and Murphy, 2006).

As an example of parallel replacement, Keles and Hartman (2004) studied the fleet replacement policies for a city transit bus operator in Europe. The main factors that were included in their replacement decisions were the ability to select from multiple manufacturers, as well as purchase price, government regulations, capital budgeting constraints and economies of scale.

Although the machine or vehicle replacement literature is replete with models that deal with budget constraints (Chand et al., 2000), variable utilisation (Bethuyne, 1998), stochastic demands (Hartman, 2001), heterogeneous vehicle types (Hartman, 2004), and humanitarian operations (Pedraza-Martinez et al., 2011, 2013; Eftekhar et al., 2013), to our knowledge, flexible lease contracts and risk management issues have yet to be considered.

Because in this article we use real options as a tool to increase contract flexibility, we briefly examine this concept and its importance in the management science literature. The concept of real options can be traced back to Myers (1977) who used options to study investment projects in real assets. Indeed, a real option is a right to make a decision, as in for instance, an option to make a capital investment. In contrast to financial options, a real option is not tradable. For example, a firm's owner cannot sell the option to extend his/her company to another person; only s/he can make this decision for the company. In addition, short-term lease contracts are a characteristic of many business companies; for example, apartment leasing or service operations generally involve expensive facilities for doing special tasks (e.g., Grenadier, 1995). With the real options methodology, a project is assumed to incorporate an option which determines the generation of cash flow, and the optimal investment policies are just the optimal rules for exercising the option (e.g., Anderson, 2014).

Real options have been used for capital planning in many applications in different industries. Specifically, early real options literature focused mainly on the oil and gas upstream industry (e.g., Brennan and Schwartz, 1985; Paddock and Siegel, 1988). For example, Murphy and Oliveira (2010, 2013) proposed the use of option contracts as instruments to manage the US Strategic Petroleum Reserve, as they signal the government commitment to act during a disruption, provide more risk-management opportunities to the refinery industry, and reimburse some of the costs of maintaining the reserve. Since our research relates to environmental and sustainability development in different industries, we focus on the literature related to this application of real options.

Avadikyan and Llerena (2010), by taking a real options reasoning approach, provided a more robust justification of companies' investment decisions regarding hybrid vehicles (HVs) as a technological strategy in order to have the flexibility to overcome market and policy uncertainties. Specifically, they introduced four types of growth option strategies for HVs, which are: (1) an option to keep the existing technological situation and providing a long-term hedging strategy for coping with uncertainties; (2) an option to limit HV project risks; (3) an option to diversify; and (4) a platform with an internal flexibility option.

By using a real option approach, Kleindorfer et al. (2012) have extended the EV adoption decision after considering the effects of uncertainty in fuel, carbon, and battery prices. They proposed a model for optimal EV-Internal combustion vehicles (ICV) replacement decisions in a dynamic setting in the fleet system of a company in France's postal sector. In their model, there is no flexibility within the terms of the leasing contract, and they have assumed a six-year planning horizon for it. Moreover, they have considered four replacement policies which are (1) ICV-only policy; (2) Static policy; (3) Dynamic policy; and (4) Perfect information policy. In the ICV-only policy, just ICV replacement decisions are considered. In the Static policy, decisions for the entire planning horizon are made at the beginning of the horizon, and in Dynamic policy, decisions are made in each time period, based on the awareness of the latest uncertainties. Finally, in the hypothetical case of Perfect information policy, the decision-maker has perfect information about future uncertainty realizations.

To sum up, we understand that there is also a gap in the literature as no researcher to date has tackled the issue of replacing fleet vehicles with alternative fuel vehicles, and using real options analysis to design leasing contracts that are flexible enough to accommodate existing uncertainties. Specifically, our approach is different from those in recent literature pertaining to the adoption of new vehicle technologies (e.g., Drake and Spinler, 2013; Kleindorfer et al., 2012; Wang et al., 2013), in terms of stochastic parameters in the model and different options for the leasing contracts. In this article, we extend our approach to the problem that we considered in our previous work, (Ansaripoor et al., 2014, 2016) by taking into account the uncertainties that exist in a real-world situation. These uncertainties are CO_2 prices, fuel prices, mileage covered by a vehicle, and fuel consumption. We design a lease contract that gives us three vehicle-leasing options. The first choice is the base contract that has no option over four years. However, if the car is returned, the penalty is very high. The second alternative is to lease the vehicle with the option to return the vehicle by paying a small penalty. Finally, the third choice is to lease the vehicle with the swap option where a small penalty is paid for returning the vehicle and choosing another vehicle. In order to evaluate these leasing contracts, we can use real option theory.

2.2. Measures of Risk Based on Value at Risk

In recent years, authorities have taken a greater interest in the effects of unexpected losses connected with extreme events affecting financial markets. As a consequence, more attention is being given to the risks taken by financial institutions. This is the background that explains the choice of Value at Risk (VaR) as a synthetic risk measure, which can express the market risk of a financial asset or of a portfolio. VaR (e.g., Anderson, 2014) measures the worst expected loss for a given time period under normal market confidence at a given confidence level for the period. However, VaR is unstable and difficult to work with when losses are not normally distributed (Rockafellar and Uryasev, 2000).

An alternative risk measure that does quantify the losses that might be encountered in the distribution tail is Conditional Value at Risk (CVaR); VaR calculates the maximum loss expected with a degree of confidence, whereas CVaR calculates the expected value of losses if they are greater than or equal to the VaR and it takes into account the magnitude of losses (Rockafellar and Uryasev, 2000). CVaR is a consistent risk measure since it is sub-additive and convex (Artzner et al., 1997). Moreover, it can be optimized using linear programming and non-smooth optimization algorithms, which allow the handling of portfolios with a very large number of instruments and scenarios (Rockafellar and Uryasev, 2000). However, despite these advantages, in a dynamic setting CVaR does not satisfy the time consistency principle (Shapiro, 2011; Boda and Filar, 2006).

Ansaripoor et al. (2016) introduced a recursive formulation of CVaR, the RECVaR, which takes into account the time consistency issue (Rudloff et al., 2014; Detlefsen and Scandolo, 2005) in a dynamic setting. Their approach is different from that of Shapiro (2009, 2011), where the cost-to-go function concept was used to satisfy the time consistency principle, as they provide a recursive formulation of the CVaR for a scenario tree, explicitly computing the CVaR of the parent node as a function of the CVaR of the respective children nodes. Ansaripoor et al. (2016) have concluded that the RECVaR provides more intuitive and robust results because it takes into account the risks that exist in the middle stages of the scenario tree, whereas Shapiro's (2011) formulation is not sensitive to these kinds of risks. For these reasons, we use RECVaR as a risk measure.

3. A General Model for a Fleet Management System with Flexible Lease Contracts

In this section, we introduce a multi-stage stochastic programming model in order to obtain the optimal number of vehicles to be leased, taking into account the constraints to minimize expected cost and risk, during the planning horizon. As shown in Figure 1, we consider three types of contracts including different options. The first choice is the base contract that has no option. However, if the car is returned, the penalty cost is very high. With the second type of contract, the vehicle is leased with the option to return it by paying a small penalty. Finally, the third one is the swap contract which allows the lessee to pay a very small penalty for returning the vehicle and selecting another vehicle. The notation used is summarized in Tables 1(a) and 1(b). In addition, in Figure 1, at each node, $\xi_{ni} = (c_n^p, f_{ni}, D_n, o_n)$ is a vector of stochastic processes which are CO_2 prices, fuel prices, mileage driven, and fuel consumption for fossil fuel technologies, per 100 miles, respectively.

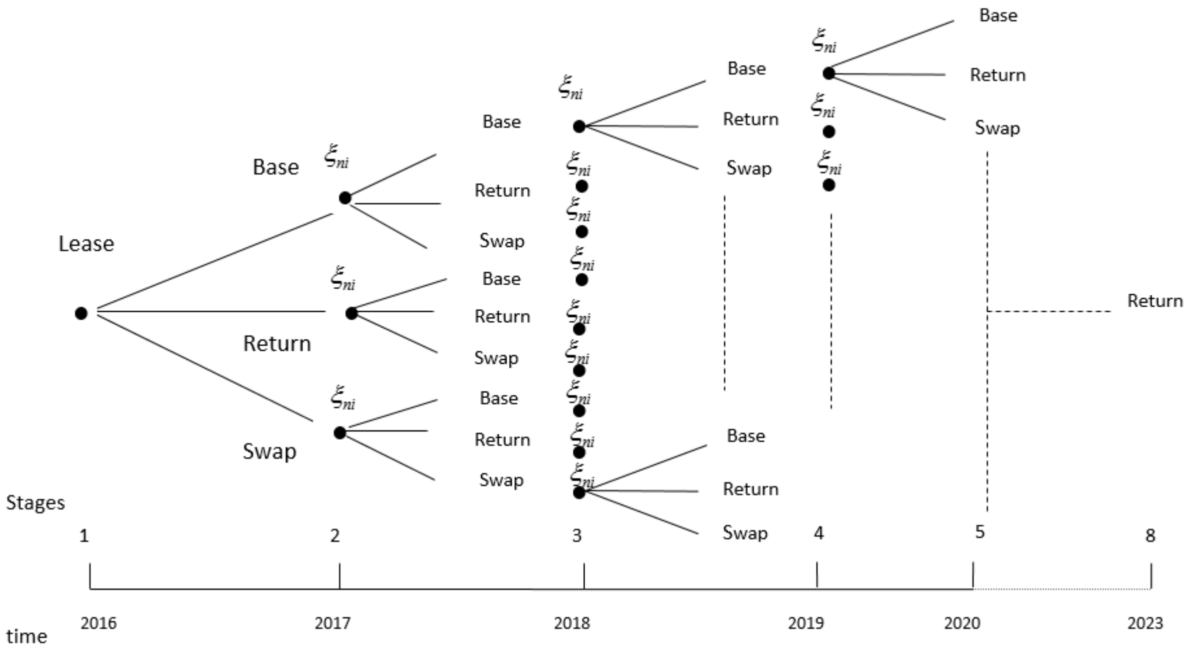

Figure 1: The node-based tree for a generic lease contract with options to choose from the base contract, the return early contract, and the swap contract

We consider five technologies: fossil fuels (petrol, diesel), hybrids (petrol, diesel), and EVs. In equation (1), r_{ni} and o_n represent the running costs and the consumption of fossil fuels, and hybrids, per 100 miles, at each node, respectively. In addition, the running cost for EVs, per 100 miles, r_{ni}, is calculated using equation (2). We also take into account the cost of fuel prices for each technology and CO_2 emissions, at each node, in (1) and (2), by including the parameters f_{ni} and c_n^p, respectively. Furthermore, c_i^g denotes the CO_2 emissions (g/litre) for fossil fuels and hybrids, and c^e shows the CO_2 emissions for EVs (g/mile).

$$r_{ni} = o_n(f_{ni} + c_n^p c_i^g) \qquad \forall n \in N,\ i = \text{fossil fuels, hybrids} \tag{1}$$

$$r_{ni} = f_{ni} + 100 c^e c_n^p \qquad \forall n \in N,\ i = \text{electric} \tag{2}$$

So, based on equations (1) and (2), we can calculate the total annual running cost per vehicle at each node, λ_{ni}, using equation (3), in which D_n represents annual mileage driven at node n.

$$\lambda_{ni} = \frac{r_{ni}}{100} D_n \qquad \forall n \in N, i \in I \tag{3}$$

Table 1(a). Indices and variables of the model

$i \in I$={fossil fuels, hybrids, and electric}

$a \in A$= {1, 2,.., A} index for age of the vehicles

$n \in N$= {1,2,...,N}index for nodes in scenario tree

$t \in T$= {1, 2, .. ,T}index for time periods in year over planning horizon

$s_t \in S$= {1, 2,.., S_t} index for number of branches (states) at each stage

c$\in C$={1,2,...,C} index for different types of contracts

$\Psi_{n,m}$: Tree structure for parent nodes n and child nodes m

$\Omega_{t,n}$: Tree structure for parent nodes n and stage t

x_{niac}: Total number of vehicles with technology i, age a, contract c currently leased at node n

y^{+}_{nic}: Number of new vehicles with technology i, contract c, which company leases at node n

y^{-}_{niac}: Number of vehicles with technology i , age a , contract c, which company returns at node n

α_n^{β}: Value at risk at confidence level of β at node n

Π_n : Auxiliary variable for linearization of minimum function

ϕ_n^{β}: Conditional value at risk at confidence level of β at node n

z_n: Positive stochastic variables for loss function at node n

L_n: Loss function at node n

Q_n: Total expected cost function at node n

The total investment (fixed) cost per vehicle is represented by (4) for fossil fuels and hybrid technologies, and by (5) for EVs. As we are taking into account the leasing contracts for different types of vehicles in the fleet system, we use the annual lease cost which is represented by l_i, to obtain the fixed cost at each node. Moreover, for EVs we have an extra investment cost, which is the annual leasing cost of batteries, represented in equation (5) by M_e.

$$\mu_i = l_i \qquad i = \text{fossil fuels, hybrid} \qquad (4)$$

$$\mu_i = l_i + M_e \qquad i = \text{electric} \qquad (5)$$

Table 1(b). Parameters of the model

ρ: Coefficient for relation between value and price of an option

ω: Parameter for trade-off of risk and cost in the objective function

β : Confidence level for calculating RECVaR and VaR

V°_{niac}: Value of the option for technology type *i*, age *a*, contract *c* at node *n*

P_{niac} : Premium of the option for technology type *i*, age *a*, contract *c* at node *n*

γ_{iac} : Penalties for returning the vehicles with technology *i,* age *a*, contract *c*

θ_c : Coefficient vector for penalties of contract *c*

δ : Annual learning rate for the technological development of batteries for EVs

h_n: Number of vehicles at node *n*

f_{ni}: Fuel price for technology *i*, at node *n*

o_n: Fuel consumption, litres per 100 mile, at node *n*

D_n: Annual mileage covered at node *n*

Q_{nia} : Expected cost per vehicle for technology *i*, age *a*, at node *n*

r_{ni}: Running cost per 100 miles for technology *i* at node *n*

c^p_n : CO_2 prices, per g, at node *n*

c^e: CO_2 emissions, g per mile, for electrical technology

c^g_i : CO_2 emissions, g per litre, for fossil fuel/hybrid technology

l_i: Annual lease cost for each technology *i*

M_e: Annual lease cost for EV batteries

λ_{ni}: Total annual running cost per vehicle for technology *i* at node *n*

μ_i: Total annual fixed cost per vehicle for technology *i*

The penalty for returning the vehicles, γ_{iac}, is calculated by equation (6), where θ_c is the coefficient vector of penalties for different type of contracts and *A* is the maximum age of the vehicle during the leasing period. Moreover, equations (7)-(9) show the value of different options.

$$\gamma_{iac} = \theta_c (A - a)\mu_i \qquad a<A \qquad (6)$$

Equation (7) represents the value of the contract without any option, i.e., the base contract. Equations (8) and (9) represent the value of the contracts that include the return and swap options, respectively. In addition, in equations (7)-(9), Q_{nia} is the expected cost per vehicle for each technology at age a, which is calculated by equation (10).

$$V^{0}_{nia1} = 0 \quad \forall n \in N, i \in I, a \in A \tag{7}$$

$$V^{0}_{nia2} = \max(0, Q_{nia} - \gamma_{ia2}) \quad \forall n \in N, i \in I, a \in A \tag{8}$$

$$V^{0}_{nia3} = \max(0, Q_{nia} - \min_{i \neq j}(Q_{nij}) - \gamma_{nia3}) \quad \forall n \in N, i \in I, a \in A \tag{9}$$

$$Q_{nia} = \mu_i + \lambda_{ni} + \frac{1}{S_t} \sum_{\psi(n,m)} Q_{mi(a+1)} \quad \forall n \in N, i \in I, a \in A \tag{10}$$

Finally, the premium of each option, P_{niac}, is obtained by equation (11), in which ρ is a coefficient for relation between value and price of an option and is between 0 and 1. In addition, V^{0}_{niac} is the value of the option for technology type i, age a, contract c, at node n.

$$P_{niac} = \rho V^{0}_{niac} \quad \forall n \in N, i \in I, a \in A, c \in C \tag{11}$$

Our objective is to minimize the weighted average of RECVaR and the expected cost at the root node. The firm aims to address the mixed integer multi-stage stochastic programming (MIP) model using equations (1)-(25). The objective function (12) minimizes the weighted average of expected cost, Q_1, and RECVaR, ϕ_1^{β}, at the root node. If ω ($0 \le \omega \le 1$) equals 1, only the expected cost is minimized, and if ω is equal to zero, only RECVaR is minimized. We have four decision variables. The first decision variable is y^{+}_{nic} which denotes the number of new vehicles with technology i and contract c which are replaced at node n. The second one is y^{-}_{niac} which represents the number of vehicles with technology i, age a, contract c that are returned at node n. The third one is VaR, at each node, α_n^{β}. Finally, the last one is z_n, which denotes the losses that are beyond VaR at each node.

$$\underset{y^{+}_{nic}, y^{-}_{niac}, \alpha_n^{\beta}, z_n}{Min} \quad \omega Q_1 + (1-\omega)\phi_1^{\beta} \tag{12}$$

s.t.

$$\sum_a \sum_c x_{1i1c} = h_n \quad \forall n \in N, i \in I \tag{13}$$

$$y^{+}{}_{nic} = x_{ni1c} \quad \forall n \in N, i \in I, c \in C \tag{14}$$

$$y^{+}{}_{nic} \text{ and } y^{-}{}_{niac} = 0 \quad \forall n \in \Omega_{t,n}, i \in I, a \in A, c \in C \text{ if } t \geq T\text{-}A\text{-}1,\ a\text{=}1 \tag{15}$$

$$x_{miac} = y^{+}_{mic} + (x_{ni(a-1)c} - y^{-}_{miac}) \quad \forall i \in A, a \in A, c \in C, (n,m) \in \Psi_{n,nn} \tag{16}$$

$$\sum_{i}\sum_{a}\sum_{c} x_{niac} \geq h_n \qquad \forall n \in N \tag{17}$$

$$\begin{aligned} L_n = {} & \sum_{i}\sum_{a}\sum_{c}(\lambda_{ni} + \mu_i + P_{niac})x_{niac} / 10^6 + \sum_{i}\sum_{a=2}^{A}\gamma_{ia1}(y^{-}_{nia1} + y^{-}_{nia3}) \\ & + \sum_{i}\sum_{a=2}^{A}\gamma_{ia2}(y^{-}_{nia2}) \text{ -} \sum_{i}\sum_{a=2}^{A}(\gamma_{ia1} - \gamma_{ia3})\Pi_n \qquad \forall n \in N \end{aligned} \tag{18}$$

$$\Pi_n \leq \sum_{i}\sum_{a=2}^{A} y^{-}_{nia3} \quad \forall n \in N \tag{19}$$

$$\Pi_n \leq \sum_{i}\sum_{c} y^{+}_{nic} \quad \forall n \in N \tag{20}$$

$$Q_n = L_n + \frac{1}{S_t}\sum_{\Psi(n,m)}(Q_m) \quad \forall (n,m) \in \Psi_{n,m} \tag{21}$$

$$z_m \geq L_m - \alpha_n^{\beta} \quad \forall (n,m) \in \Psi_{n,m} \tag{22}$$

$$\phi_n^{\beta} = \alpha_n^{\beta} + \frac{1}{S_t(1-\beta)}\sum_{\Psi(n,m)}(z_m) + \frac{1}{S_t}\sum_{\Psi(n,m)}(\phi_m^{\beta}) \qquad \forall (n,m) \in \Psi_{n,m} \tag{23}$$

$$\alpha_n^{\beta} = 0 \qquad \forall n \in N \tag{24}$$

$$x_{niac}, y^{+}_{nic}, y^{-}_{naic}, \Pi_n \in Z^{+}, \text{ and } \alpha_n^{\beta},\ z_n \in R^{+} \tag{25}$$

Constraints (13) show the initial condition of the fleet system at each node, h_n, and should be equal to the total number of the vehicles under different types of contracts, $\sum_{c} x_{1i1c}$, at the root node. In constraints (14), we determine the number of new leased vehicles with a specific contract, at each node, $y^{+}{}_{nic}$, required to replace the retired vehicles at the corresponding node, x_{ni1c}. In addition, constraints (15) show that the planning horizon for decision variables, $y^{+}{}_{nic}$ and y^{-}_{niac} is A years and after that there will be no new leased vehicles, and no returned vehicles in the fleet system. In addition, the vehicles that are at age one (a=1) cannot be returned. Constraints (16) show that the total number of vehicles, at each child node, x_{miac}, is equal to the number of new leased vehicles, y^{+}_{mic}, plus the number of vehicles which are left, $x_{ni(a-1)c} - y^{-}_{miac}$, after returning vehicles, y^{-}_{miac}, with age more than one. Moreover, constraints (17) indicate that the total number of vehicles for all technologies, contracts, and ages, $\sum_{i}\sum_{a}\sum_{c} x_{niac}$, at each node, should be greater than or equal to the number of vehicles which are needed, h_n, at the corresponding node. Constraints (18) show the total loss function (total cost), L_n, at each node. The first term in constraints (18), $\sum_{i}\sum_{a}\sum_{c}(\lambda_{ni} + \mu_i + P_{niac})x_{niac} / 10^6$, represents the sum of running cost, λ_{ni}, fixed cost, μ_i, premium of the options, P_{niac}, at the corresponding node. The second term, $\sum_{i}\sum_{a=2}^{A}\gamma_{ia1}(y^{-}_{nia1} + y^{-}_{nia3})$, shows the penalty cost for returning the vehicles for base and swap contracts, and the third term, $\sum_{i}\sum_{a=2}^{A}\gamma_{ia2}(y^{-}_{nia2})$, represents the penalty cost for contracts with the return option. Finally, the fourth term, $\sum_{i}\sum_{a=2}^{A}(\gamma_{ia1} - \gamma_{ia3})\Pi_n$, shows the amount of money that is refunded when the swapping option is selected. Constraints (19-20) are represented for the linearization of minimum function, $\Pi_n = \min\left(\sum_{i}\sum_{c} y^{+}_{nic}, \sum_{i}\sum_{a=2}^{A} y^{-}_{nia3}\right)$, used in (18). Constraints (21) represent the recursive formula for calculating the total expected cost function, at each node, Q_n, which is equal to the loss function, L_n, at the corresponding node plus the average of cost functions, $\frac{1}{S_t}\sum_{\Psi(n,m)}(Q_m)$, in successor nodes. In order to take into account the time consistency issue of CVaR (Shapiro, 2011), we have used constraints (22) and (23), which have been proven to be time-consistent (Ansaripoor et al., 2016). In addition, constraints (24) show that the Value at Risk (VaR), α_n^{β}, in the final stage should be zero because, in this stage, there is no uncertainty and all values of stochastic processes have been realized. Finally, constraints (25) show the integer values of, x_{niac}, y^{+}_{nic}, y^{-}_{niac}, Π_n, and non-negative, α_n^{β} and z_n, decision variables.

4. Analysing the Main Properties of the Model

In this section, we consider the impact of using option contracts on the expected cost and risk (RECVaR) of the fleet. Firstly, we test whether we can reduce the expected cost of the fleet system by using option contracts. This is the conventional goal of risk-neutral fleet managers. In order to provide a formal proof for it, in Proposition 1, we firstly provide a prerequisite proposition and then we proceed with the main proposition related to the effect of using option contracts on the expected cost of the fleet.

Let ΔL_{nic} stand for the change in the loss function per vehicle, type *i*, for contract *c*, at node *n*, compared to the case without using any contract *c*; let ΔQ_{niac} represent the change of expected cost per vehicle for contract *c*, at age *a*, at node *n*, compared to when there is no contract *c*; let $E(\Delta L_{mic})$ be the expected change of loss function per vehicle, type *i*, at child *m*, at age one, compared to the case without using any contract *c*, and $EE(\Delta L_{m^2c})$ represent the expected of expected change of loss function per vehicle, type *i*, at child node of child node *m*, at age 2, compared to the case without using any contract *c*, and $EE....E(\Delta L_{m^Aic})$ represent the expected of expected change of loss function per vehicle, type *i*, at child node of child node *m*, at age *A*, compared to the case where there is no contract *c*.

Proposition 1: *If* $0 < \gamma_{ia3} < \gamma_{ia2} < \gamma_{ia1}$ *then the change of the expected cost at parent node n,* ΔQ_{niac}*, for contract c , at age a, compared to the case without using any contract c is :*

$$\Delta Q_{niac} = \Delta L_{nic} + E(\Delta L_{mic}) + EE(\Delta L_{m^2ic}) + ...EE....E(\Delta L_{m^{A-1}ic}) + EE....E(\Delta L_{m^Aic})$$

Proposition 2: *If* $0 < \gamma_{ia3} < \gamma_{ia2} < \gamma_{ia1}$ *then*

$$V^b_{niac} = (\Delta Q_{niac} - \gamma_{niac})^+ = (A-a)[\bar{\lambda}_{(A-a)ic} + \Delta\mu_i - \theta_c \mu_i)]^+ \quad \forall n \in N, i \in I, a \in A, c \in C,\ a \geq 2$$

Proposition 2 explains that if we return a vehicle with contract *c*, at age *a*, at node *n*, the amount which is saved, compared to the case where there is no contract *c*, is equal to the change of expected cost per vehicle of type *i*, contract *c,* at parent node *n*, and at age *a*, minus the penalty, γ_{iac}, which is incurred if the vehicle is returned under contract *c*. In proposition 2, $\bar{\lambda}_{(A-a)ic}$ denotes the average running costs for the remaining periods at age *a*, for technology *i,* and option contract *c*. The penalty can be obtained by equation (6) for each contract. The amount which is saved is equal to the ex-ante value of the option contract *c*, i.e., V^b_{niac}.

Therefore, we can conclude that the total cost of the fleet system decreases by using option contract *c*, compared to the case without any contract *c*, if the sum of the ex-ante values of V^b_{niac} for all the vehicles for any contract *c*, has a positive value.

Next, we consider the situation where one of the option contracts is chosen instead of the other one. The general conclusion is that by selecting the appropriate values of the parameters, we have the flexibility to choose from different contract options. This is a very important issue which gives managers the flexibility to choose the contract which matches the condition of their fleet management system.

Let V^g_{niac} denote, for the ex-post value of option contract *c*, technology *i*, at age *a*, at node *n*, and τ_c represent the probability that contract *c* is used. Then, for ex-post value of option contracts, we use (26). In equation (26), P_{niac} is the premium of option contract *c*, which can be obtained with equation (11).

$$V^g_{niac} = (V^b_{niac} - P_{niac})^+ \tau_c \quad \forall n \in N, i \in I, a \in A, c \in C \tag{26}$$

Proposition 3: *If* $0 < \gamma_{ia3} < \gamma_{ia2} < \gamma_{ia1}$, $1 \le k, j \le c$ *, then the contract with ex-post value of* V^g_{niaj} *is used instead of the contract with ex-post value of* V^g_{niak} *when*

$$\frac{[\bar{\lambda}_{(A-a)ij} + \Delta\mu_{ij} - \theta_j \mu_i - P_{niaj}]^+}{[\bar{\lambda}_{(A-a)ik} + \Delta\mu_{ik} - \theta_k \mu_i)]^+} > \frac{\tau_k}{\tau_j} \quad \forall n \in N, i \in I, a \in A, c \in C$$

We now explain the effect of using contracts on the risk (RECVaR) of the fleet management system. The next proposition proves that, by using option contracts, the value of RECVaR decreases.

Proposition 4: *Let* ϕ^β_n *and* $\phi^\beta_{n(0)}$ *represent the value of RECVaR with and without contracts, respectively. Then we have,* $\phi^\beta_n - \phi^\beta_{n(0)} \le 0 \quad \forall\, n \in N$

So far, we have considered the effect, on the expected cost and on the RECVaR, of using the option contracts. Now we want to answer the question: what is the change of value per year for the swap option of leasing EVs instead of fossil vehicles, taking into consideration the technological need to change the batteries of EVs, which is expected to occur in the future? This is also an interesting issue which can be a good justification for considering EVs in the fleet replacement decisions for managers by using the swap option. Before answering this question, we provide a proposition whereby we can obtain the time that it takes EVs to be more efficient than other technologies, and then we present another proposition to study the idea that we have mentioned.

Now, we are ready to provide a proposition for calculating the time during our planning horizon in which EVs are more cost efficient than other technologies. Let μ_{0e} stand for the annual fixed cost of the EVs, at time zero, μ_{te} denote the annual fixed cost of EVs in year *t*, and $g(\mu^*_{te}, t)$ represent the density function *of the First Passage Time* (*FTP*) for the stochastic process μ_{te}. The *FTP* is the expected time when μ_{te} crosses a threshold (Withmore, 1986).

Proposition 5: *Let i stand for fossil fuel or hybrid vehicles. The expected number of years for EVs to be more cost efficient than technology i is:*

$$\int_0^{\infty} tg\left[(l_i + D_t r_{ti}/100)\text{-}D_t r_{te}/100,\ t\right]dt$$

Next, we want to analyze the change of the value of the option to swap fossil fuel or hybrid vehicles in favour of leasing EVs per year, taking into consideration the technological issue of having to change EV batteries. The reason for doing this is that we can swap a fossil fuel vehicle for EVs and we can reduce the expected cost.

Proposition 6: *Let* $\Delta\mu_{tie}$ *denote the difference of change of annual fixed cost of EVs and technology i, in year t, and T represents the length of the planning horizon. The value of the option to swap a fossil fuel or hybrid vehicle for an EV increases*, *per year*, *by* $\Delta V_{tia3} = T\Delta\mu_{tie} \quad \forall a \in A$.

5. Implementation and Computational Experiments

This article is motivated by the problem faced by a large firm in the UK that aims to reduce, simultaneously, the costs and risks associated with its vehicle fleet. The firm leases a large fleet of vehicles used by its engineers. Generally, an engineer is assigned a vehicle for the whole lifetime of the lease. Vehicles are assigned depending on the engineer's specialization. Currently, their fleet comprises only diesel vehicles with different capacities (small, light, medium size vehicles). The size of the vehicle is an important feature because, depending on their specialization, the engineers have to carry different materials, and therefore require a vehicle with sufficient capacity. For instance, power engineers need light vans with enough carrying capacity. We consider small vans to be those weighing 300 kg; light vans weigh 500 kg and have a greater carrying capacity. Medium vans can be used for any purpose, but there are only a few of these because they are far more expensive to lease and maintain.

We use the historical data for fuel prices from Jan. 2000 to Dec. 2015. As can be seen from Table 2, when we consider the average correlation coefficient in each year between fuel prices from Jan. 2000 to Dec. 2015, the diesel and petrol prices have a very high correlation coefficient of 0.876 but a negative correlation of (-0.126) and (-0.162) with electricity, respectively. (In this article, we use electricity and electricity charge prices interchangeably. However, the price of electricity is the price of each charge for a 22 kWh battery). Based on this observation, we can generate the simulated scenarios for fuel prices with the above correlation matrix using the expected forecasted prices from 2016 to the end of 2023 (Table 3).

Table 2. Correlation matrix for fuel prices from Jan. 2000 to Dec. 2015

	Petrol	Diesel	Electricity
petrol	1	0.876	-0.126
Diesel	0.876	1	-0.162
Electricity	-0.126	0.162	1

At each stage (year), we want to have the minimum number of vehicles in the fleet system, taking into account that the contracts for the vehicles will be retired by the end of the fourth year. Moreover, we assume an initial condition where there is a 2016 fleet system consisting of 2369 diesel vehicles with different capacities (small, light, medium), all of which are at age one, i.e., new leased vehicles. It is assumed that all vehicles are vans. There are options for returning or swapping the vehicles that are at age two, three, and four. Indeed, the vehicles that are at age one or new leased vehicles cannot be returned or swapped. We also consider four technologies: petrol, hybrid-petrol, hybrid-diesel, and EVs. We assume that other technologies have different capacities (small, light, and medium), such as diesel vans. The leasing costs for each capacity and technology are summarised in Table 4.

Table 3: Average forecasted fuel prices (£) from 2016 to 2023

	2016	2017	2018	2019	2020	2021	2022	2023
Petrol	1.338	1.386	1.435	1.483	1.532	1.58	1.628	1.677
Diesel	1.387	1.439	1.49	1.542	1.593	1.645	1.696	1.748
Electricity	2.476	2.581	2.685	2.789	2.894	2.998	3.102	3.206

Table 4: The annual leasing costs (£) for vehicles with different capacities and technologies

Technology	Small (£)	Light(£)	Medium(£)
Petrol	2508	2736	3192
Diesel	2640	2880	3360
Hybrid-Petrol	3588	3912	4572
Hybrid-Diesel	3828	4176	4872
EVs	4572	4980	5808

For the petrol and hybrid technologies, we use their expected values of fuel consumption as a base for diesel technology for different type of vehicles as represented in Table 5. By multiplying the fuel consumption coefficient vector, [1.31, 1, 0.93, 0.8], we obtain the corresponding fuel consumption of petrol, diesel, petrol-hybrid, and diesel-hybrid for each capacity, respectively. The benchmark is indicated by the number 1 and corresponds to diesel technology. For EVs, we take into account the forecasted electricity prices shown in Table 3 for small capacity as a base, and by multiplying the ratio of power of batteries, [1, 1.18, 1.77], for small, light and medium, respectively, by which we can obtain the electricity prices for each type of EV. The benchmark is the first component of the vector and is shown by 1. Furthermore, for the monthly lease cost of batteries for EVs, we assume that the base price for small capacity (22 kWh) is £79, and for obtaining the monthly cost of batteries for other capacities, we multiply by the same ratio vector: [1, 1.18, 1.77]. The CO_2 emissions for petrol, diesel, hybrid-petrol and hybrid-diesel are 2310, 2680, 1719, and 2177 (g/litre), respectively. Moreover, the CO_2 emissions for EVs are 81 g/mile and for obtaining CO_2 emissions for other capacities of vehicles we use the same ratio vectors as was explained before.

Given these constraints, we want to minimize the weighted average of total expected costs and RECVaR during the planning horizon. Our goal is to determine the optimal policy from 2017 to 2020. Note that in order to calculate this policy, we need to continue the calculation for the stochastic variables from 2021 to the end of 2023 (until the end of life of the vehicles leased during the period of analysis).

Table 5: Fuel consumption and mileage data per vehicle

Types of vehicles	Annual Mileage		Fuel Consumption (litres/100 miles)		Vehicles	
	Mean	S.D.	Mean	S.D.	Number	%
Small vans	12140.9	5646.4	8.2	1.1	1137.0	48.0
Light vans	13212.0	5734.4	9.7	1.2	1077.0	45.5
Medium vans	13106.9	5093.0	14.5	2.5	155.0	6.5

5.1. Solving the Fleet Replacement Problem

The model presented in Section 3 has been implemented and solved using CPLEX running on a laptop with a 4.2 GHz processor and 30 GB RAM. For the means and standard deviations of corresponding distributions, we have used the information in Table 5. For simulating the CO_2 prices, we have assumed a uniform distribution between £5×10^{-6}/g and £20×10^{-6}/g. The number of vehicles at each node for each type of vehicle has a normal distribution with the mean equal to the number of vehicles in each type (Table 5).

We used the method of Høyland and Wallace (2001) to generate scenarios which is the general method of scenario generation by matching statistical properties to specific targets. The method generates a set of discrete scenarios so that statistical properties of the random variables match specified target values. We have used Tables 2 and 3 for fuel prices and Table 5 for mileage and fuel consumption.

We have solved the problem for all vehicle types but we have presented only the results for light vans. Moreover, for value of ω, we assumed ω equals to 0.5 in which the weights for expected cost and RECVaR in the objective function are equal.

We considered three values of ρ equal to 0, 0.1, and 0.25 for relation between value and premium of an option. Moreover, the coefficient vector for penalties of different contracts, θ_c, is equal to 1, 0.5, and 0.1 for contracts with no option (c = 1), return option (c = 2), and swap option (c = 3), respectively. We denote different technologies by D, P, H-P, H-D, and E for Diesel, Petrol, Hybrid-Petrol, Hybrid-Diesel, and Electric, respectively. Finally, the results are converged by using 10165 scenarios. The results are shown in Table 6.

As seen from Table 6, when ρ equals zero, i.e., the options are free, we have different combinations of contracts for the leasing of vehicles. In addition, diesel technology has the biggest portion of leased vehicles in different contracts. For example, in 2018 we have 932 diesel vehicles at age 2, which are swapped for other contracts. However, we do not have any returned vehicles with base contracts. Now we change the value of ρ to 0.1, i.e., we should pay for return and swap contracts which is 10% of their value, calculated in equations 8-9. As seen, the vehicles are leased with base contracts and contracts with the swap option. In addition, we have returned vehicles with base contracts and contracts with the swap option. However, there are no returned vehicles with the option to return. An interesting observation regarding the returned vehicles with base contracts is the fact that there is a trade-off between the penalty and the price of an option. Finally, when ρ is 0.25 or more, we have just leased and returned vehicles with base contracts. In addition, there is no vehicle swap. We have provided a formal proof for these results in Proposition 3.

Moreover, as seen from Figure 2, the values of RECVaR and expected cost when ρ is equal to zero, are less than the case when ρ is not equal to zero. The reason is that using contracts with options decreases the whole RECVaR and the expected cost. In other words, when more contracts are used in the fleet system, the total cost and the RECVaR are minimized. Indeed, the lower the price of the options, the lower is the RECVaR and the expected cost. For example, when ρ decreases from 0.25 to 0, we have £3.56M (12%) decrease in RECVaR and £2.99M (11%) in expected cost. We have proved these results in Propositions 2 and 4 regarding the effect of option contracts on the expected cost and on RECVaR, respectively.

Table 6: Optimal policy for light vans, ω=0.5, 10165 scenarios, θ_c= (1, 0.5, 0.1), and ρ=0, 0.1, and 0.25

				Number of Vehicles For Each Contract				
				Leased			Returned	Swapped
	Year	Tech.	age	C1	C2	C3	C2	
ρ=0	2017	D	1		34	1043		
	2018	D	1	210	212	551		
			2				6	932
	2019	P	1	1	1			
		D	1	38	39	31		
			2				4	7
			3				11	90
	2020	P	1	1				
		D	1	31				
			2				2	2
			3				4	2
			4				8	12
				Leased		Returned	Swapped	
				C1	C3	C1		
ρ=0.1	2017	D	1	84	993			
	2018	D	1	1028	3			
			2			7	990	
	2019	P	1	1				
		D	1	20				
			2			3	1	
			3			17	3	
	2020	P	1	1				
			3			2		
			4			18		
		D	1	20				
				Leased	Returned			
				C1	C1			
ρ=0.25	2017	D	1	1077				
	2018	D	1	41				
			2		7			
	2019	P	1	5				
		D	1	20				
			3		29			
	2020	P	1	3				
		D	1	50				
			4		54			

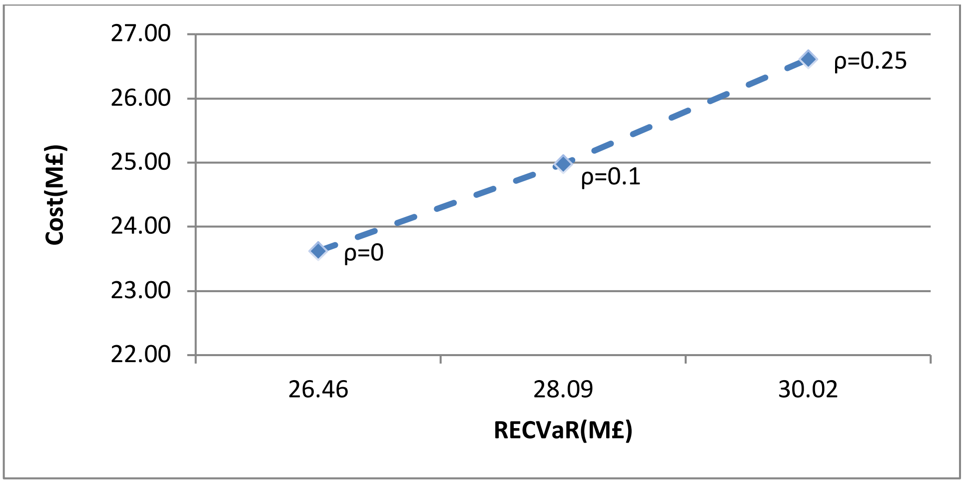

Figure 2: The values of RECVaR and expected cost for Light Vans, ω=0.5, 10165 scenarios, θ_c= (1, 0.5, 0.1)

5.2 Modelling the Effect of Technology Development

In this section, we consider the progress that is expected in the next decade regarding the technology of batteries and the EVs market, and its subsequent effect on the suggested portfolio system of the company.

Dinger et al. (2010) mention that the current cost of an automotive lithium-ion battery pack is between \$1000 and \$1200 per kWh. They further predict that this price will decrease to between \$250 and \$500 per kWh at scaled production. Hence, for the 15kWh battery (Figure 3), the price is expected to drop from \$16000 to \$6000.

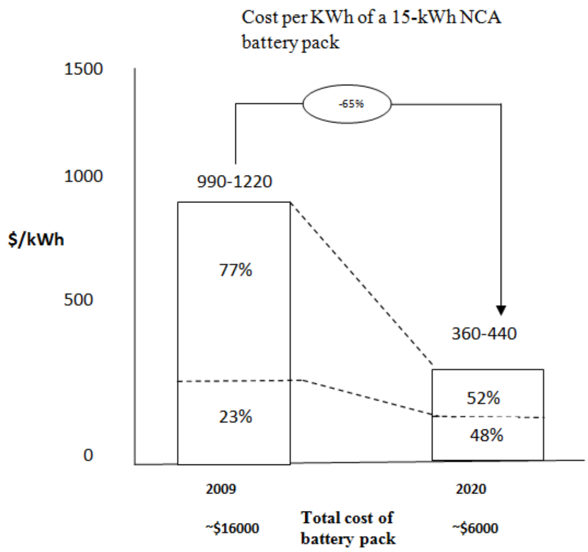

Figure 3: BCG outlook for battery costs from 2009 to 2020, Dinger et al. (2010)

As mentioned in Proposition 5, we can model the effect of technology development of EVs using a *FTP* process. So, we can use the Ornstein–Unhlenbeck process (e.g., Kleindorfer et al., 2012) as the *FTP* process to generate the scenarios at each node for lease costs for EVs with appropriate parameters. We have used this stochastic process because, in the long run, the battery costs and leasing costs for EVs will gradually decrease and stabilise, as is the case with the other technologies. The Ornstein-Unhlenbeck process equation is represented by (41).

$$\mu_{ne} = \bar{\mu}_e(1-\exp(-\delta)) + \mu_{1e}\exp(-\delta) + \sigma\sqrt{\frac{1-e^{-2\delta}}{2\delta}}.z \tag{41}$$

In (41) μ_{ne} is the total annual fixed (investment) cost of EVs at each parent node *n*, μ_{1e} is the total annual fixed cost of EVs per vehicle at root node, and $\bar{\mu}_e$ is the average total annual fixed cost of EVs per vehicle in the long run. According to Dinger et al. (2010), we assume $\bar{\mu}_e$ to be 40% of the current total annual fixed cost of EVs. In addition, δ is the annual learning rate, *z* is the *quantile* for standard normal distribution and, finally, σ is the standard deviation of μ_{ne}.

The results are represented in Table 7. The learning rate, δ, is equal to 1.2 (Dinger et al., 2010). For other parameters, we have taken into account the previous assumptions presented in Section 5. As seen from Table 7, the preferred leased vehicles are those with EV technology. In some cases, diesel is chosen. But there are no optimal choices for other types of leased vehicles. Next, we consider each case in more detail and examine the corresponding optimal policies regarding the contracts.

When ρ equals zero, i.e., the options are free, we have different combinations of contracts for the leasing of vehicles. For instance, in 2018 we have 808 EVs at age 2, which are swapped. However, we do not have any returned vehicles under the base contract. Next, we change the value of ρ to 0.1. As seen, the vehicles are leased under the base contract, and contracts with the swap option, except in 2017 when 32 diesel vehicles are leased under the return option. In addition, we have vehicles returned under the base contract, and contracts with the swap option. However, there are no returned vehicles under the return option contract. Finally, when ρ is 0.25 or more, we have just leased and returned vehicles under the base contract. In addition, there is no vehicle swap.

Like the previous section, the values for the RECVaR and expected cost are reduced when ρ is decreased for three values of annual learning rate of technological change for EVs (Figure 4). For example, if δ is equal to 1.2, when ρ reduces from 0.25 to 0, we have a decrease of £2.11M (8%) in RECVaR and £1.63M (7%) in expected cost. We have proved this result in Propositions 2 and 4. In addition, as seen in Figure 4, by increasing the learning rate from 1.2 to 2, because we have more EVs in the fleet system and they are more expensive than diesel, the slope of increase in expected cost, when ρ increases from 0 to 0.25, is decreased.

If we also compare the RECVaR and expected cost for each corresponding ρ in Figures 2 and 4, we conclude that we have a reduction in values of RECVaR and expected cost for all values ρ, in Figure 4. For example, when ρ equals 0, and δ is equal to 1.2, we have £3.03M (11%) reduction in RECVaR and £0.83M (4%) in expected cost. We have found that the greater the number of EVs in the fleet system, the lower is the RECVaR. So, the reduction in RECVaR is higher than the expected cost. This is also true for other values ρ and δ.

Table 7: Optimal policy for light vans with technological development of EVs, ω=0.5, 10165 scenarios, θ_c= (1, 0.5, 0.1), δ=1.2, and ρ=0, 0.1, and 0.25

				Number of Vehicles For Each Contract					
				Leased			Returned	Swapped	
	Year	Tech.	age	C1	C2	C3	C2		
ρ=0	2017	D	1		46				
		E	1		5	1026			
	2018	D	2				8		
		E	1	110	270	471			
			2					808	
	2019	D	3				24		
		E	1	47	51	100			
			2					7	
			3					171	
	2020	D	4				9		
		E	1	54	2	1			
			2				1	5	
			3				2	3	
			4				2	33	
				Leased			Returned		Swapped
				C1	C2	C3	C1	C2	
ρ=0.1	2017	D	1	15	32				
		E	1	105		925			
	2018		1						
		D	2				2	21	
		E	1	948					
			2						890
	2019	D	3				8	10	
			1	59					
		E	3				7		33
	2020	D	4				3	1	
		E	1	21					
			4				19		1
				Leased	Returned				
				C1	C1				
ρ=0.25	2017	D	1	45					
		E	1	1032					
	2018		1						
		D	2		17				
		E	1	51					
	2019	D	3		20				
		E	1	26					
			3		6				
	2020	D	4		5				
		E	1	19					
			4		19				

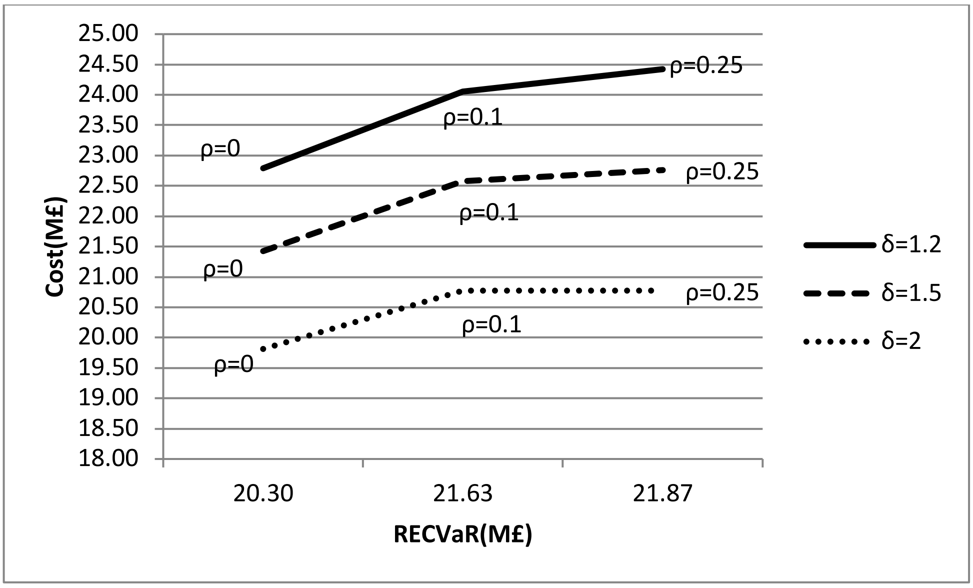

Figure 4: The values of RECVaR and expected cost for light vans with technological development of EVs, ω=0.5, 10165 scenarios, θ_c= (1,0.5,0.1), δ=1.2,1.5, and 2

6. Conclusions

In this article, we have developed a new model in a dynamic setting in order to give vehicle leasing contracts some flexibility that takes into consideration several uncertainties including CO_2 prices, fuel prices, mileage covered and fuel consumption, using the real options methodology and RECVaR. Our approach is different from that of Kleindorfer et al. (2012), in terms of the model's stochastic parameters, the use of RECVaR, and the different options available for the leasing contracts.

The main findings of this study are:

1. When the coefficient for the relationship between value and price of an option (ρ), equals zero, i.e., the options are free, we have different combinations of contracts for the leasing of vehicles.
2. When we change the value of the coefficient for the relationship between value and price of an option (ρ) to 0.1, i.e., we should pay for return and swap contracts, which is 10% of their value, the results are changed. The vehicles are leased on the terms of the base contract and the contract with the swap option. In addition, we have returned vehicles with the base contract and the contract with the swap option. However, there are no returned vehicles with the option to return. Finally, when ρ is 0.25 or more, we have just the leased and returned vehicles with the base contract. In addition, there is no vehicle swap.
3. When considering the technological change of EVs, the optimal technologies are different in the two cases. Indeed, when the effects of technology are taken into account, EVs are the preferred technology for leasing. However, in the absence of EV technology, the preferred

technology is diesel. Petrol technology is selected in a few cases when we assume that there is no EV option. Finally, hybrid technology is never selected as the optimal choice.

4. The values of RECVaR and expected cost, when the coefficient for the relation between value and price of an option (ρ) equals zero are less than when ρ is not equal to zero. The reason is that using contracts with options decreases the whole RECVaR and expected cost. In other words, when all contracts are used in the fleet system, the total cost and the RECVaR are minimized. Indeed, the lower the price of the options, the lower is the RECVaR and the expected cost.

Appendix A

Proposition 1: *If* $0<\gamma_{ia3}<\gamma_{ia2}<\gamma_{ia1}$ *then the change of the expected cost at parent node n,* ΔQ_{niac}*, for contract c , at age a, compared to the case without any contract c is :*

$$\Delta Q_{niac}=\Delta L_{nic}+E(\Delta L_{mic})+EE(\Delta L_{m^2ic})+...EE....E(\Delta L_{m^{A-1}ic})+EE....E(\Delta L_{m^Aic})$$

Proof: In order to calculate the values of ΔQ_{niac}, based on equation (21), we can obtain (A.1). Because (A.1) is a recursive equation, we can replace the value of $E(\Delta Q_{miac})$ by using equation (21) for child node *m*, and if we continue it until the maximum age of the contract, *A*, we can derive (A.2).

$$\Delta Q_{niac}=\Delta L_{nic}+\frac{1}{S_t}\sum_{\Psi(n,m)}(\Delta Q_{miac})=\Delta L_{nic}+E(\Delta Q_{miac})$$

(A.1)

$$\Delta Q_{niac}=\Delta L_{nic}+E(\Delta L_{mic})+EE(\Delta L_{m^2c})+...+EE...E(\Delta L_{m^Aic}) \blacksquare \tag{A.2}$$

We proceed to the main proposition related to the effect of using option contracts for the expected cost of the fleet.

Let V^b_{niac} denote the ex-ante value of the option contract *c*, for technology *i,* at age *a*, and at node *n* and $\bar{\lambda}_{(A-a)ic}$ denote the average running costs for the remaining periods at age *a*, for technology *i,* and option contract *c*.

Proposition 2: *If* $0<\gamma_{ia3}<\gamma_{ia2}<\gamma_{ia1}$ *then*

$$V^b_{niac}=(\Delta Q_{niac}-\gamma_{niac})^+=(A-a)[\bar{\lambda}_{(A-a)ic}+\Delta\mu_i-\theta_c\mu_i)]^+ \quad \forall c,\ a\geq 2$$

Proof: In order to calculate the values of V^{b}_{niac} for each contract, we use equation (A.3) as we have benefited from the result of Proposition 1 in equation (A.2). In equation (A.3), ΔL_{nic}, ΔL_{mic},…, and $\Delta L_{m^{A}ic}$ in all terms have two components. The first one is $\Delta\mu_{ic}$ which denotes the change in the annual fixed cost per vehicle type *i*, for contract *c* compared to the case without any contract *c* , and the second component is $\Delta\lambda_{nic}$, $\Delta\lambda_{mic}$, $\Delta\lambda_{m^{2}ic}$,…, and $\Delta\lambda_{m^{A}ic}$, which represent the change of the annual running cost per vehicle, at node parent *n*, child node *m*, child node of child node *m* , and so on, for contract *c* compared to the case without contract *c*. Now, based on equation (A.3), we can derive (A.4) in which the first term, $\Delta\mu_{ic}+\Delta\lambda_{nic}$, is equal to zero because we do not return any vehicle at age 1, and the second term denotes the expected change of cost per vehicle which is incurred by having contract *c*, age 2, and so on.

Moreover, if we let $E(\Delta\lambda_{nic})=\bar{\lambda}_{(A-a)ic}$, we can derive (A.5) from which we obtain, V^{b}_{niac}, (A.6).

$$\begin{aligned}\Delta Q_{niac} &= \Delta L_{nic} + E(\Delta L_{mic}) + EE(\Delta L_{m^{2}c}) + \ldots EE\ldots.E(\Delta L_{m^{A-1}ic}) + EE\ldots E(\Delta L_{m^{A}ic}) \\ &= \Delta\mu_{ic} + \Delta\lambda_{nic} + E(\Delta\mu_{ic} + \Delta\lambda_{mic}) + EE(\Delta\mu_{ic} + \Delta\lambda_{m^{2}ic}) + EE\ldots.E(\Delta\mu_{ic} + \Delta\lambda_{m^{A}ic})\end{aligned} \quad \text{(A.3)}$$

$$\Delta Q_{niac} = E(\Delta\mu_{ic} + \Delta\lambda_{mic}) + EE(\Delta\mu_{ic} + \Delta\lambda_{m^{2}ic}) + EE\ldots.E(\Delta\mu_{ic} + \Delta\lambda_{m^{A}ic}), \forall a \geq 2$$

(A.4)

$$\Delta Q_{niac} = (A-a)[\bar{\lambda}_{(A-a)ic} + \Delta\mu_{ic}], \quad \forall a \geq 2 \quad \text{(A.5)}$$

$$V^{b}_{niac} = (\Delta Q_{niac} - \gamma_{iac})^{+} = (A-a)[\bar{\lambda}_{(A-a)ic} + \Delta\mu_{ic} - \theta_{c}\mu_{i})]^{+}, \ \forall a \geq 2 \quad \blacksquare \quad \text{(A.6)}$$

Proposition 3: *If* $0 < \gamma_{ia3} < \gamma_{ia2} < \gamma_{ia1}$, $1 \leq k, j \leq c$ *, then the contract with ex-post value of* V^{g}_{niaj} *is used instead of the contract with ex-post value of* V^{g}_{niak} *when*

$$\frac{[\bar{\lambda}_{(A-a)ij} + \Delta\mu_{ij} - \theta_{j}\mu_{i} - P_{niaj}]^{+}}{[\bar{\lambda}_{(A-a)ik} + \Delta\mu_{ik} - \theta_{k}\mu_{i})]^{+}} > \frac{\tau_{k}}{\tau_{j}}$$

Proof: The general condition where the contract with the option *j* is used instead of the contract with option *k* can be written as (A.7). Equation (A.7) explicitly indicates that if the contract with option *j* is used instead of the contract with option *k*, then its ex-post value should be higher. Then, by using equations (A.6) and (A.7), we can derive (A.8).

$$V^{g}_{niaj} > V^{g}_{niak} \Rightarrow (V^{b}_{niak} - P_{niak})^{+}\tau_{k} > (V^{b}_{niaj} - P_{niaj})^{+}\tau_{j} \quad \text{(A.7)}$$

$$\frac{[\bar{\lambda}_{(A-a)ij} + \Delta\mu_{ij} - \theta_j \mu_i - P_{niaj}]^+}{[\bar{\lambda}_{(A-a)ik} + \Delta\mu_{ik} - \theta_k \mu_i - P_{niak})]^+} > \frac{\tau_k}{\tau_j} \quad \blacksquare \tag{A.8}$$

Proposition 4: *Let ϕ_n^β and $\phi_{n(0)}^\beta$ represent the value of RECVaR with and without contracts, respectively. Then we have, $\phi_n^\beta - \phi_{n(0)}^\beta \le 0 \quad \forall\, n \in N, c \in C.$*

Proof: We proceed by using proof by contradiction. So, let's assume that (A.9) is true.

$$\phi_n^\beta - \phi_{n(0)}^\beta \ge 0, \;\; \forall\, n \in N, c \in C. \tag{A.9}$$

Then the optimal choice is to not use contracts, and we have a lower RECVaR without options, $\phi_{n(0)}^\beta$. On the other hand, if we have the option to use the contracts and $\phi_n^\beta < \phi_{n(0)}^\beta$ then we have a lower RECVaR, when we exercise the options, and this will equal, ϕ_n^β. Therefore, by having the option to exercise contracts, we obtain $\phi_n^\beta \le \phi_{n(0)}^\beta$. ■

Proposition 5: *Let i stand for fossil fuel or hybrid vehicles. The expected number of years for EVs to be more cost efficient than technology i is:*

$$\int_0^\infty tg\left[(l_i + D_t r_{ti}/100)\text{-}D_t r_{te}/100,\; t\right]dt$$

Proof: Because in the tree structure*,* in each stage *t*, we can map the set of nodes *n,* using $\Omega_{t,n}$, we can use *t* and *n* and in (A.10), interchangeably. Now, in order to calculate the first passage time, the total annual cost of EVs should equal that of other technologies, i.e.

$$\mu_{te}^* + \lambda_{te} = \mu_{ti} + \lambda_{ti} \quad \forall t, n \in \Omega_{t,n}, i = \text{fossil fuels, hybrids} \tag{A.10}$$

Then by using equations (3), (4), and (A.10), we derive,

$$\mu_{te}^* = \mu_{ti} + \lambda_{ti} - \lambda_{te} = l_i + D_t r_{ti}/100\text{-}D_t r_{te}/100 \quad \forall t, n \in \Omega_{t,n}$$

(A.11)

All the parameters in (A.11) are defined in Tables 1(a) and 1(b). Then, the expected first passage time equals:

$$\int_0^\infty tg(\mu_{te}^*, t)dt = \int_0^\infty tg\left[l_i + D_t r_{ti}/100\text{-}D_t r_{te}/100,\; t\right]dt \quad \blacksquare \tag{A.12}$$

Proposition 6: *Let* $\Delta\mu_{tie}$ *denote the difference of change in the annual fixed cost of EVs and technology i, in year t, and T represents the length of the planning horizon. The value of the option to swap a fossil fuel or hybrid vehicle for EVs increases, per year, by* $\Delta V_{tia3} = T\Delta\mu_{tie}$ **.**

Proof: The value of option at time zero to swap fossil fuel or hybrid vehicles for EVs by using equations (9) and (10) is:

$$V_{nia3}^{t_0} = (Q_{nia} - \min_{i\neq j}(Q_{nij}) - \gamma_{nia3})^{+}$$
$$= (\mu_i + \lambda_{ni} + \frac{1}{S_t}\sum_{\psi(n,m)} Q_{mi(a+1)} - \mu_e - \lambda_{ne} - \frac{1}{S_t}\sum_{\psi(n,m)} Q_{me(a+1)} - \gamma_{nia3})^{+} \quad (A.13)$$

Because in the tree structure, in each stage *t*, we can map the set of nodes *n,* using $\Omega_{t,n}$, we can use *t* and *n* and in (A.14), interchangeably. In order to calculate the change in the value of swap option at time *t*, because the only change during the planning horizon, in (A.13), is the fixed cost of EVs, we can write:

$$\Delta V_{tia3} = V_{nia3}^{t_t} - V_{nia3}^{t_0} = \Delta\mu_{tie} + \frac{1}{S_t}\sum_{t=2}^{T}\Delta\mu_{tie} = \Delta\mu_{tie} + \frac{1}{S_t}(T-1)\Delta\mu_{tie}S_t = T\Delta\mu_{tie} \quad \blacksquare \quad (A.14)$$

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