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Modelling the Generalised Median Correspondence through an Edit Distance

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Abstract. On the one hand, classification applications modelled by structural pattern recognition, in which elements are represented as strings, trees or graphs, have been used for the last thirty years. In these models, structural distances are modelled as the correspondence (also called matching or labelling) between all the local elements (for instance nodes or edges) that generates the minimum sum of local distances. On the other hand, the generalised median is a well-known concept used to obtain a reliable prototype of data such as strings, graphs and data clusters. Recently, the structural distance and the generalised median has been put together to define a generalise median of matchings to solve some classification and learning applications. In this paper, we present an improvement in which the Correspondence edit distance is used instead of the classical Hamming distance. Experimental validation shows that the new approach obtains better results in reasonable runtime compared to other median calculation strategies.

Keywords: Generalised Median, Edit distance, Optimisation, Weighted Mean.

1 Introduction

A correspondence is defined as the result of a bijective function which designates a set of one-to-one mappings between elements representing the local information of two structures i.e. sets of points, strings, trees, graphs or data clusters. Each element (a point for sets of points; a character for strings, or a node and its edges for trees or graphs) has a set of attributes that contain specific information. Correspondences are usually generated, either manually or automatically, with the purpose of finding the similarity or a distance between two structures. In the case that correspondences are deduced through an automatic method, this is most commonly done through an optimisation process called matching. Several matching methods have been proposed for set of points [32], strings [25], trees and graphs [29].

Correspondences are used in various frameworks such as measuring the accuracy of different graph matching algorithms [4], [31], improving the quality of other correspondences [5], learning edit costs for matching algorithms [6], estimating the pose of a fleet of robots [7], performing classification [17] or calculating the consensus of a set of correspondences [21], [19], [18], [20]. While most of these methods use the classical Hamming distance (HD) to calculate the dissimilarity between a pair of correspondences, in [23] authors have shown that this distance does not always reflect the dissimilarity between a pair of correspondences, and thus, a new distance called Correspondence Edit Distance (CED) was defined.

The median of a set of structures is roughly defined as a sample that achieves the minimum sum of distances (SOD) to all members of such set. This concept has been largely considered as a suitable representative prototype of a set [13] because of its robustness. For the case of strings [3], graphs [2], and data clusters [11], computing the median is an NP-complete problem. Thus, some suboptimal methods have been presented to calculate an approximation to the median. For instance, an embedding approach has been presented for strings [14], graphs [8] and data clusters [10]. Likewise, a strategy known as the evolutionary method for strings [9] and correspondences [22] has proven to obtain fair approximations to the median in reasonable time. Moreover, [22] presented a minimisation method which obtains the median using optimisation functions based on the HD. This work proved that it is possible to obtain the exact median for a set of correspondences using this framework, provided that the distance considered between the correspondences is the HD. In this paper our work is devoted towards revisiting the median calculation frameworks presented in [22], this time using the CED.

The rest of the paper is structured as follows. Section 2 establishes the basic definitions. Afterwards, in Section 3 we present the method to calculate the generalised median based on the CED. Then, Section 4 provides an experimental validation of the method. Finally, Section 5 is reserved for the conclusions and further work.

2 Basic Definitions

2.1 Distance between structures

Consider a structure $G = (\Sigma, \mu)$, where $v_i \in \Sigma$ denotes the elements (i.e. local information) and μ is a function that assigns a set of attributes to each element. This structure may contain null elements which have a set of attributes that differentiate them from the rest. We refer onwards to these null elements of G as $\hat{\Sigma} \subseteq \Sigma$. Moreover, given $G = (\Sigma, \mu)$ and $G' = (\Sigma', \mu')$ of the same order n (naturally or due to the aforementioned null element presence), we define the set of all possible correspondences T, such that each correspondence in T maps all elements of G to elements of G', $f : \Sigma \to \Sigma'$ in a bijective manner.

For structures such as strings [30], trees [1] and graphs [26], [12], [28], one of the most widely used frameworks to calculate the distance is the edit distance. The edit distance is defined as the minimum amount of required operations that transform one object into the other. To this end, several distortions or edit operations, consisting of insertion, deletion and substitution of elements are defined. Edit cost functions are introduced to quantitatively evaluate the edit operations. The basic idea is to assign a penalty cost to each edit operation considering the amount of distortion that it introduces in the transformation. Substitutions simply indicate element-to-element mappings. Deletions are transformed to assignments of a non-null element of the first structure to a null element of the second structure. Insertions are transformed to assignments of a non-null element of the second structure to a null element of the first structure. Given G and G' and a correspondence f between them, the edit distance is obtained as follows:

$$EditCost(G,G',f) = \sum_{\substack{v_i \in \Sigma - \hat{\Sigma} \\ v'_j \in \Sigma' - \hat{\Sigma}'}} d(v_i, v'_j) + \sum_{\substack{v_i \in \Sigma - \hat{\Sigma} \\ v'_j \in \hat{\Sigma}'}} K + \sum_{\substack{v_i \in \hat{\Sigma} \\ v'_j \in \Sigma - \hat{\Sigma}}} K \tag{1}$$

where $f(v_i) = v'_j$ and function d is a distance function between the mapped elements. Moreover, K is a penalty cost for the insertion and deletion of elements. Thus, the edit distance ED is defined as the minimum cost under any bijection in T:

$$ED(G,G') = \min_{f \in T} EditCost(G,G',f) \tag{2}$$

2.2 Mean, Weighted Mean and Median

In its most general form, the mean of two structures G and G' is defined as a structure $\bar{G}$ such that:

$$Dist(G,\bar{G}) = Dist(\bar{G},G') \quad \text{and} \quad Dist(G,G') = Dist(G,\bar{G}) + Dist(\bar{G},G') \tag{3}$$

where $Dist$ is any distance metric defined on the domain of these structures. Moreover, the concept of weighted mean is used to gauge the importance or the contribution of the involved structures in the mean calculation. The weighted mean between two structures is defined as:

$$Dist(G,\bar{G}) = \lambda \quad \text{and} \quad Dist(G,G') = \lambda + Dist(\bar{G},G') \tag{4}$$

where λ is a constant that controls the contribution of the structures and holds $0 \leq \lambda \leq Dist(G,G')$. G and G' satisfy this condition, and therefore are also weighted means of themselves.

From the definition of the median, two different approaches are identified: the set median (SM) or the generalised median (GM). The first one is defined as the structure within the set which has the minimum SOD. Conversely, the GM is the structure out of any element in the set which obtains the minimum SOD.

2.3 Distance Between Correspondences

Given structures G and G' and two correspondences f^1 and f^2 between them, we proceed to define the HD and the CED.

Hamming Distance The HD is defined as:

$$HD(f^1, f^2) = \sum_{i=1}^{n} (1 - \delta(v'_a, v'_b)) \tag{5}$$

where a and b such that $f^1(v_i) = v'_a$ and $f^2(v_i) = v'_b$, and δ being the Kronecker Delta function:

$$\delta(x, y) = \begin{cases} 1 & \text{if } x = y \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Correspondence Edit Distance The CED is defined, in a similar way to Equations 1 and 2, as:

$$CED(f^1, f^2) = \min_{h \in H} Corr\_EditCost(f^1, f^2, h) \tag{7}$$

where

$$\begin{aligned} Corr\_EditCost(f^1, f^2, h) = \sum_{\substack{m_i^1 \in M^1 - \hat{M}^1 \\ m_k^2 \in M^2 - \hat{M}^2}} d(m_i^1, m_k^2) + \sum_{\substack{m_i^1 \in M^1 - \hat{M}^1 \\ m_k^2 \in \hat{M}^2}} K \\ + \sum_{\substack{m_i^1 \in \hat{M}^1 \\ m_k^2 \in M^2 - \hat{M}^2}} K \end{aligned} \tag{8}$$

where M^1 and M^2 are the sets of all possible mappings, $\hat{M}^1$ and $\hat{M}^2$ are the sets of null mappings.

The distance between mappings, $d(m_i^1, m_k^2)$ was defined using Equation 9 as:

$$d(m_i^1, m_k^2) = dn(v_i, v_k) + dn(f^1(v_i), f^2(v_k)) \tag{9}$$

where dn is a distance between the local parts of the structures, which is application dependent.

Notice that the elements used by CED are the mappings within f^1 and f^2. More formally, correspondences f^1 and f^2 are defined as sets of mappings $f^1 = m_1^1, ..., m_i^1, ..., m_n^1$ and $f^2 = m_1^2, ..., m_k^2, ..., m_n^2$, where $m_i^1 = (v_i, f^1(v_i))$ and $m_k^2 = (v_k, f^2(v_k))$.

2.4 Generalised Median Correspondence based on the Hamming Distance

In [22], authors presented a method to calculate the exact GM $\hat{f}$ of a set of correspondences based on the HD. Such method is based on converting a set of correspondences $f^1, ..., f^i, ..., f^m$ into correspondence matrices $F^1, ..., F^i, ..., F^m$.

Afterwards, a linear solver [16], [24], [15] is applied to the sum of these matrices as follows:

$$\hat{f} = argmin \sum_{i=1}^{n} (C \circ F^i[x,y]) \tag{10}$$

where $[x,y]$ is a specific cell and C is the following matrix:

$$C = \sum_{i=1}^{m} (\mathbf{1} - F^i[x,y]) \tag{11}$$

where

$$F^i[x,y] = \begin{cases} 1 & \text{if } f^i(v_x) = {v^i}_y \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

The idea is that by introducing a value of either 0 or a 1 in the correspondence matrix, the HD is being considered and thus minimised by the method.

3 Methodology

The aim of this paper is to model the GM of a set of correspondences through the CED. As commented in the introduction, it only has been modelled through the HD and we supposed that through the CED, much more interesting or useful median could be generated from an application point of view. Therefore, we only want to redefine matrix C in Equation 11 since the current one makes the median to be generated through the HD. Equation 13 shows our proposal:

$$C = \sum_{i=1}^{n} B^i[x,y] \tag{13}$$

where

$$B^i[x,y] = Dist(v_x, {f^i}^{-1}(v'_y)) + Dist(v'_y, f^i(v_x)) \tag{14}$$

Suppose that m is the mapping $m = \{v_x, v'_y\}$. Then, $B^i[x,y]$ is defined as the distance between this supposed mapping $f(v_x) = v'_y$ and the mappings imposed by correspondence f^i that relates elements v_x and v'_y. That is,

$$B^i[x,y] = d(m, m^i_x) + d(m, m^i_p) \tag{15}$$

As the distance between two mappings becomes higher, so does the value of $B^i[x,y]$. Likewise, the value of $(\mathbf{1} - F^i[x,y])$ in Equation 11 is higher for mappings that are not present in any correspondence of the set. As a result, matrix C in Equation 13 is a generalisation of matrix C in Equation 11.

Finally, considering Equations 9 and 15, we arrive to Equation 14. Figure 1 graphically shows the computation of $B^i[x,y]$:

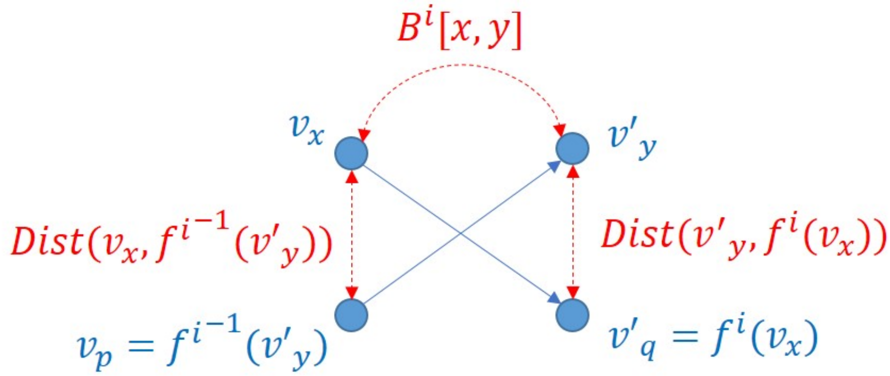

Fig. 1. $\circ \rightarrow$: Mappings in correspondences. $\rightarrow$: Computation of the distance.

Notice that the first part of the expression is similar to how the bijective function h is calculated in Equation 7, in the sense that it only computes the distance between mappings that have the same element on the output structure G. Moreover, notice that according to the $Dist$ measure used, null elements (and thus null mappings) are considered accordingly. Finally, matrix C is minimised in the same way as in Equation 10.

4 Validation

The experimental validation was carried out as follows. We have generated two repositories S^5 (with graphs/correspondences of a cardinality of 5 nodes/mappings) and S^{30} (with graphs/correspondences of a cardinality of 30 nodes/mappings), with the attributes of the nodes being real numbers, and edges being unattributed and conformed through the Delaunay triangulation. Each repository is integrated by 3 datasets consisting of 60 8-tuples $s_1 = \{G_1, G'_1, f^1_1, ..., f^6_1\}, .., s_i = \{G_i, G'_i, f^1_i, ..., f^6_i\}, ..., s_{60} = \{G_{60}, G'_{60}, f^1_{60}, ..., f^6_{60}\}$. All correspondences for each dataset are obtained through the following three correspondence generation scenarios:

- Completely at random: Six bijective correspondences are randomly generated for each tuple.
- Evenly distributed: From a "seed" bijective correspondence generated using [27], two mappings are swapped randomly and a new correspondence is created. This process is repeated six times for each tuple. The seed correspondence is not included in the tuple.
- Unevenly distributed: From a "seed" bijective correspondence generated using [27], pairs of mappings are swapped a random number of times and a new correspondence is created. This process is repeated six times for each tuple. Due to the randomness of the swaps, the seed correspondence may be included in the tuple.

The median was calculated for HD and CED by using the following methods:

1. SM as the correspondence in the set with the lowest SOD (A* method).
2. Evolutionary method for GM correspondence approximation presented in [22] (EVOL1).
3. Evolutionary method for GM correspondence approximation presented in [22] using a modified weighted mean search strategy (EVOL2).
4. Minimisation method (Min-GM). Method presented in [22] for HD and the method presented in this paper for CED.

Tables 1 to 3 shows the average SOD of the mean with respect to the set (SOD_{AVG}), the reduction percentage of SOD of methods 2, 3 and 4 with respect to 1 (RED) and the average runtime in seconds (RUN) for the three datasets in the two repositories. Notice that since the HD and the CED are distances which exist in different spaces, a comparison of SOD_{AVG} results between HD and CED methods is not viable. Moreover, RED scores are mostly meant to illustrate the improvement of each method with respect to the SM in its own distance space, since the increment of HD is linear while CED depends on the attributes of the graphs.

For the "Completely at random" datasets, Table 1 shows lower SOD_{AVG} values for Min-GM than for the rest of methods on both S^5 and S^{30}. Moreover, it can be observed that Min-GM achieves a 10% RED on the dataset in the S^{30} repository. However, this case is also the one that takes the most time to be computed. In contrast, although RED is not that considerable for Min-GM in the HD case, the runtime for this method is always comparable to the SM calculation. Finally, it can be noticed that EVOL1 never outperforms the SM, while EVOL2 does for the dataset in S^{30}. Both EVOL1 and EVOL2 have similar runtimes.

Table 1. Average SOD (SOD_{AVG}), reduction percentage of average SOD with respect to SM (RED) and runtime (RUN) using the "Completely at random" scenario.

		Completely at random					
		S^5			S^{30}		
		SOD_{AVG}	RED	RUN	SOD_{AVG}	RED	RUN
HD	SM	19	-	0.0009	141	-	0.01
	MIN-GM	18	6	0.002	137	3	0.008
	EVOL1	19	0	0.004	141	0	0.1
	EVOL2	19	0	0.009	139	1.5	0.2
CED	SM	62000	-	0.01	642000	-	4.4
	MIN-GM	60000	4	0.02	580000	10	9.3
	EVOL1	62000	0	0.014	642000	0	4.7
	EVOL2	62000	0	0.007	628000	3	4.8

In the "Evenly distributed" datasets shown in Table 2, the best SOD_{AVG} and RED results are obtained by Min-GM. In fact, this experiment proves that

Table 2. Average SOD (SOD_{AVG}), reduction percentage of average SOD with respect to SM (RED) and runtime (RUN) using the "Evenly distributed" scenario.

		Evenly distributed					
		S^5			S^{30}		
		SOD_{AVG}	RED	RUN	SOD_{AVG}	RED	RUN
HD	SM	13	-	0.006	19	-	0.01
	Min-GM	12	8	0.002	12	37	0.003
	EVOL1	13	0	0.003	15	22	0.004
	EVOL2	13	0	0.007	14	27	0.02
CED	SM	18400	-	0.02	63100	-	4.1
	Min-GM	18100	2	0.03	49300	22	9
	EVOL1	18400	0	0.003	63100	0	3.5
	EVOL2	18400	0	0.007	59000	7	3.5

Min-GM always obtains the exact GM, given that the median calculated for S^5 and S^{30} always has a SOD of 12 towards the correspondences in the set. This value results from multiplying the number of correspondences (six) times the mappings swapped from the seed correspondence (two), which is known in advance to be the GM. Given the attribute dependant nature of the CED, this rule is not visible for the SOD_{AVG} and thus RED scores of Min-GM using CED appear to be lower compared to Min-GM using HD.

Finally, Table 3 shows the results for the "Unevenly distributed" datasets, where although the GM may be included in the set, larger SOD_{AVG} values are obtained compared to the previous two scenarios. In this case, it is observed that RED is larger for Min-GM using CED than for HD. Nonetheless, the computation of Min-GM using CED for the S^{30} dataset conveys the largest runtime. Meanwhile, EVOL1 and EVOL2 maintain a similar trend to the previous two scenarios.

Table 3. Average SOD (SOD_{AVG}), reduction percentage of average SOD with respect to SM (RED) and runtime (RUN) using the "Unevenly distributed" scenario.

		Unevenly distributed					
		S^5			S^{30}		
		SOD_{AVG}	RED	RUN	SOD_{AVG}	RED	RUN
HD	SM	17	-	0.006	66	-	0.001
	MIN-GM	16	6	0.002	53	20	0.003
	EVOL1	17	0	0.003	65	22	0.006
	EVOL2	17	0	0.007	64	27	0.02
CED	SM	76500	-	0.005	839000	-	4.9
	MIN-GM	69100	10	0.002	669000	21	9.9
	EVOL1	76500	0	0.006	839000	0	5.3
	EVOL2	765000	0	0.01	779000	8	5.3

The following conclusions can be drawn from these experiments. If the correspondences have a low number of mappings or high precision is required, then Min-GM with CED is the best option. In contrast, HD has a better accuracy to runtime trade-off for correspondences with a high mapping order. It is also interesting to notice that the evolutionary methods, regardless of the weighted mean strategy, only outperformed the SM approach on the S^{30} repository, since the low amount of mappings in S^5 did not allow an effective weighted mean computation.

5 Conclusions and Further Work

In this paper, we presented a method for computing the GM correspondence based on an edit distance for correspondences called CED, which is a generalisation of a method based on the HD. Experimental validation shows that this approach is the best option to find the exact GM in three different correspondence scenarios, considering that by using the CED, a better represented GM is obtained at the cost of a larger computational complexity, especially as the number of mappings in correspondences increases. As future work, we are interested in comparing our method with more options for the GM calculation, putting particular emphasis in embedding approaches. It is also necessary to perform more experiments on real life repositories which contain structures and correspondences.

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